AN APPLICATION OF THE INTUITIONISTIC FUZZY PROMETHEE METHOD IN THE SELECTION OF TRANSPORTATION SERVICE

by

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Many of the problems we encounter involve uncertainties. Intuitionistic fuzzy logic works well in solving problems with uncertainties. Transportation problems, which are needed find the most appropriate and best, also contain uncertainty. A solution is found by using one of the decision making methods by determining the weight values of the criteria. The PROMETHEE method has an important place among the multi-criteria decision-making methods as it allows the decision maker to observe both positive and negative rankings. We may solve problems with uncertainties through using intuitionistic fuzzy logic. In this study, a new and different solution method is presented to find the most suitable transportation service provider by using the intuitionistic fuzzy PROMETHEE method. Criterion weight values were determined by controlled sets. Decision makers may apply this method to their problems by changing the weights we use depending on the importance of their criteria.

Key words: *fuzzy sets, intuitionistic fuzzy theory, multi criteria decision making, PROMETHEE method, intuitionistic fuzzy PROMETHEE*

Introduction

Fuzzy logic was defined by Zadeh [1]. Intuitionistic fuzzy logic was defined by Atanassov [2]. In the present paper, intuitionistic fuzzy expressions will be denoted by IF for convenience. Multi-criteria decision-making (MCDM) processes are a challenging processes for decision-makers. Because many factors come into play while making the decision. Decision-makers want to make decisions by considering all factors at the same time. Therefore, MCDM methods make it easier for decision-makers in many ways [3]. The PROMETHEE method, one of the multi-criteria decision making methods, was developed by Brans [4]. The PROMETHEE method is the focus of attention of most researchers because of its advantages such as giving the chance to observe positive and negative rankings at the same time and assigning specific importance to each criterion. This method varies across other decision methods since it evaluates each alternative within itself. Many researchers have developed applications through using PROMETHEE methods and multi criteria decision making [5-9]. Intuitionistic fuzzy sets and fuzzy sets provide ease of application in numerous areas such as education, medicine, engineering, supplier selection, personnel selection, agriculture, computer algebraic structures, topologic spaces [10-13]. Transportation service provider is one of these

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areas. Wu *et al.* [14] used the TOPSIS method to find the most suitable transportation service provider and also used the entropy method to determine the criterion weights. Many factors of the transportation involve uncertainty. For instance, the cost of transportation is affected by increases in needs, fuel and products. There is no certainty in such cases as adverse weather conditions, road, traffic conditions will affect the cost and delivery time. In this paper, the same case example was applied through the intuitionistic fuzzy PROMETHEE method. Therefore, the use of intuitionistic fuzzy logic in problems with uncertainty gives more satisfactory and more accurate results since there are situations involving uncertainty in this problem. This is a new and different study to choose the most suitable transportation service provider in terms of the method used for determining the criterion weights. Transportation services are an application area that has attracted the attention of many researchers [15, 16]. The quality of transportation services plays a significant role in people's lives. Attention should be paid to many features such as the cost of transportation services and customer expectations. It is a vital issue as the life of all people depends on transportation.

Preliminaries

Definition 1. [2] Let X = Ø. An intuitionistic fuzzy set A in X:

$$A = \{\langle x, \mu_{\mathrm{A}}(x), \nu_{\mathrm{A}}(x)\rangle \mid x \in X\} \tag{1}$$

$$\mu_{\mathrm{A}}(x), \nu_{\mathrm{A}}(x), \pi_A(x) : X \to [0,1] \tag{2}$$

$$\mu_{\mathrm{A}}(x), \nu_{\mathrm{A}}(x), \pi_A(x) = 1 \tag{3}$$

Intuitionistic fuzzy value (IFV) defined by Xu [17] and is: $\tilde{a} = (\mu_{\tilde{a}}, \nu_{\tilde{a}}, \pi_{\tilde{a}})$, where $\mu_{\tilde{a}}, \nu_{\tilde{a}}, \pi_{\tilde{a}} \in [0,1]$.

