A NOVEL AND UNIQUE APPROACH FOR DETERMINING THE SURVIVOR EQUIVALENCE FACTORS FOR COMPLEX SYSTEMS

by

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In this paper, we provide a novel method to determine the survivor equivalence factors for coherent systems, whatever the unit's lifetime distribution and the structure of the system. In order to determine the survivor equivalence factors for complex systems with independent units, we employ the idea of a survival signature and the ReliabilityTheory R package. Survival functions and mean times to failure are computed using the ReliabilityTheory package for systems were improved using: reduction technique and duplication techniques, including: hot duplication, cold duplication with reliable switch, and cold duplication with unreliable switch. Survival survivor equivalence factors and mean survivor equivalence factors are taken into consideration as measures for comparing system improvements. For the new survivor equivalence factors method to be understood, numerical example for complex system is provided.

Key words: *survivor equivalence factor, reduction strategy, duplication strategy, survival signature and ReliabilityTheory R package*

Introduction

In engineering, the duplication approach, which means adding further units in parallel with the existing system's units, can be used to improve system architecture. Three strategies can be used to expand the system: hot duplication, cold duplication with a reliable switch, and cold duplication with an unreliable switch. For a variety of reasons, including space restrictions and high costs, the redundancy duplication approach is unable to increase the performance of some systems. Examples of the architecture of an air space system include a costly unit and a small amount of space. The reduction approach, which entails increasing the system's performance by lowering the failure rate for particular system units by a factor v, where $v \in (0, 1)$, can be used to get around these restrictions.

Reliability equivalency factors were first discussed by [1, 2]. The majority of research on reliability equivalency factors assume systems with identical, distributed units and have certain structures [3-10]. Pogany *et al*. [11, 12] introduced the survivor equivalence to get more general and substantially simpler approach.

Samaniego gave a very thorough explanation of the idea of a signature including system signature theory and determining signatures for systems with few units in [13]. Coolen and Tahani [14] performed numerous extensions on the survival signature and signature. Aslett [15] developed a software tools in R to determine the survival signature, which are quite beneficial,

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particularly in systems with multiple units. A very good study on the survival signature for complex systems was recently presented by Aslett *et al*. [16].

In this study, we introduce a novel method that uses the notion of a survival signature to calculate the survivor equivalence factors for any system. As far as we are aware, this is the first attempt to determine the survivor equivalence factors for various systems using the concept of the survival signature. The system's structure and the lifetime distribution of each unit are the only pieces of information we require to use the survival signature to generate the survivor equivalence factors. Therefore, using the survival signature to generate the survivor equivalence factors is practical and suitable for use in real-world applications, which may provide significant advantages. Flexibility is the first feature of applying the survival signature to generate the survivor equivalence factors. It provides a fundamental approach that may be used for several system structures. It is possible to use the survival signature to generate survivor equivalence factors for systems with different structures and different units, which more closely resembles practical applications than all earlier research in this area. The second reason is that, as we will show in this work, there is a specialized computer package that makes it easy to derive survival signatures for any complicated systems.

In this paper, we consider studying a system with multiple different units and a complex structure. Firstly, using the ReliabilityTheory R package [15], we determine the survival function (SF) as well as the system's mean time to failure (MTTF) of the considered system. Secondly, we use the same software to determine the SF and MTTF of the system after optimizing the performance of its units based on reduction, hot duplication, and cold duplication (reliable and unreliable) strategies. Thirdly, we independently equate the SF and MTTF of the system that has been optimized using the reduction approach with the SF and MTTF, respectively, of the system that has been optimized using the duplication strategies, in order to generate the corresponding survivor equivalence factors. Finally, we use summary tables and figures to clarify the results obtained from an application example.

Survival signature

As previously mentioned, the survival signature was first proposed by Samaniego [13]. They looked at survival signatures for systems with identical units as well as systems with various unit kinds. The probability that a system will work if a certain number of its units are working is known as the survival signature.

