THE DI-TOPOLOGICAL TEXTURE GRAPHS

by

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In this study, n-point graphs and n-point texture spaces are examined and graphs that we will call Texture Graphs are obtained. In addition, it is shown how a di-topology can be obtained on the given texture space with the help of this graph. It is shown that a di-topological texture space $(S,\mathcal{S},\tau,\kappa)$ *associated with di-graph* (*S*, *G*) *and each di-graph* (*S*, *G*) *with n points associated with a unique di-topology on texture space. With the graph obtained from co-topology, it has been seen that there is alternative information for the solution of many mathematical and non-mathematical problems in terms of application.*

Key words: *graph, di-topology, texture space*

Introduction

Graph theory, which was founded by Euler in 1736 with the Konigsberg bridge problem, is used in many fields today, especially with its applications. This theory, whose applications we cannot finish counting, has brought solutions to many different problems in every space where it is defined. Texture spaces, which consist of mathematical concepts and structures independent of the complement operation, are more general structures than topological spaces. The concept of texture is also known as fuzzy structures as a point-set-based counterpart of fuzzy sets. These structures were later developed and called texture spaces. In the following years, many mathematicians carried many mathematical concepts to these spaces and examined them. However, the concept of the graph has been never used. In this study, we present the concept of di-topological texture graph to the literature by moving graphs to texture spaces. Thus, we think that we have added a geometric interpretation to the texture spaces. Thus, it is thought that examining the graph concept to texture spaces, will contribute to the solution of many mathematical and non-mathematical problems.

The texture space we want to move the graph to is first defined by Brown in 1993 [1, 2]. Texture spaces are point-set-based and independent from complement spaces that contain many mathematical concepts. In addition, a generalized complement operation on texture spaces has been defined, and the relationship of texture spaces with known general structures has been revealed. In the work titled *Di-topological texture spaces and fuzzy topology II, Topological considerations* [4], the relationship of di-topologies with topologies and bi-topologies has been revealed. Detailed information about the general definitions and concepts used in this paper regarding texture spaces and di-topologies can be found in [1-5].

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The relationship between graphs and topologies in general topological spaces was first described in 1967 by J. W. Evans, F. Harary, and M. S. Lynn. In their work titled *On the Computer Enumeration of Finite Topologies* [6], they showed that there is a one-to-one mapping between *n*-point topologies and n-point transitive directed graphs with the help of open sets. Bhargav and Ahlborn's [7] article titled *On Topological Space Associated With Di-graph* published in 1968 showed the existence of relations between closed sets and directed graphs for general topological spaces. In Kannan's article titled *A note on some generalized closed sets in bi-topological spaces associated with di-graphs* [8] showed the existence of relations between closed sets and directed graphs for bi-topological spaces. The article titled *Bi-topological space associated with di-graphs* [9] presented by Girija and Pilakat in 2013 again showed that there is an alternative mapping between directed graphs and bi-topologies. Unlike Kannan's study [8], Girija and Pilakat [9] obtained bi-topology from any di-graph using a quasi-pseudo metric. When these studies are examined, it is seen that the graphs could be moved to texture spaces. In 2003, in the study called *Di-uniform Texture Space* [10] by Selma Ozcag and Lawrence Brown, it was proved that di-topologies were obtained using quasi pseudo-dimetrics. In this study, metric neighborhoods are defined for texture spaces. Thus, it is seen that graphs and di-topologies can be matched with the help of metric neighborhoods.

It can be seen that n-point graphs can be matched to n-point topologies. Similarly, n-point graphs can be matched to n-point bi-topologies. Therefore, by matching n-point graphs with n-point di-topologies, the graphs we will call di-topological texture graphs are obtained in this. In this paper, since we work with finite graphs, the space is taken as plain texture.

Preliminaries

This section contains the notations which are needed in the sequel.