For IFV $\tilde{a} = (\mu_{\tilde{a}}, \nu_{\tilde{a}})$ and $\tilde{b} = (\mu_{\tilde{b}}, \nu_{\tilde{b}})$ the following operations have been carried out [17, 18]:

$$(\tilde{a} \oplus \tilde{b} = (\mu_{\tilde{a}} + \mu_{\tilde{b}} - \mu_{\tilde{a}}\mu_{\tilde{b}}, \nu_{\tilde{a}}\nu_{\tilde{b}}) \tag{4}$$

$$\tilde{a} \otimes \tilde{b} = (\mu_{\tilde{a}}\mu_{\tilde{b}}, \nu_{\tilde{a}} + \nu_{\tilde{b}} - \nu_{\tilde{a}}\nu_{\tilde{b}}) \tag{5}$$

$$\oplus_{j=1}^{m} \tilde{a}_j = (1 - \prod_{j=1}^{m}(1 - \mu_j), \prod_{j=1}^{m}\nu_j) \tag{6}$$

$$\otimes_{j=1}^{m} \tilde{a}_j = [\prod_{j=1}^{m}\mu_j, \prod_{j=1}^{m}(1 - \nu_j)] \tag{7}$$

Many researchers have suggested approaches for comparing the IFV [18, 19]. The following method was used in this paper for to rank IFV [19]:

$$\rho(\alpha) = 0.5(1 + \pi_\alpha)(1 - \mu_\alpha) \tag{8}$$

As the $\rho(\alpha)$ value decreases, the preferred value α increases.

The intuitionistic fuzzy PROMETHEE

The concept of weight represents the importance of that criteria. The weights are expressed as IFV in the intuitionistic fuzzy PROMETHEE. It is of great importance for decision-makers to determine the specific importance level for each criterion. In this paper, controlled sets were used. Some methods may help decision makers in determining intuitionistic fuzzy weights [20-23]. The basic definitions for controlled sets are as follows.

Definition 2. [21] Let E be an $\alpha-$ set. The following mapping on E was defined so that:

$$\alpha^*(x) = \begin{cases} 1-\alpha(x), & x \in E_\alpha \\ \sup_y \alpha(y), & y \in E \ni \alpha(x) < 1-\alpha(y) \\ 0, & \text{otherwise} \end{cases} \tag{9}$$

where $E_\alpha = \cup_{a \in E} \bar{a}$.

Definition 3. [21] Let E be $\alpha-$ set. Then the set $A = \{\langle x, \alpha(x), \alpha^*(x)\rangle \mid x \in E\}$ is called $(\alpha, \alpha^*)-$ controlled set.

The V-shape with indifference criterion type was utilized [24]:

$$P(d) = \begin{cases} 0, & d \le q \\ \frac{d-q}{p-q}, & q < d \le p \\ 1, & d > p \end{cases} \tag{10}$$

The deviations between $x_i (i = 1, 2, ..., n)$ and $c_j (j = 1, 2, ..., m)$ were determined:

$$d_j(x, y) = c_j(x) - c_j(y) \tag{11}$$

Definition 4. [25] An intuitionistic fuzzy preference relation R on the set $X = x_1, x_2, ..., x_n$ is represented by a matrix $R = (r_{ik})_{nn}$, where $r_{ik} = \langle (x_i, x_k), \mu(x_i, x_k), \nu(x_i, x_k)\rangle$ for all $i, k = 1, 2, ..., n$ with the condition:

$$\mu_{ik}, \nu_{ik} \in [0,1], \quad \mu_{ik} + \nu_{ik} \le 1, \quad \mu_{ik} = \nu_{ki}, \quad \mu_{ki} = \nu_{ik} \tag{12}$$

$$\mu_{ii} = \nu_{ii} = 0.5, \quad \pi_{ik} = 1 - \mu_{ik} - \nu_{ik},$$

$$\text{for all } i, k = 1, 2, ..., n$$

The preference matrix was obtained [26]:

$$U^{(j)} = [\mu_{ik}^{(j)}]_{nn} = \begin{bmatrix} - & \mu_{12}^{(j)} & \cdots & \mu_{1n}^{(j)} \\ \mu_{21}^{(j)} & - & \cdots & \mu_{2n}^{(j)} \\ \vdots & \vdots & - & \vdots \\ \mu_{n1}^{(j)} & \mu_{n2}^{(j)} & \cdots & - \end{bmatrix} \tag{13}$$

The IF preference relation was obtained:

$$R^{(j)} = [r_{ik}^{(j)}]_{nn} = \begin{bmatrix} - & [\mu_{12}^{(j)}, \nu_{12}^{(j)}] & \dots & [\mu_{1n}^{(j)}, \nu_{1n}^{(j)}] \\ [\mu_{21}^{(j)}, \nu_{21}^{(j)}] & - & \dots & [\mu_{2n}^{(j)}, \nu_{2n}^{(j)}] \\ \vdots & \vdots & - & \vdots \\ [\mu_{n1}^{(j)}, \nu_{n1}^{(j)}] & [\mu_{n2}^{(j)}, \nu_{n2}^{(j)}] & \dots & - \end{bmatrix} \tag{14}$$