For every coherent system that has n independent, identical units, where each unit has a continuous lifetime distribution. Let $\Psi(u)$ be the probability that the system will function if exactly u of its units are functional for $u = 0, 1, 2, ..., n.$ When every system unit fails the system will not work, which means $\Psi(u) = 0$ and when all system units are operating properly, the system should work, which means $\Psi(u) = 1$. For u units function which means u units with state $x_j = 1$, there are $\binom{n}{u}$ state vectors $\underline{x}$, so $\sum_{j=1}^{n} x_j = u.$ The set of these vectors will be referred as X_u. The following $\Psi(u)$ is a representation of the system survival signature:

$$\Psi(u) = \binom{n}{u}^{-1} \sum_{\underline{x} \in X_u} \psi(\underline{x}) \tag{1}$$

where $\psi(\underline{x})$ is the system stat for each X_u state vector.

For a system with several different kinds of units, Frank and his team considered a coherent system with n independent units are categorized into m type where type j has n_j identical units for $j = 1, 2,..., m$. Let $\Psi(u_1, u_2,..., u_m)$, for $u_j = 0, 1,..., n_j$ be the probability that the system will function if exactly u_j of its units of type j are functional, for $j = 1, 2,..., m$. For u_j units of type j function which means u_j of its n_j units with state $x_i^j = 1$, there are $n_j u_j$ statevectors $\underline{x}^j$, so $\sum_{i=1}^{n_j} x_j^i = u_j$. Let $X_{u1,u2,...,um}$ be the set of all system's state vectors for which $\sum_{i=1}^{n_j} x_j^i = u_j$, $j = 1, 2, 3,...,m$. Then the survival signature of such a system:

$$\Psi(u_1,u_2,u_3,...,u_m) = \left[\prod_{j=1}^{m}\binom{n_j}{u_j}^{-1}\right] \times \sum_{\underline{x}\in X_{u_1,...,u_m}} \psi(\underline{x}) \tag{2}$$

Standard system

Considering that there is an interconnected system consisting of n independent units divided into m different kinds, where kind j consists of n_j similar units for $j = 1, 2, 3,..., m$. The entire system has a total unit count of $\sum_{j=1}^{m} n_j = n$. The probability that the system will operate if exactly u_j of its units of type j operate at time x is defined as the survival signature of the system, which is denoted by $\Psi(u_1, u_2, u_3,..., u_m)$, for $u_j = 0, 1, 2,..., n_j$. Assuming that for every unit, the lifetime distribution and the survival function $S_j(x)$ are known for all units i ($i = 1, 2, 3,..., n_j$) of type $j = 1, 2, 33,..., m$. Then the system's survival function can be expressed according to [14, 16]:

$$S(x) = \sum_{u_1=0}^{n_1}...\sum_{u_m=0}^{n_m}\left[\Psi(u_1,...,u_m)\prod_{j=1}^{m}\left\{\binom{n_j}{u_j}[1-S_j(x)]^{n_j-u_j}[S_j(x)]^{u_j}\right\}\right] \tag{3}$$

Then, using the following formula, we can determine this system's mean time to failure:

$$MTTF = \int_0^\infty S(x)\mathrm{d}x = \int_0^\infty \left\{\sum_{u_1=0}^{n_1}...\sum_{u_m=0}^{n_m}\left[\Psi(u_1,...,u_m)\prod_{j=1}^{m}\left\{\binom{n_j}{u_j}[1-S_j(x)]^{n_j-u_j}[S_j(x)]^{u_j}\right\}\right]\right\}\mathrm{d}x \tag{4}$$

Structure of improved system

Reduction and standby redundancy are the two main ways to make a system better. The latter includes the hot duplication and cold duplication versions. Furthermore, reliable switch or unreliable switch can be used for cold duplication. Here, we get the survival function as well as the mean time to failure for a complex system that has been modified using the techniques mentioned previously. The system design that is improved using the reduction approach should be equivalent to the design of the system improved according to one of the redundancy strategies.

Reduction strategy

We indicated at the start of this research paper that the system's performance can be increased by scaling the hazard function for certain of the system's units by a factor $\upsilon \in (0, 1)$. The construction of the standard system is already known, as are the lifetime distribution of each unit of the standard system. Therefore, the performance of the system can be improved if we know the mechanism used to reduce the failure rate of some or all of the system's units. Many studies in this field have discussed the mechanism of reducing the failure rate of units with known lifetime distributions. The mechanism of reducing the failure rate of the basic lifetime distribution (Exponential lifetime distribution) has already been studied in many papers, including [1-5, 17, 18]. The mechanism of reducing the non-constant failure rate has been studied in some papers like Wiebull distribution in [6], gamma distribution in [7], exponentiated exponential distribution in [19], exponentiated Weibull distribution in [20], and Burr type X distribution in [21]. It is worth mentioning that all previous studies assumed the construction of a regular system and identical units with a known life distribution. In this study, we circumvent both of these limitations by utilizing a general framework.