Texture spaces

Definition 1. Let S be a set. Then $\mathcal{S} \subseteq P(S)$ is called a texturing of *S*, and $\mathcal{S}$ is said to be textured by *S* if

1. $(\mathcal{S}, \subseteq)$ is a complete lattice containing *S* and $\varnothing$ for any index set *I* and $A_i \in \mathcal{S},\ i \in I$ the meet $\Lambda_{i\in I} A_i$ and the join $\mathrm{V}_{i\in I} A_i$ in $\mathcal{S}$ are related with the intersection and union in *P*(*S*) by the equalities:

$$\bigwedge_{i\in I} A_i = \bigcap_{i\in I} A_i$$

for all *I*, while:

$$\bigvee_{i\in I} A_i = \bigcup_{i\in I} A_i$$

for all finite *I*.

2. $\mathcal{S}$ is completely distributive.
3. $\mathcal{S}$ separates the points of *S*. That is, given $s_1 \neq s_2$ in *S* we have $A \in \mathcal{S}$ with $s_1 \in A, s_2 \notin A$ or $A \in \mathcal{S}$ with $s_2 \in A, s_1 \notin A.$

If *S* is textured by $\mathcal{S}$ then $(S, \mathcal{S})$ is called a *texture space*, or simply a *texture* [5].

A texturing on a set *S* is a point separating, complete, completely distributive lattice $\mathcal{S}$ of subsets of *S* concerning inclusion which contains $S, \varnothing$ and, for which arbitrary meet coincides with intersection and finite joins coincide with the union. Then $(S,\mathcal{S})$ is called a texture space. The sets:

$$P_s = \bigcap\{A: s \in A \in \mathcal{S}\} \text{ and } Q_s = \vee\{P_t : s \notin P_t\}$$

are known as *p*-sets and *q*-sets and they are important tools for textures as we will see in the sequel. If for all $s \in \mathcal{S}$, we have $P_s \not\subseteq Q_s$, then $(S,\mathcal{S})$ is called a plain texture space. This is equivalent to saying that $(S,\mathcal{S})$ is closed under arbitrary unions.

Definition 2. Let $(S,\mathcal{S})$ be a texture space. If τ and κ satisfy the following conditions, thus τ is a topology on $(S,\mathcal{S})$ and κ is a co-topology on $(S,\mathcal{S})$, a di-topology (τ,κ) pair on $(S,\mathcal{S})$, and $(S,\mathcal{S},\tau,\kappa)$ is di-topological texture space.

– A topology on $(S,\mathcal{S})$ to family a $\tau \subseteq \mathcal{S}$ that satisfies the following conditions, and the elements of τ are called open sets of the topology [5]:

T-1) $\varnothing, S \in \tau$

T-2) $G_1, G_2 \in \tau \Rightarrow G_1 \bigcap G_2 \in \tau$ and

T-3) $i \in I, G_i \in \tau \Rightarrow \vee_{i \in I} G_i$

– A co-topology on $(S,\mathcal{S})$ to family a $\kappa \subseteq \mathcal{S}$ that satisfies the following conditions, and the elements of κ are called closed sets of the co-topology [5]:

CT-1) $\varnothing, S \in \kappa$

CT-2) $K_1, K_2 \in \kappa \Rightarrow K_1 \bigcup K_2 \in \kappa$ and

CT-3) $i \in I, K_i \in \kappa \Rightarrow \bigcap_{i \in I} K_i$

Here, there is generally no link between open sets and closed sets.

How di-topologies are obtained with the help of pseudo-dimetrics is expressed as follows.

Definition 3. Let (τ,κ) be a di-topology on $(S,\mathcal{S})$.

1. If $s \in S^b$, a neighborhood of *s* is a set $N \in \mathcal{S}$ for which there exists $G \in \tau$ satisfying $P_s \subseteq G \subseteq N \not\subseteq Q_s$.
2. If $s \in S$, a co-neighborhood of *s* is a set $M \in \mathcal{S}$ for which there exists $K \in \kappa$ satisfying $P_s \not\subseteq M \subseteq K \subseteq Q_s$.