There are a number of aggregation operators for IFS [17, 18]. The IFWA operator was used in this paper:

$$r(x_i, x_k) = r_{ik} = \bigoplus_{j=1}^{m} [\tilde{w}_j \bigotimes r_{ik}^{(j)}] \tag{15}$$

$$\tilde{w}_j \bigotimes r_{ik}^{(j)} = [\mu_{ik}^{(j)} \mu_{\tilde{w}j}, \nu_{ik}^{(j)} + \nu_{\tilde{w}j} - \nu_{ik}^{(j)} \nu_{\tilde{w}j}] \tag{16}$$

$$r(x_i, x_k) = \overset{m}{\underset{j=1}{\oplus}} [\tilde{w}_j \otimes r_{ik}^{(j)}] = \left\{ 1 - \prod_{j=1}^{m} \left[1 - \mu_{ik}^{(j)} \mu_{\tilde{w}j} \right], \quad \prod_{j=1}^{m} \left[\nu_{ik}^{(j)} + \nu_{\tilde{w}j} - \nu_{ik}^{(j)} \nu_{\tilde{w}j} \right] \right\} \tag{17}$$

Overall intuitionistic fuzzy preference relationship is established:

$$R = (r_{ik})_{nn} = \begin{bmatrix} - & (\mu_{12}, \nu_{12}) & \dots & (\mu_{1n}, \nu_{1n}) \\ (\mu_{21}, \nu_{21}) & - & \dots & (\mu_{2n}, \nu_{2n}) \\ \vdots & \vdots & - & \vdots \\ (\mu_{n1}, \nu_{n1}) & (\mu_{n2}, \nu_{n2}) & \dots & - \end{bmatrix} \tag{18}$$

Every alternative is compared to option (n –1). As a result of intuitionistic fuzzy positive and negative outranking flow can be achieved as follows.

– The intuitionistic fuzzy positive outranking flow:

$$\tilde{\varphi}^{+}(x_i) = \frac{1}{n-1} \overset{n}{\underset{k=1, k \neq i}{\oplus}} r(x_i, x_k) \tag{19}$$

– The intuitionistic fuzzy negative outranking flow:

$$\tilde{\varphi}^{-}(x_i) = \frac{1}{n-1} \overset{n}{\underset{k=1, k \neq i}{\oplus}} r(x_k, x_i) \tag{20}$$

The relationship between $\tilde{\varphi}^{+}(x_i)$ and $\tilde{\varphi}^{-}(x_i)$ can be explained. The intuitionistic fuzzy net flow calculate using the function defined by Szmidt and Kacprzyk:

$$\rho[\tilde{\varphi}(x_i)] = \rho[\tilde{\varphi}^{+}(x_i)] - \rho[\tilde{\varphi}^{-}(x_i)] \tag{21}$$

An application of intuitionistic fuzzy PROMETHEE

Step 1: Selection and evaluation was performed among 10 alternatives of transportation service providers. Of all alternatives, intuitionistic fuzzy PROMETHEE method was used to choose the best transportation service provider and comparisons were made with the results

obtained from the TOPSIS method [14]. The C_2 and C_4 are profit criteria and the C_1 and C_3 are cost criteria $\{C_1, C_2, C_3, C_4\}$ be set of criteria represent, respectively:
– C_1: transportation price
– C_2: transportation capacity
– C_3: delivery time
– C_4: company reputation

$\{A_1, A_2, A_3, A_4, A_5, A_6, A_7, A_8, A_9, A_{10}\}$ be set of alternatives (the set of criteria are the ten candidate transportation service providers) in tab. 1.

Table 1. Alternatives and criteria

	C_1	C_2	C_3	C_4
A_1	100	400	10	0.9
A_2	145	600	19	0.99
A_3	85	500	20	0.84
A_4	90	300	8	0.88
A_5	120	700	6	0.86
A_6	110	900	9	0.75
A_7	80	800	12	0.8
A_8	105	500	8	0.9
A_9	95	1000	23	0.95
A_{10}	115	1000	11	0.99

Step 2: The weights of the criteria were calculated with the help of controlled sets in tab. 2.