Improvement plan for system units

For any coherent system composed of n independent units split into m different types, the survivor equivalence factors can be calculated for unit i (i = 1, 2, ..., n_j) of type j (j = 1, 2, ..., m) if the reduction improvement technique is known and specified for this unit. If the performance of q_j units of the system is improved based on the strategy of reduction, the number of types of systems in the improved system increases by one and becomes $m + 1$, for $q_j \in \{1,..., n_j - 1\}$. The system whose performance was developed based on the theory of reduction strategy has all the characteristics of the standard system, but with an extra type consisting of q_j units that replace the same number of units of type j. The survival function of the new optimized unit is written as $S_j^A(x)$.

Using the reduction approach, it is possible to derive the system's survival function and mean time to failure by applying the attributes of the optimized system described by eqs. (3) and (4), respectively.

Improvement plan for system types

When all units of type j are improved depending on the reduction strategy, the optimized system has all the characteristics of the standard system with the exception of swapping out the the survival function $S_j(x)$ of type j with $S_j^A(x)$. These steps are repeated with each improvement if there are multiple types of system units to be improved or if units of different types are being improved.

Duplication strategy

Here we want to show ways to get some of the survival measurements, like the survival function and the mean time to failure, for a complex system whose performance has been improved based on one of the strategies of duplication. We will calculate these measurements for a system whose performance has been improved based on a hot duplication strategy, a cold duplication strategy with a reliable switch, or a cold duplication strategy with an unreliable switch.

Hot duplication improvement

Improving the performance of the system depending on hot duplication means adding a similar unit in parallel to the original unit so that the two units work together.

For system unit improvement, we once more take into account a coherent system with n independent units split into m different types. Where the system structures and the unit's lifetime distribution as well as the survival function $S_j(x)$ are known for all unit i (i = 1, 2, ..., n_j) of type j (j = 1, 2, ..., m). Then the survival function of the unit whose performance has been optimized based on the hot duplication takes the form:

$$S_j^B(x) = 1 - \left\{1 - S_j(x)\right\}^2 \tag{5}$$

If the performance of q_j units of the system is improved based on the strategy of hot duplication, the number of unit types increases and becomes m +1, for $q_j \in \{1,..., n_j - 1\}$.

The system whose performance was developed based on hot duplication strategy has all the characteristics of the standard system, but with an extra type consisting of q_j units that replace the same number of units of type j. The number of units of type j has decreased and become $n_j - q_j$ instead of n_j. The survival function of the improved units in the improved system, which improved based on the strategy of hot duplication, is denoted by $S_j^B(x)$.

To improve the performance of certain types in the system, we improve the performance of all units of type j depending on hot duplication for $j = 1, 2, 3..., m$. The survival functions $S_j^B(x)$ of units whose performance has been optimized replace the survival functions $S_j(x)$ of the original units. These steps are repeated with each improvement if there are multiple types of system units to be improved or if units of different types are being improved.

Cold duplication with reliable switch improvement

This approach means that a comparable unit is linked to the original unit in such a way that when the original unit fails, it is immediately activated.

For system unit improvement, we once more take into account a coherent system with n independent units split into m different types. Where the system structures and the unit's lifetime distribution as well as the survival function $S_j(x)$ are known for all unit i ($i = 1, 2, 3,..., n_j$) of type j ($j = 1, 2, 3,..., m$). Regarding to a definition of cold duplication with reliable switch, this improvement can be compared to a renewal process that just requires one renewal [20]. Then the survival function of the unit whose performance has been optimized based on the cold duplication with reliable switch takes the form:

$$S_j^C(x) = 1 - \int_0^x \frac{-\mathrm{d}S_j(y)}{\mathrm{d}y} \left\{1 - S_j(x-y)\right\} \mathrm{d}y \tag{6}$$

If the performance of q_j units of the system is improved based on the strategy of cold duplication with reliable switch, the number of unit types increases and becomes $m + 1$, for $q_j \in \{1,..., n_j - 1\}$. The system whose performance was developed based on cold duplication with reliable switch has all the characteristics of the standard system, but with an extra type consisting of q_j units that replace the same number of units of type j. The number of units of type j has decreased and become $n_j - q_j$ instead of n_j. The survival function of the improved units in the improved system, which improved based on the strategy of cold duplication with a reliable switch, is denoted by $S_j^C(\mathrm{x})$.