Note that although co-neighborhood are defined for all $s \in S$ it will be apparent later that we need only consider the co-neighborhood of points in S^b (in here, the core of a set *A* in $\mathcal{S}$ is the set A^b defined by:

$$A^b = \bigcup\left\{\bigcap\{A_j : j \in J\} : \{A_j : j \in J\} \subseteq \mathcal{S}, A = \vee\{A_j : j \in J\}\right\}$$

for plain ($(S,\mathcal{S})$ space is called plain texture space if $P_s \not\subseteq Q_s$ for every $s \in S$) textures, we have $A^b = A$).

Again the formal duality between these two concepts is clear. We denote the set of neighborhoods (co-neighborhood) of *s* by $\eta(s)[\mu(s)]$, respectively. We will also refer to $[\eta(s),\mu(s)]$, $s \in S^b$, as the di-neighborhood system of (τ,κ).

How di-topologies are obtained with the help of di-neighborhood is expressed as follows.

Theorem 1. For a di-topology (τ,κ) on $(S,\mathcal{S})$ let the families $\eta(s), s\in S^b$ and $\mu(s), s\in S$ be defined as above.

1. For $s\in S^b$ we have $\eta(s)\neq\varnothing$ and these families satisfy the following conditions:
   i. $N\in\eta(s)\Rightarrow N\not\subseteq Q_s$
   ii. $N\in\eta(s), N\subseteq N'\in\mathcal{S}\Rightarrow N'\in\eta(s)$
   iii. $N_1,N_2\in\eta(s)$, $N_1\cap N_2\not\subseteq Q_s\Rightarrow N_1\cap N_2\in\eta(s)$
   iv.
      a) $N\in\eta(s)\Rightarrow\exists N^*\in\mathcal{S}, P_s\subseteq N^*\subseteq N,$ so that $N^*\not\subseteq Q_t\Rightarrow N^*\in\eta(t),\forall t\in S^b$
      b) For $N\in\mathcal{S}$ and $N\not\subseteq Q_s,$ if there exists, $N^*\in\mathcal{S}$, $P_s\subseteq N^*\subseteq N$ which satisfies $N^*\not\subseteq Q_t\Rightarrow N^*\in\eta\,(t),\ \forall s,t\in S^b,$ then $N\in\eta\,(s).$

Moreover, the sets G in τ are characterized by the condition that $G\in\eta(s)$ for all s with $G\not\subseteq Q_s$.

1. For $s\in S$ we have $\mu(s)\neq\varnothing$ and these families satisfy the following conditions [5]:
   i. $M\in\mu(s)\Rightarrow P_s\not\subseteq M$
   ii. $M\in\mu(s), M\supseteq M'\in\mathcal{S}\Rightarrow M'\in\mu(s)$
   iii. $M_1,M_2\in\mu(s)\Rightarrow M_1\cup M_2\in\mu(s)$
   iv.
      a) $M\in\mu(s)\Rightarrow\exists M^*\in\mathcal{S}, M\subseteq M^*\subseteq Q_s,$ so that $P_t\not\subseteq M^*\Rightarrow M^*\in\mu(t),\forall t\in S.$
      b) For $M\in\mathcal{S}$ and $P_s\not\subseteq M,$ if there exists $M^*\in\mathcal{S}, M\subseteq M^*\subseteq Q_s$ which satisfies $P_t\not\subseteq M^*\Rightarrow M^*\in\mu(t),\forall t\in S,$ then $M\in\mu(s).$

Moreover, the sets K in κ are characterized by the condition that $K\in\mu(s)$ for all s with $P_s\not\subseteq K.$

How di-topologies are obtained with the help of pseudo-dimetrics is expressed as follows.

Definition 4. Let $(S,\mathcal{S})$ be a texture, $\rho=(\overline{\rho},\underline{\rho}):S\times S\to[0,+\infty)$ two-point functions.

