ANALYTICAL ANALYSIS OF STEADY FLOW OF NANOFLUID, VISCOUS DISSIPATION WITH CONVECTIVE BOUNDARY CONDITION

by

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We focused on the analytical analysis of steady flow of nanofluid, viscous dissipation with convective boundary condition in this work. The MWCNT and SWCNT are used to described the nanofluid. A similarity transformation is applied to convert nonlinear PDE from their dimensional form to dimensionless nonlinear ODE. The developed nonlinear ODE for velocity and temperature profiles are solved by using an approximate analytical technique called the homotopy asymptotic method. Graphs are used to discuss and illustrate the results. Graphs are used to interpret the effects of several factors. Finally, the skin friction and Nusselt number are illustrated in the form of table.

Keywords*: MHD (SWCNT, MWCNT), stretching surface, viscous dissipation, homotopy asymptotic method*

Introduction

Due to the high rate of energy and heat transmission ratio of nanofluids have some important applications in manufacturing and sciences. The scientists investigating the mass and heat transfer of nanofluid from the past few years from different channels. Here, we focused on the study of the energy and heat transfer ratio of a CNT nanofluid over a stretching surface along with viscous dissipation. Chen [1] used un-stretched sheet to study the liquid film of the power law model with the effect of Marangoni convection. Kumar *et al*. [2] study the nanofluid flow with the effect of Marangoni convection and heat source sink.

The CNT is the well-known class of carbon family for the boost of heat transfer devices. Haq *et al*. [3] used trapezoidal cavity to study nanofluid thermal management, with the effect of Marangoni convection. Iijima [4] used Krastschmer and Huddman technique discuss for the first time (MWCNT) multi wall carbon nanotubes. In 1993, Ajayan [5] studied single

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wall carbon nanotubes (SWCNT). The SWCNT are rolled sheet in a cylinder or tube shape of graphene with range of diameter from 0.4×10^{-9} m to 3×10^{-9} m and thickness is about 0.34×10^{-9} m [6]. In [7], MWCNT is of 2 to 50 coaxial nanotubes with 0.34×10^{-9} m layer spacing and the range of diameter is 3×10^{-9} m to 30×10^{-9} m. Hone in [8] found that the thermal conductivity is up to 6600 W/mK for SWCNT and 3000 W/mK for MWCNT at room temperature. Haq *et al.* [9] investigated the assessment of engine oil base CNT and fund that the greater value of heat transfer and skin friction is more as compare to kerosene oil and water base CNT. The MWCNT and SWCNT are both studied by Khan *et al.* [10] using the Navier slip boundary condition. Using a CNT base nanofluid and a continuous heat flux, Kamli and Binesh [11] investigated convective heat analysis. The thermal efficiency of CNT in engine oil nanofluid was explored by Lie *et al.* [12], and similar results were achieved for ethylene glycol. The effectiveness of a nanofluid made of coolant water base CNT is studied by Halelfadel *et al.* [13], Liquid metals, electrolytes, and plasma all are examples of MHD fluids [14].

There are some important and useful applications of MHD fluids in industrial sector and engineering sector for example, reactor cooling, drug targeting and power generation, *etc*, due to this important application a lot of the researchers and scientist's efforts on MHD fluids. The authors in [15] studied the impact of inner thermal generation/absorption MHD using perpendicular plate. Krishna and Chamka [16] discussed the impact of Hall and ion slip for a MHD nanofluids. Lund *et al.* [17] study the influence of MHD flow using shrinking and perpendicular surface. Islam *et al.* [18] inspected the thermal radiation and Hall current impact on MHD hybrid nanofluid. Due to numerous potential applications of nanofluids in various fields of science and technology, nanotechnology has attracted researchers.

Motivated from the aforementioned literature, we opt to study analytical analysis of steady flow of nanofluid, viscous dissipation with convective boundary condition, for the improvement of the ratio of heat transfer and to reduce the energy consumption which is the requirement of industries sector. The novelty of this work is for the first time unsteady CNT along with viscous dissipation and convective boundary condition are investigated approximate analytical on this model. Graphs are used to interpret the effects of several factors.