To improve the performance of certain types in the system, we improve the performance of all units of type j depending on cold duplication strategy with reliable switch for j ($j = 1, 2, 3,..., m$). The survival functions $S_j^C(\mathrm{x})$ of units whose performance has been optimized replace the survival functions $S_j(x)$ of the original units. These steps are repeated with each improvement if there are multiple types of system units to be improved or if units of different types are being improved.

Cold duplication with unreliable switch improvement

This strategy to improve system performance means that by using a random switch with a constant rate of failure, a comparable unit is connected to the original unit by a cold standby in order to increase system performance.

For system unit improvement, we once more take into account a coherent system with n independent units split into m different types. Where the system structures and the unit's lifetime distribution as well as the survival function $S_j(x)$ are known for all unit i ($i = 1, 2, ..., n_j$)

of type j (j = 1, 2, ..., m). Regarding the definition of cold duplication with an unreliable switch, this improvement can be achieved using the same method we employed for cold duplication with a reliable switch, but with the additional condition that the switch is not completely perfect [22]. Then the survival function of the unit whose performance has been optimized based on the cold duplication with unreliable switch takes the form:

$$S_j^D(x) = 1 - \int_0^x \frac{-\mathrm{d}S_j(y)}{\mathrm{d}y}\left\{1 - S_j(x-y)s(y)\right\}\mathrm{d}y \tag{7}$$

where $s(y)$ represents the unreliable switch's survival function. The lifetime distribution of an unreliable switch was chosen to be exponential with a constant failure rate μ and survival function $s(y) = \mathrm{e}^{-\mu y}$.

If the performance of q_j units of the system is improved based on the strategy of cold duplication with unreliable switch, the number of unit types increases and becomes $m + 1$ for $q_j \in \{1,..., n_j - 1\}$. The system whose performance was developed based on cold duplication with unreliable switch has all the characteristics of the standard system, but with an extra type consisting of q_j units that replace the same number of units of type j. The number of units of type j has decreased and become $n_j - q_j$ instead of n_j. The survival function of the improved units in the improved system, which improved based on the strategy of cold duplication with a unreliable switch, is denoted by $S_j^D(x)$.

Survivor equivalence factors

The survivor equivalence factor is a factor that must be multiplied by a unit characteristic of a system design in order to achieve equality with a better design. Equivalence between several system designs in terms of a reliability characteristic like the survival function and the mean time to failure, [4].

This study computes two reliability equivalence factor measurements. In the first, survival survivor equivalence factors (SSEF) are calculated using the survival function. In the second, the mean time to failure is used to calculate the mean survivor equivalence factors (MSEF).

Survival survivor equivalence factors

To calculate the SSEF, the following set of equations must be resolved:

$$S_R(x) = S_D(x) = \eta \tag{8}$$

where $S_R(x)$ is the survival function of the system whose performance has been improved based on the reduction strategy and $S_D(x)$ – the survival function of the system whose performance has been improved based on one of the redundancy strategies.

Mean survivor equivalence factors

To calculate the MSEF, the following set of equations must be resolved:

$$\mathrm{MTTF}_R = \mathrm{MTTF}_D \tag{9}$$

where MTTF_R is the mean time to failure of the system whose performance has been improved based on the reduction strategy and MTTF_D – the mean time to failure of the system whose performance has been optimized based on one of the redundancy strategies.

Numerical results and analysis

To clarify the mechanism of using the previous theoretical part in real applications, we apply the aforementioned theory to a coherent system of eleven independent units classified into four types, where the connections between system units are completely perfect. Figure 1 displays the construction form, and tab. 1 lists the properties of the studied system. The steps to determining the survival signature for this system have already been carefully studied by Aslett *et al*. [16].