Then $\rho=(\overline{\rho},\underline{\rho})$ is called a pseudo dimetric on $(S,\mathcal{S})$ if

M1	$\overline{\rho}(s,t)\le\overline{\rho}(s,u)+\overline{\rho}(u,t)$	$\forall s,t,u\in S$
M2	$P_s\not\subseteq Q_t\Rightarrow\overline{\rho}(s,t)=0$	$\forall s,t\in S$
DM	$\overline{\rho}(s,t)=\underline{\rho}(t,s)$	$\forall s,t\in S$
CM1	$\underline{\rho}(s,t)\le\underline{\rho}(s,u)+\underline{\rho}(u,t)$	$\forall s,t,u\in S$
CM2	$P_t\not\subseteq Q_s\Rightarrow\underline{\rho}(s,t)=0$	$\forall s,t\in S$

In this case, $\overline{\rho}$ is called the pseudo metric, $\underline{\rho}$ the pseudo cometric of ρ. If the DM condition is removed, then $\rho=(\overline{\rho},\underline{\rho})$ is called a quasi pseudo dimetric on $(S,\mathcal{S})$. If ρ is a pseudo dimetric (quasi pseudo dimetric) which satisfies the conditions

M3 $P_s\not\subseteq Q_u$, $\overline{\rho}(u,v)=0$, $P_v\not\subseteq Q_t\Rightarrow P_s\not\subseteq_t\forall s,t,u\in S$

CM3 $P_u\not\subseteq Q_s$, $\overline{\rho}(u,v)=0$, $P_t\not\subseteq Q_v\Rightarrow P_u\not\subseteq Q_v\,\forall s,t,u\in S$

It is called a dimetric (quasi dimetric).

For a pseudo dimetric to be dimetric must be $\rho(s,t)=0\Rightarrow P_s\not\subseteq Q_t$ [10].

Theorem 2. Let $\rho,$ a pseudo dimetric on $(S,\mathcal{S}),$ $s\in S^b$ and for $\varepsilon>0$:

$$N_\varepsilon^\rho(s)=\vee\{P_t:P_s\not\subseteq Q_u \text{ with } \exists u\in S,\underline{\rho}(u,t)<\varepsilon\}$$

and

$$M_\varepsilon^\rho(s)=\bigcap\{Q_t:P_u\not\subseteq Q_s \text{ with } \exists u\in S,\underline{\rho}(u,t)\le\varepsilon\}$$

In this case, $\beta_\rho = \{N_\varepsilon^\rho(s) : s \in S^b, \varepsilon > 0\}$ is a base for di-topology (τ_ρ, κ_ρ) on $(S, \mathcal{S})$ and $\gamma_\rho = \{M_\varepsilon^\rho(s) : s \in S^b, \varepsilon > 0\}$ is a co-base [10].

For more details see [1-5, 10].

Graphs

A graph is a pair of sets $G(V, E)$ where *V* is the set of vertices and *E* is the set of edges, formed by pairs of vertices. The two vertices *u* and *v* are end vertices of the edge (u, v). The two edges (u,v) and (v,u) are the same. In other words, the pair is not ordered.

A directed graph or di-graph is formed by vertices connected by directed edges or arcs. A di-graph is a pair $D(V, E)$ where *V* is the vertex set and *E* is the set of vertex pairs graphs. The difference is that now the elements of *E* are ordered pairs: the arc from vertex *u* to vertex v is written as (*u*, *v*) and the other pair (v,u) is the opposite direction arc. Vertex *u* is the initial vertex and vertex v is the terminal vertex of the arc (u, v).

Let $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ are two graphs. The combination of these graphs is indicated by:

$$G_1 \cup G_2 = [V(G_1 \cup G_2), E(G_1 \cup G_2)]$$

and it is defined:

$$V(G_1 \cup G_2) = V_1 \cup V_2$$

$$E(G_1 \cup G_2) = E_1 \cup E_2$$

For more details see [11, 12].