Table 2. Weights of criteria

Weights	$\mu_{\tilde{\omega}_j}$	$\nu_{\tilde{\omega}_j}$
$\tilde{\omega}_1$	0.10	0.5072
$\tilde{\omega}_2$	0.38	0.5072
$\tilde{\omega}_3$	0.5072	0.38
$\tilde{\omega}_4$	0.02	0.5072

Step 3: Deviation $d_j(x_i, x_k)$ for each criterion; *V*-shape was calculated using generalization criteria:

$$d_j(x_i, x_k) = c_j(x_i) - c_j(x_k)$$

$$\mu_{ik}^{(j)} = \begin{cases} 0, & d_j(x_i, x_k) \le q \\ \frac{d_j(x_i, x_k) - q}{p - q}, & q < d_j(x_i, x_k) \le p \\ 1, & d_j(x_i, x_k) > p \end{cases}$$

Step 4: The IF preference matrices were determined.
Step 5: Overall IF preference relation was created.
Step 6: The IF positive and negative outranking flows were calculated.
Step 7: The values were ranked in tab. 3. The graphs of the alternatives in the positive and negative outranking flows are shown in figs. 1 and 2.

Table 3. The intuitionistic fuzzy positive and negative outranking flows

$\rho[\tilde{\varphi}^{+}(x_1)] = 0.987255246$	$\rho[\tilde{\varphi}^{-}(x_1)] = 0.955037943$
$\rho[\tilde{\varphi}^{+}(x_2)] = 0.953980212$	$\rho[\tilde{\varphi}^{-}(x_2)] = 0.974653673$
$\rho[\tilde{\varphi}^{+}(x_3)] = 0.955247725$	$\rho[\tilde{\varphi}^{-}(x_3)] = 0.966948519$
$\rho[\tilde{\varphi}^{+}(x_4)] = 0.996967882$	$\rho[\tilde{\varphi}^{-}(x_4)] = 0.951691402$
$\rho[\tilde{\varphi}^{+}(x_5)] = 0.976717303$	$\rho[\tilde{\varphi}^{-}(x_5)] = 0.955260117$
$\rho[\tilde{\varphi}^{+}(x_6)] = 0.963954352$	$\rho[\tilde{\varphi}^{-}(x_6)] = 0.964617824$
$\rho[\tilde{\varphi}^{+}(x_7)] = 0.964248237$	$\rho[\tilde{\varphi}^{-}(x_7)] = 0.967235655$
$\rho[\tilde{\varphi}^{+}(x_8)] = 0.989845837$	$\rho[\tilde{\varphi}^{-}(x_8)] = 0.955162146$
$\rho[\tilde{\varphi}^{+}(x_9)] = 0.949673098$	$\rho[\tilde{\varphi}^{-}(x_9)] = 0.995806919$
$\rho[\tilde{\varphi}^{+}(x_{10})] = 0.95709825$	$\rho[\tilde{\varphi}^{-}(x_{10})] = 0.972438752$

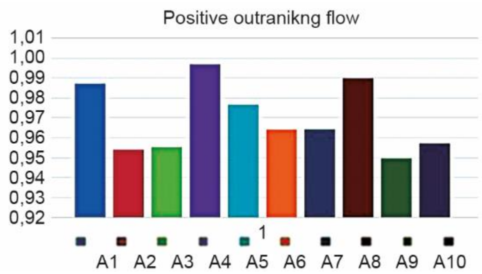

Figure 1. Graph of positive outranking flow

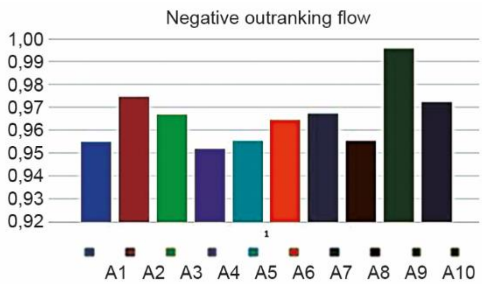

Figure 2. Graph of negative outranking flow

Step 8: In order to make a general ranking, a net outranking is needed since positive outranking and negative outranking are not the same. The net outranking flow in tab. 4 is calculated and its graph in fig. 3.