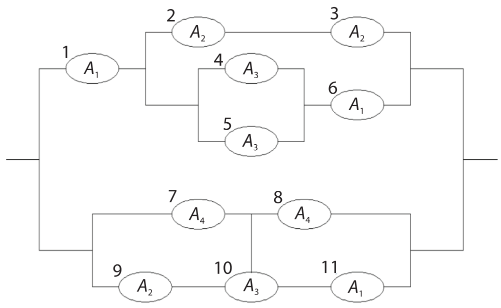

Figure 1. The considered system with eleven units divided into four different types

Table 1. Properties of the considered system

System unit types	lifetimes of system units	Improved technique for reduction
$A_1 = \{1, 6, 11\}$	A_1 ~ Exponential ($\lambda = 0.56$)	$S_1^{(A)}(x) = e^{-\upsilon\lambda x}$ [3, 4]
$A_2 = \{2, 3, 9\}$	A_2 ~ Weibull ($\beta = 0.273$, $\alpha = 2.21$)	$S_2^{(A)}(x) = e^{-\upsilon\beta x\alpha}$ [6]
$A_3 = \{4, 5, 10\}$	A_3 ~ Exponential Weibull ($\beta = 0.112$, $\alpha = 2.1$, $\theta = 1.21$)	$S_3^{(A)}(x) = 1 - (1 - e^{-\upsilon\beta x})^{\theta}$ [20]
$A_4 = \{7, 8\}$	A_4 ~ Gamma ($\alpha = 3.1$, $\lambda = 1.12$)	$S_4^{(A)}(x)= \int_x^{\infty}[(\upsilon\lambda)^{\alpha} x^{\alpha-}1/ \Gamma\alpha]$ e−υλxdx [7]

Table 2. The hot SSEF for units of the considered system

Unit	$\eta = 0.1$	$\eta = 0.5$	$\eta = 0.9$
1	0.6858	0.5690	0.4419
2	0.8504	0.6289	0.3439
3	0.8504	0.6289	0.3439
4	0.6003	0.3552	0.1615
5	0.6003	0.3552	0.1615
6	0.6858	0.5680	0.4333
7	0.7675	0.6458	0.4936
8	0.7655	0.6414	0.4989
9	0.8455	0.6407	0.3634
10	0.6041	0.3656	0.1690
11	0.6855	0.5691	0.4394

Table 3. The hot MSEF and MTTF for units of the considered system

Unit	MSEF	MTTF
1	0.5726	2.4088
2	0.5427	2.3482
3	0.5427	2.3482
4	0.5431	2.3483
5	0.5431	2.3483
6	0.5942	2.3920
7	0.6985	2.6179
8	0.6956	2.5993
9	0.6176	2.3856
10	0.4459	2.3877
11	0.5748	2.4045

Table 4. The hot SSEF for types of the considered system

Type	$\eta = 0.1$	$\eta = 0.5$	$\eta = 0.9$
A_1	0.6941	0.5863	0.4659
A_2	0.8456	0.6453	0.3795
A_3	0.6086	0.3679	0.1693
A_4	0.7907	0.6848	0.5571

Table 5. The hot MSEF$_s$ and MTTF$_s$ for types of the considered system

Type	MSEF	MTTF
A_1	0.5983	2.5502
A_2	0.6075	2.4054
A_3	0.4761	2.4009
A_4	0.7183	2.9942

Table 7. The cold MSEF with reliable switch and MTTF for units of the considered system

Unit	MSEF	MTTF
1	0.3664	2.4600
2	0.2646	2.3569
3	0.2646	2.3569
4	0.1543	2.3605
5	0.1543	2.3605
6	0.3817	2.4349
7	0.3939	2.9376
8	0.3951	2.8895
9	0.4744	2.4140
10	0.1182	2.4251
11	0.3670	2.4531

Table 9. The cold MSE with reliable switch and MTTF for types of the considered system

Type	MSEF	MTTF
A_1	0.3966	2.7653
A_2	0.2414	2.5660
A_3	0.1270	2.4491
A_4	0.4508	4.5310

Table 11. The cold MSEF with unreliable switch and MTTF for units of the considered system