Mathematical formulation

Consider steady incompressible MHD 2-D flow of CNT nanofluid over a non-linear stretching surface along with the influence of magnetic field and viscos dissipation. The influence of thermophoresis and also Brownian diffusion are also considering. The Cartesian coordinate arrangement so that *x*- and *y*-directions are taken along an moving surface, with velocities of $U_w(x,y) = a(x + y)^n$ and $V_w(x,y) = b(x + y)^n$, where a,b are constants and $n > 0$, the heat transfer coefficient is h_f and T_f is the temperature of fluid under the surface are used to characterize the convective heating process, which controls the surface temperature. The equations are given as:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu_f\frac{\partial^2 u}{\partial y^2}-\frac{\sigma_f}{\rho}B^2up-\frac{\nu_f}{K}u-F_bu^2-\lambda\left(u^2\frac{\partial^2 u}{\partial x^2}+v^2\frac{\partial^2 u}{\partial y^2}+2uv\frac{\partial^2 u}{\partial x\partial y}\right) \tag{2}$$

Then energy equation is as:

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}+\lambda_e\Gamma_e=\alpha_f\frac{\partial^2 T}{\partial y^2}+\tau\left[\frac{D_T}{T_\infty}\left(\frac{\partial T}{\partial y}\right)^2\right]+\frac{Q_0}{(\rho C_p)_f}(T-T_\infty)+\frac{\mu_{\text{nf}}}{\rho_{\text{nf}}c_p}\left(\frac{\partial u}{\partial y}\right)^2 \tag{3}$$

with the following:

$$u = U_w,\ v = V_w,\quad -k\frac{\partial T}{\partial y} = h_f\left(T_f - T\right)\quad \text{at } y = 0$$
$$u \to 0,\ v \to 0,\ T \to T_\infty \quad \text{at } y \to \infty \tag{4}$$

Selecting the similarity transformations as:

$$\eta = \left[\frac{a(n+1)}{2\nu_f}\right]^{1/2}(x+y)^{n-1/2},\quad u = a(x+y)^n f'(\eta)$$
$$v = a(x+y)^n,\quad \theta(\eta) = \frac{T - T_\infty}{T_f - T_\infty} \tag{5}$$

$$f'''(\eta) + f(\eta)f''(\eta) - \frac{2n}{1+n}\left[f'(\eta)\right]^2 - \frac{2}{1+n}(\text{Ha})^2 f'(\eta) - \frac{2}{1+n}\Lambda f'(\eta) - \frac{2}{1+n}F_r f'(\eta)^2 -$$
$$-\beta\left\{f'(\eta)\left[2f'(\eta) - \frac{\eta}{2}f''(\eta)\right] + \frac{1}{2}f(\eta)f'''(\eta)\right\} - 3f(\eta)f'(\eta)f''(\eta) = 0 \tag{6}$$

$$\frac{1}{\text{Pr}}\theta''(\eta) + f(\eta)\theta'(\eta) + \frac{2}{(1+n)}S\theta(\eta) + Nt\theta'(\eta)^2 + \text{Ec}\left[f''(\eta)\right]^2 = 0 \tag{7}$$

$$f(0) = f'(0) = 1,\ \theta(0) = -\gamma\left[1 - \theta(0)\right],\ f'(\infty) \to 0,\ \theta(\infty) \to 0 \tag{8}$$

Prandtl number, sink parameter, Eckert number, magnatic filed, and Couple stress parameter are defined:

$$\text{Pr} = \frac{\nu_f}{\alpha_f}, S = \frac{Q(x+y)^{1-n}}{a(\rho c)_p}, \text{Ec} = \frac{U_\varpi^2}{C_p(T_\varpi - T_0)}, M = \frac{\sigma_{nf}B_0^{\ 2}}{\rho_{nf}u_0}, K$$