Table 4. Values of net outranking flow

$\rho[\tilde{\varphi}(x_1)] = 0.032217303$	$\rho[\tilde{\varphi}(x_6)] = -0.000663471$
$\rho[\tilde{\varphi}(x_2)] = -0.020673461$	$\rho[\tilde{\varphi}(x_7)] = -0.002987419$
$\rho[\tilde{\varphi}(x_3)] = -0.011700793$	$\rho[\tilde{\varphi}(x_8)] = 0.034683691$
$\rho[\tilde{\varphi}(x_4)] = 0.04527648$	$\rho[\tilde{\varphi}(x_9)] = -0.046133822$
$\rho[\tilde{\varphi}(x_5)] = 0.021457186$	$\rho[\tilde{\varphi}(x_{10})] = -0.015340502$

As a result, the ranking from the best alternative to the worst alternative is:

$$A_9, A_2, A_{10}, A_3, A_7, A_6, A_5, A_1, A_8, A_4$$

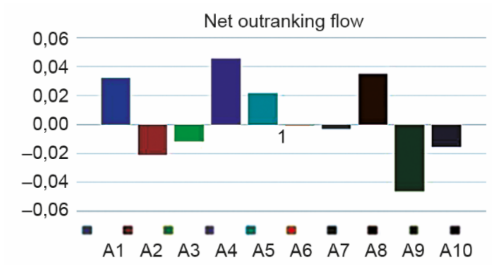

Figure 3. Graph of net outranking flow

During selection and evaluation process, not only positive but also negative sense alternatives are compared and a net ranking is obtained, the most exact consequence is obtained. Upon examining the results obtained with the TOPSIS method different results were obtained. The reasons for this difference can be explained as follows. First of all, it means to include the degrees of membership, non-membership and sensitivity in evaluating the case study in an intuitionistic fuzzy sense. However, thanks to the PROMETHEE method defined in intuitionistic fuzzy sets, both this sensitivity was activated and positive-negative rankings were obtained. In addition, a supervised system was established by means of controlled sets while determining the criteria weights. Thus, it may be wise the mention that the intuitionistic fuzzy PROMETHEE method, whose algorithm is given in present paper, offers a more rational and sensitive solution.

Conclusion

Intuitionistic fuzzy PROMETHEE is a method that attracts my attention thanks to its benefits such as allowing the researcher to observe the positive and negative rankings simultaneously. In this paper, a case study was evaluated to identify and evaluate the most suitable transportation service provider. Wu's article was revised. A new and different method was applied to this subject and different results were achieved. This method provided better results since there are uncertainties in this problem. This paper employed the intuitionistic fuzzy PROMETHEE method. The weights of the criteria were determined using the controlled sets. The results of the same case sample were compared with the results obtained by the TOPSIS method. New perspective was gained by including membership, non-membership, and sensitivity degrees thanks to intuitionistic fuzzy sets. Multi-criteria decision making methods have enabled us to obtain innovative results in the field of selection and evaluation in many areas.

Nomenclature

MCDM – multi criteria decision making
IF – intuitionistic fuzzy
PROMETHEE – preference ranking organization method for enrichment of evaluations
TOPSIS – technique for order of preference by similarity to ideal solution

References

[1] Zadeh, L. A., Fuzzy Sets, *Information and Control*, *8* (1965), 3, pp. 338-353
[2] Atanassov, K. T., Intuitionistic Fuzzy Sets, VII ITKR Session, Sofia, 20-23 June 1983. (Deposed in Centr. Sci.-Techn. Library of the Bulg. Acad. of Sci., 1697/84) (in Bulgarian), Reprinted: *Int. J. Bioautomation*, 20 (2016), S1
[3] Majumder, M., *Multi Criteria Decision Making*, Chapter 2, Springer, New York, USA, 2015, Jan., pp. 35-47
[4] Brans, J. P., L'ingéniérie de la Décision. Elaboration D'instruments D'aide à la Decision, Méthode PROMETHEE, in: *Laidea la decision: Nature, Instruments et Perspectives d avenir*, R. Nadeau, M. Landry, ed., Presses de lUniversite Laval, Quebec, Canada, (1982), pp. 183-213