Unit	MSEF	MTTF
1	0.3904	2.4532
2	0.3364	2.3543
3	0.3364	2.3543
4	0.2124	2.3583
5	0.2124	2.3583
6	0.4067	2.4291
7	0.4507	2.8800
8	0.4514	2.8369
9	0.5896	2.3905
10	0.1795	2.4177
11	0.3912	2.4467

Table 6. The cold SSEF with reliable switch for units of the considered system

Unit	$\eta = 0.1$	$\eta = 0.5$	$\eta = 0.9$
1	0.4521	0.3556	0.2637
2	0.3924	0.2693	0.1061
3	0.3924	0.2693	0.1061
4	0.1429	0.0685	0.0271
5	0.1429	0.0685	0.0271
6	0.4521	0.3544	0.2550
7	0.4361	0.3327	0.2261
8	0.4339	0.3261	0.2295
9	0.4021	0.2998	0.1193
10	0.1468	0.0723	0.0287
11	0.4516	0.3558	0.2612

Table 8. The cold SSEF with reliable switch for types of the considered system

Type	$\eta = 0.1$	$\eta = 0.5$	$\eta = 0.9$
A_1	0.4709	0.3830	0.2928
A_2	0.6147	0.3164	0.1325
A_3	0.1501	0.0730	0.0288
A_4	0.5128	0.4260	0.3276

Table 10. The cold SSEF with unreliable switch for units of the considered system

Unit	$\eta = 0.1$	$\eta = 0.5$	$\eta = 0.9$
1	0.4810	0.3800	0.2828
2	0.4361	0.3455	0.1671
3	0.4361	0.3455	0.1671
4	0.2085	0.1390	0.0866
5	0.2085	0.1390	0.0866
6	0.4810	0.3787	0.2740
7	0.4845	0.4071	0.3427
8	0.4826	0.4131	0.3446
9	0.4415	0.3732	0.1805
10	0.2115	0.1428	0.0889
11	0.4805	0.3801	0.2803

Table 12. The cold SSEF with unreliable switch for types of the considered system

Type	$\eta = 0.1$	$\eta = 0.5$	$\eta = 0.9$
A_1	0.4987	0.4066	0.3118
A_2	0.7301	0.3874	0.1930
A_3	0.2142	0.1435	0.0890
A_4	0.5430	0.4658	0.3891

Table 13. The cold MSEF with unreliable switch and MTTF for unit types of the considered system

Type	MSEF	MTTF
A_1	0.4207	2.7330
A_2	0.2811	2.5374
A_3	0.1880	2.4407
A_4	0.4919	4.1762

The SSEF were found using a special software package in *R* called *ReliabilityTheor* based on the aforementioned equations for all units whose performance was improved according to the previous equations for all units whose performance was improved according to hot and cold (reliable and unreliable) duplication. The outcomes when η was set to 0.1. 0.50, and 0.9 are presented in tabs. 1, 5, 9. The constant failure rate has been chosen to be $\mu = 0.05$ for an unreliable switch. With the same mechanism, it was found and calculated the SSEF for unit types and are presented in tabs. 4, 8, and 12. For further clarification regarding the results presented in the those tables, It is possible to see that:

– Lowering unit number one's failure rate by setting $\upsilon = 0.6858$ increases the system's performance like adding a second unit in parallel with unit 1 based on the hot duplication strategy where the system's survival function is decided to be $\eta = 01$, tab. 2.
– Lowering type one's failure rate A_1 by setting $\upsilon = 0.3830$ increases the system's performance like adding extra unit in parallel according to a cold duplication strategy with reliable switch, to each unit in type A_1 where the system's survival function is decided to be $\eta = 0.5$, tab. 8.
– Similar to that, the other outcomes shown in those tables can also be interpreted.

The MSEF and MTTF for the system units are presented in tabs. 3, 7, and 11, for hot and cold (reliable and unreliable) duplication. Tables 5, 9, and 13 present the MSEF for unit types. For further clarification regarding the results presented in the those tables, It is possible to see that:

– The system mean time to failure increases from 2.3395 to 24088 by improving unit number 1 through hot duplication, and the same result can be obtained by lowering the same unit's failure rate by setting $\upsilon = 0.5726$, tab. 3.
– The best unit improvement is achieved by improving the performance of unit number 7, tabs. 3, 7, and 11.
– The second best unit improvement is achieved by improving the performance of unit number 8, tab. 3, 7, and 11.
– Whether unit 2 or unit 3 is improved, the result is the same as for unit 4 or unit 5, tabs. 3, 7, and 11.
– The best type improvement is achieved by improving the performance of each unit of type A_4, tab. 5, 9, and 13.
– Similar to that, the other outcomes shown in those tables can also be interpreted.