The dimensionaless form of $\text{Re}_x^{1/2} C_{fx}$ and $\text{Re}_x^{-1/2}\ \text{Nu}_x$:

$$\text{Re}_x^{1/2} C_{fx} = \left(\frac{n+1}{2}\right)^{1/2} f'''(0), \text{Re}_x^{-1/2}\ \text{Nu}_x = -\left(\frac{n+1}{2}\right)^{1/2}\theta'(0) \tag{9}$$

When the mass flux is equal to zero then, the Sherwood number will also equals to 0 and

$$\text{Re}_x = \frac{U_w(x+y)}{\nu_f}$$

represents the Local Reynolds number.

Solution methodology

The series solutions which is from eqs. (6) and (7) have been established by taking the homotopy asymptotic method. We select:

$$f_o(\eta) = 1 - e^{-\eta},\ \theta_o(\eta) = \frac{1}{1+\gamma}e^{-\eta} \tag{10}$$

$$f_o(\eta) = 1 - e^{-\eta},\ \theta_o(\eta) = \frac{1}{1+\gamma}e^{-\eta} \tag{11}$$

The linear operators are:

$$L_f\left[D_1^* + D_2^* e^{\eta} + D_3^* e^{-\eta}\right] = 0,\; L_\Theta\left[D_7^* e^{\eta} + D_8^* e^{-\eta}\right] = 0 \tag{12}$$

where $D_i^*\left(i = 1\text{–}10\right)$ clarify the random constants. The problems have been constructed for zero$^{\text{th}}$ and m^{th} order distortion as given the aforementioned linear operators. The problems have been resolved with the help of MATHEMATICA.

Figure 1 illustrated the impact of nanoparticle volume fraction via velocity filed, from the obtained result we see that velocity profile is having inverse relation to the nanoparticle, that is the increasing value of nanoparticles volume friction decrees the velocity filed. Figure 2 showed the influence of permeability porous factor via velocity filed, from the obtained result we see that velocity profile is having inverse relation to the permeability porous parameter, that is the increasing value of permeability porous parameter decrees the velocity filed.

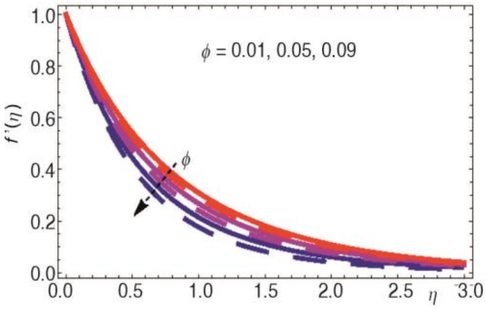

Figure 1. Impact of nanoparticle volume frication via velocity filed

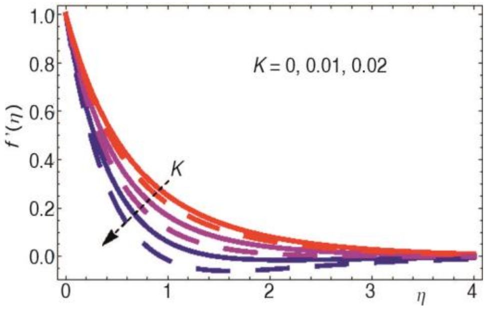

Figure 2. Impact of permeability porous factor via velocity filed

Figure 3 represented the influence of magnetic field factor via velocity filed, from the obtained result we see that velocity profile is having inverse relation to the magnetic field parameter, that is the increasing value of permeability porous factor decrees the velocity filed. Figure 4 showed the results of Eckert number on temperature filed, from fig 5 we see that temperature filed has direct relation to Eckert number, that is the increasing value of Eckert number increase the temperature filed.