[5] Albadvi, A., Formulating National Information Technology Strategies: A Preference Ranking Model Using PROMETHEE Method, *European Journal of Operational Research*, *153* (2004), 2, pp. 290-296
[6] Brans, J. P., *et al.*, PROMETHEE: A New Family of Outranking Methods in Multicriteria Analysis, *Operational Research IFORS*, *3*, (1984), 84, pp. 477-490
[7] Tugrul, F., *et al.*, Interpretation of Physical Conditions of Schools with Fuzzy Multi Criteria Decision Making, *Journal of Universal Mathematics, 3* (2020), 1, pp. 46-52
[8] Tugrul, F., Çitil, M., A New Perspective on Evaluation System in Education with Intuitionistic Fuzzy Logic and PROMETHEE Algorithm, *Journal of Universal Mathematics, 4* (2021), 1, pp. 13-24
[9] Tugrul, F., Çitil, M., Application of Mathematical Modeling in Multi Criteria Decision Making Process: Intuitionistic Fuzzy PROMETHEE, *Journal of Mathematical Sciences and Modelling, 5* (2022), 2, pp. 48-56
[10] Tugrul, F., *et al.*, Application of Intuitionistic Fuzzy Sets in High School Determination Via Normalized Euclidean Distance Metod, *Notes on Intuitionistic Fuzzy Sets, 23* (2017), 1, pp. 42-47
[11] Tugrul, F., *et al.*, School Success Ranking in Multi Criteria Decision Making, *Turkish Journal Mathematics and Computer Science, 10* (2018), Nov., pp. 1-6
[12] Tugrul, F., *et al.*, Application of Ranking with Similarity Measure in Multi Criteria Decision Making, *Konuralp Journal of Mathematics, 7* (2019), 2, pp. 438-441
[13] Tugrul, F., *et al.*, On the Determination Students' Aptitude Using Intuitionistic Fuzzy Logic, *Journal of Universal Mathematics, 2* (2019), 1, pp. 36-41
[14] Wu, X., *et al.*, Evaluation and Selection of Transportation Service Provider by TOPSIS Method with Entropy Weight, *Thermal Science, 25* (2021), 2B, pp. 1483-1488
[15] Aydin, S., Kahraman, C., Vehicle Selection for Public Transportation Using an Integrated Multi Criteria Decision Making Approach: A Case of Ankara, *Journal of Intelligent & Fuzzy Systems, 26* (2014), 5, pp. 2467-2481
[16] Sharma, S. K., Kumar, V., Optimal Selection of Third-Party Logistics Service Providers Using Quality Function Deployment and Taguchi Loss Function, *Benchmarking: An International Journal, 22* (2015), 7, pp. 1281-1300
[17] Xu, Z. S., Intuitionistic Fuzzy Aggregation Operators, *IEEE Transactions on Fuzzy Systems, 15* (2007), 6, pp. 1179-1187
[18] Xu, Z. S., Yager, R. R. Some Geometric Aggregation Operators Based on Intuitionistic Fuzzy Set, *International Journal of General Systems, 35* (2006), 4, pp. 417-433
[19] Szmidt, E., Kacprzyk, J., Amount of Information and Its Reliability in the Ranking Of Atanassov's Intuitionistic Fuzzy Alternatives, in: (E. Rakus-Andersson, *et al.*, ed.) (Studies in Computational Intelligence), *Recent Advances in Decision Making*, Springer, Berlin, Germany, 2009, pp. 7-19
[20] Cuvalcioglu, G., *Controlled Set Theory*, Bogolyubov Readings DIF-2013, National Academy of Science, Kiev, Ukraine, 2013
[21] Cuvalcioglu, G., Some Properties of Controlled Set Theory, *Notes on Intuitionistic Fuzzy Set, 20* (2014), 2, pp. 37-42
[22] Liao, H. C., Xu, Z. S., Priorities of Intuitionistic Fuzzy Preference Relation Based on Multiplicative Consistency, *IEEE Transactions on Fuzzy Systems, 22* (2014), 6, pp. 1669-1681
[23] Liao, H. C., Xu, Z. S., Some algorithms for group decision making with intuitionistic fuzzy preference information, *International Journal of Uncertainty Fuzziness and Knowledge-Based Systems, 22* (2014), 4, pp. 505-529
[24] Vincke, P. H., Brans, J. P., A Preference Ranking Organization Method, the PROMETHEE Method for MCDM, *Management Science*, *31* (1985), 6, pp. 647-656
[25] Xu, Z. S., Intuitionistic Preference Relations and their Application in Group Decision Making, *Information Sciences, 177* (2007), 11, pp. 2363-2379
[26] Liao, H. C., Xu, Z. S., Multi-Criteria Decision Making with Intuitionistic Fuzzy PROMETHEE, *Journal of Intelligent Fuzzy Systems, 27* (2014), 4, pp. 1703-1717

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