Texture spaces associated with graphs

Girija and Pilakat [9] showed that bi-topologies are associated with directed graphs and bi-topologies. In their study, they defined a path using a quasi-pseudo metric as follows. $p : V \times V \to R$ and $q : V \times V \to R$, respectively:

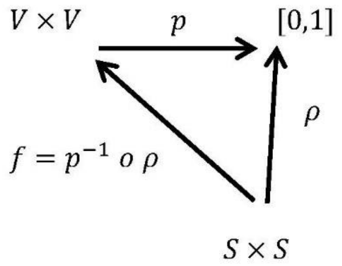
$$p(x, y) = \begin{cases} 0, x \text{ is reachable from } y \\ 1, \text{otherwise} \end{cases}, \quad q(x, y) = \begin{cases} 0, y \text{ is reachable from } x \\ 1, \text{otherwise} \end{cases}$$

Ozcag and Brown [10] defined quasi-pseudo dimetric as follows:
for $\forall s, t \ (s \neq t) \in S$

$$\rho = (\bar{\rho}, \underline{\rho}) : S \times S \to [0, +\infty)$$

$$\bar{\rho}(s,t) = \begin{cases} 0, P_s \not\subseteq Q_t \\ 1, \text{otherwise} \end{cases} \quad \underline{\rho}(s,t) = \begin{cases} 0, P_t \not\subseteq Q_s \\ 1, \text{otherwise} \end{cases}$$

Considering these two studies:



…(*)

$$f = p^{-1} o\, \rho : S\times S \to S\times S$$

$$f = p^{-1} o\, \rho[(s,t)] = \begin{cases} (s,t) & P_s \nsubseteq Q_t \\ \text{there is not a way} & \text{otherwise} \end{cases}$$

$$f = (\overline{f}, \underline{f}) : S\times S \to S\times S$$

$$\overline{f}(s,t) = \begin{cases} \text{there is a way from } t \text{ to } s, & P_s \nsubseteq Q_t \\ \text{there is not a way from } t \text{ to } s, & \text{otherwise} \end{cases}$$

$$\underline{f}(s,t) = \begin{cases} \text{there is a way from } s \text{ to } t, & P_t \nsubseteq Q_s \\ \text{there is not a way from } s \text{ to } t, & \text{otherwise} \end{cases}$$

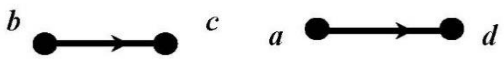
The path, called the *texture path*, is obtained. Allows us to obtain a path from t to s if $P_s \nsubseteq Q_t$ and a path from s to t if $P_t \nsubseteq Q_s$. With the help of the edges (texture paths) formed by these paths, the graphs that we called texture di-graph is obtained.

Example 1. Let $S = \{a,b,c,d\}$ be a set:

$$\mathcal{S} = \{\varnothing, S, \{a\}, \{b\}, \{a,b\}, \{a,d\}, \{b,c\}, \{a,b,c\}, \{a,b,d\}\}$$

be a texture on *S*. It is obtained by following the texture di-graph with the help of the sets P_s and Q_t:

$$f = (\overline{f}, \underline{f}) : S\times S \to S\times S$$



$P_c \nsubseteq Q_b$ $P_d \nsubseteq Q_a$

Corollary 1. Let *S* be a finite set and $\mathcal{S}$ be any texture. Then $(S,\mathcal{S})$ texture space associated with a di-graph.

The method of obtaining di-topology by using texture graph

Evans *et al.* [6] conceived this idea and he proved that there is a one-to-one correspondence between the set of all topologies on a set X with n points and the set of all transitive di-graphs with n points. He established his results as follows. Let V be a finite set and τ be a topology on V. The transitive di-graph corresponding to this topology is got by drawing a line from u to v, if and only if, u is in every open set containing v. Conversely, let D be a transitive di-graph on V; the family $B=\{Q(a):a\in V\}$ forms a base for a topology on V, where $Q(a)=\{a\}\cup\{b\in V:(b,a)\in E(D)\}$

It is obtained di-topology by using Evans's method, *Theorem 1* and *Theorem 2*, texture graph, as follows.

Definition 5. For $\forall s,t\in S\ (s\neq t)$ $\eta(s)=P_s\cup\{P_t: s \text{ is reachable from } t\}$:

$$\beta_\tau=\vee\{\eta(s):s\in S\}$$

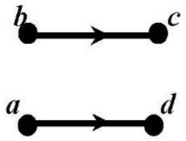
is a base for τ topology. For $\forall s,t\in S\ (s\neq t)$:

$$\mu(t)=Q_t\bigcap\{Q_s: t \text{ is reachable from } s\}$$

$$\beta_\kappa=\bigcap\{\mu(t):t\in S\}$$

is a co-base for κ co-topology. Thus, a di-topology is associated with the texture space with the help of a texture di-graph. A di-topology in the space where this texture graph is obtained is found, thus the di-graph is called the di-topological texture di-graph.