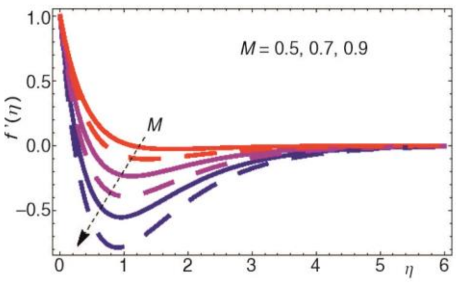

Figure 3. Influence of magnetic field parameter via velocity filed

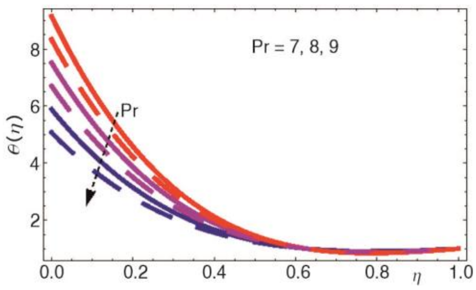

Figure 4. Illustration of Eckert number via temperature filed

Figure 5 showed the results of parental number on temperature filed, from fig. 5 we see that temperature filed has inverse relation to Prandtl number, that is the increasing value of Prandtl number decrease the temperature field.

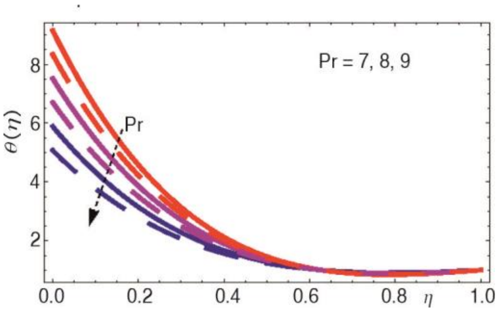

Figure 5. Influence of Prandtel number via temperature filed

Conclusion

This paper investigates analytical analysis of study flow of nanofluid, viscous dissipation with convective boundary condition. The key theme of this research is to increase the heat transfer ration to solve the energy feeding problem. To find the approximate solution we used homotopy asymptotic method for the non-Newtonian casson fluid model. The Impacts of different parameter which are obtained from velocity and temperature equations are presented in the form of graphs. The Influences of skin friction C_f is discus in the form of tabs. 1 and 2. The principal determined points are as follows.

Table 1. Influence skin friction $-\mathrm{Re}_x^{-1/2} C_{fx}$ via Ha, ω, Λ, and F_r when n = 1.3

Ha	ω	Λ	F_r	$-\mathrm{Re}_x^{-1/2} C_{fx}$
0.01	0.5	0.03	0.4	0.3172
0.03				0.4721
0.05				0.4981
	0.6			0.5312
	0.7			0.5753
	0.8			0.6432
		0.05		0.6982
		0.07		0.7123
		0.09		0.7766
			0.5	0.8100
			0.6	0.8932
			0.7	0.9921

Table 2. Influence of local Nusselt number $\mathrm{Re}_x^{-1/2}\,\mathrm{Nu}_x$ on different parameter when *n* = 1.2

Ha	ω	γ	Nt	Nb	S	Pr	β_T	$\mathrm{Re}_x^{-1/2}\,\mathrm{Nu}_x$
								Active control
0.01	0.05	0.1	0.10	0.70	0.35	1	0.10	0.3218
0.03								0.5478
0.05								0.5512
	0.06							0.5973
	0.07							0.6001
	0.08							0.6743
		0.2						0.6941
		0.3						0.7321
		0.4						0.7942
			0.15					0.8321
			0.20					0.9371
			0.25					0.9541
				0.75				0.9931
				0.80				1.2713
				0.85				1.3201
					0.40			1.4052
					0.45			1.4197
					0.50			1.4367
						2		1.4567
						3		1.4719
						4		1.4998
							0.20	1.5002

- By enhancing Magnetic field velocity is decreasing.
- By enhancing permeability parameter velocity filed is decreasing.
- By enhancing Eckert number temperature filed is increasing.
- By enhancing prandtl number temperature filed is decreasing.
- By enhancing nanoparticle volume friction parameter decreases the velocity profile.

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