For *Example 1*.



$\beta_\tau=\{\{a\},\{b\},\{b,c\},\{a,d\}\}$ is a base and τ topology

$$\tau=\{S,\varnothing,\{a\},\{b\},\{b,c\},\{a,d\},\{a,b,c\},\{a,b,d\}\}$$

For co-topology

$\beta_\kappa=\{\varnothing,\{b,c\},\{a,d\}\}$ is a co-base and κ co-topology

$$\kappa=\{S,\varnothing,\{b,c\},\{a,d\}\}$$

Remark 1. A graph we call texture graph can be associated with every texture space defined on S set different from the empty set. Note that the texture space basically contains di-topologies with the help of metrics and neighborhoods. The di-topologies found with the help of neighborhoods are more than one. Therefore, a different graph is associated with each di-topology. However, the di-topology found with the help of the composite transformation(*) and the graph associated with to it are unique. So, the following theorem is obtained.

Theorem 3. Following statements are equivalent.

i. A di-topological texture space $(S,\mathcal{S},\tau,\kappa)$ associated with di-graph (S, G).
ii. Each di-graph (S, G) with n points associated with a unique di-topology on texture.

Proof: We show that with any topology τ of any di-topology one can associate a texture di-graph (S, G), as follows. The point set of (S, G). is that of τ. For two distinct points s and t of τ, s will be adjacent to t in (S, G) provided s is in every neighborhood of $t,\eta(t)$ (open sets containing t). If $s \in S^b$, a neighborhood of s is a set $N \in \mathcal{S}$ for which there exists $G \in \tau$ satisfying $P_s \subseteq G \subseteq N \nsubseteq Q_s$. But if we choose this collection $\eta(s) = \{B : P_s \subseteq B \subseteq G \subseteq N \nsubseteq Q_s,\ s \in S^b \text{and } G \in \tau,\ N \in \mathcal{S}\}$, then we get the base for di-topology. Thus, a family $\eta(s)$ of di-graph (S, G). is open if s is in every neighborhood of t imply that $st \in G,$, where $st \in G$ is a path from s to t.

Clearly, (S, G) is uniquely determined by τ. Similarly, we can show that any co-topology κ of any di-topology one can associate a texture di-graph (S, G). For two distinct points s and t of κ. The s will be adjacent to t in (S, G) provided s is not in every co-neighborhood of $t, \mu(t)$ (closed sets containing t). The (S, G) is uniquely determined by κ. If $s \in S$, a co-neighborhood s is a set $M \in \mathcal{S}$ for which there exists $K \in \kappa$ satisfying $P_s \nsubseteq M \subseteq K \subseteq Q_s$. But if we choose this collection $\mu(s) = \{B : P_s \nsubseteq M \subseteq B \subseteq K \subseteq Q_s,\ s \in S \text{and} K \in \kappa,\ M \in \mathcal{S}\}$, then we get the co-base for di-topology. Thus a family $\mu(s)$ of di-graph (S, G) is closed if s is not in every neighborhood of t implying that $st \in G$.

We next show that each texture di-graph (S, G) with n points corresponds to a unique di-topology on texture $(S, \mathcal{S})$. Define τ as that topology with the same point set as (S, G) in which the basic open set are all sets $\eta(s),\ s \in S^b$. The $\eta(s)$ consists of s and all points adjacent to it. We should show that τ is a topology. By definition, every open set in τ is formed from $\bigcup_{i=1}^{n} \eta(s_i)$. Immediately the union of two open sets is open. It is sufficient to show that the intersection of two open sets is open, if we prove it for two basic open neighborhoods $\eta(s_1)$ and $\eta(s_2)$. It is clear from *Theorem 1*, such that for $N \in \mathcal{S}$ and $N \nsubseteq Q_s$, if there exists, $N^* \in \mathcal{S}$, $P_s \subseteq N^* \subseteq N$ which satisfies $N^* \nsubseteq Q_t \Rightarrow N^* \in \eta(t),\ \forall s,t \in S^b$, then $N \in \eta(s)$.

Similarly, this process can be made for any co-topology κ. By definition, every closed set in κ is formed from $\mu(s)$. Thus, the proof is completed. It is easy to say that there is a 1-1 correspondence between the di-topological texture space with n points and the texture di-graph with n points. Thus, $\beta_\tau = \{\eta(s) : s \in S^b\}$ is a base for τ on texture $(S, \mathcal{S})$ and $\beta_\kappa = \{\mu(s) : s \in S\}$ is a co-base for κ on texture $(S, \mathcal{S})$. Thus each di-graph (S, G) determines a unique di-topological texture space $(S, \mathcal{S}, \tau, \kappa)$.

Applications of texture di-graphs

Graphs associated with topologies provide some information in terms of application. However, graphs associated with di-topologies offer much more information. Because in di-topological space there are two different graphs corresponding to topology and co-topology. While the graph corresponding to the topology is the same as the texture graph, the graph corresponding to the co-topology is completely different and offers alternative information in terms of application.

Definition 6. Let $(S, \mathcal{S}, \tau, \kappa)$ be a di-topological texture space. The graph corresponding to this space consists of two parts. One is the graph corresponding to τ and the other is the graph corresponding to κ. When obtaining the first of these graphs, if there is P_a in every p-set containing P_b, there is a path from a to b. This path is called the τ-path and the combination of these paths obtained from the topology is called the τ graph. If there is Q_b in every q-set containing Q_a, while obtaining the other, there is a path from a to b. This path is called a

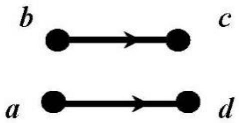
κ path, and the combination of κ paths is a κ graph obtained from co-topology. Thus, the di-topological texture graph is obtained with the τ graph and κ graph.

Remark 2. Texture graph and τ graph are the same, but κ the graph is different from this graph.

Example 2. Let $S = \{a,b,c,d\}$ be a set:

$$\mathcal{S} = \{\varnothing, S, \{a\},\{b\},\{a,b\},\{a,d\},\{b,c\},\{a,b,c\},\{a,b,d\}\}$$

be a texture on *S*. We get the following texture di-graph with the help of P_s and Q_t:



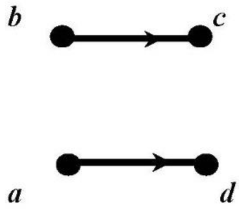
The topology of texture space is:

$$\tau = \{\{a\},\{b\},\{a,b\},\{a,d\},\{b,c\},\{a,b,c\},\{a,b,d\},S,\varnothing\}$$

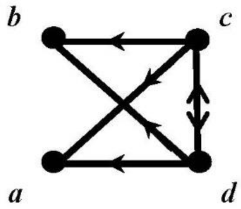
The co-topology of texture space is:

$$\kappa = \{\{a,d\},\{b,c\},S,\varnothing\}$$

Thus, we get the following τ graph and κ graph by using τ-paths and κ-paths, respectively:



τ graph



κ graph

For example, by using di-topological texture graph, alternative routes can be offered for the four settlements given in figs. 1-3.

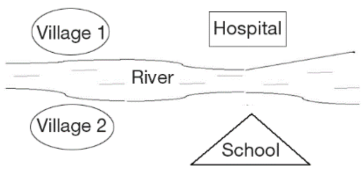


Figure 1.

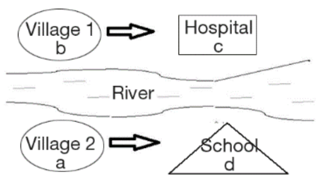


Figure 2.

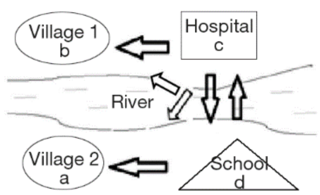


Figure 3.

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