The survival functions of some modified and standard systems are presented in figs. 2. From this figure, it may be observed that:

– The optimum unit for hot or cold duplication is unit number 7.
– The second optimum unit for hot or cold duplication is unit number 8, then unit number 1, fig. 2(b) and tabs. 3, 7, and 9 for comparison.
– The optimum type for hot or cold duplication is type A_4, then type A_1.

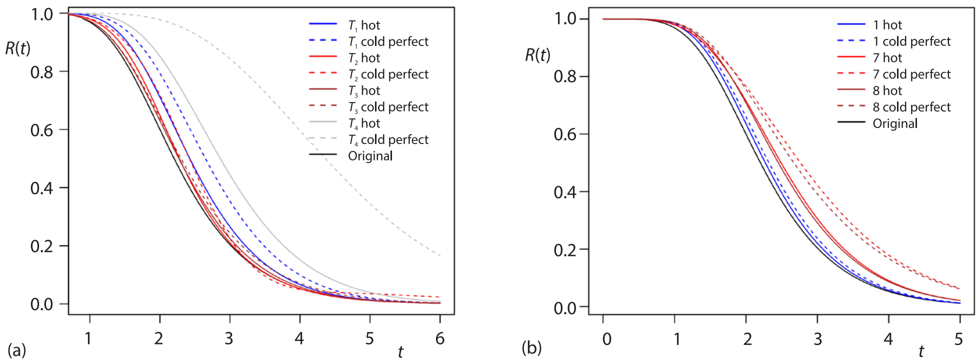

Figure 2. The survival functions of the system under consideration, both in their basic and modified forms; (a) types development and (b) units development

Figures 3 and 4 show how MTTF behaves when the failure rate is reduced by factor υ, whether it is for system units or for system types. According to these two figures, it may be observed that:

– By decreasing υ for all possible reduction improvements, the MTTF are non-decreasing.
– The best reduction unit is achieved by lowering the failure rate of unit 7, fig. 3(a).
– The same improvement results from decreasing the failure rate of any unit 2, or unit 3, similarly for unit 4 or unit 5. Figure 3(b) and tabs. 3, 7, and 11 can be compared.
– By decreasing the failure rate of all units of type A_4, we get the best type that can be improved. It leads to a huge development of the system's mean time to failure, fig. 4(a).
– The worst reduction type improvement is obtained while lowering the type A_3 failure rate, fig. 4(b).

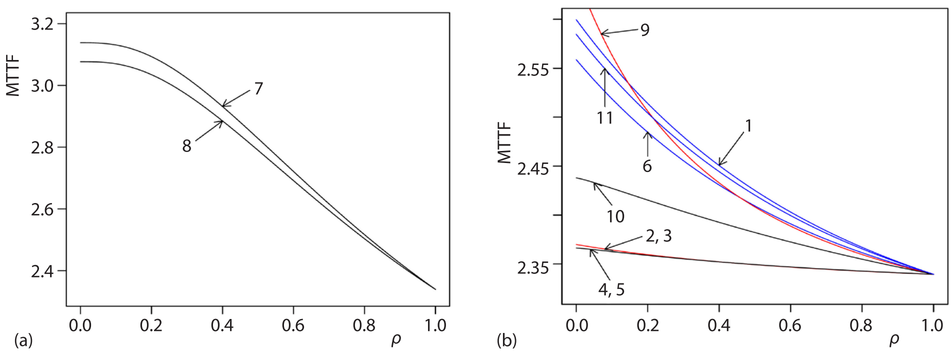

Figure 3. The mean time to failure behavior *vs.* υ, for the units of the considered system; (a) 7th and 8th units and (b) other units

Conclusion

To our knowledge, this study is the first attempt to compute the survivor equivalence factors and mean survivor equivalence factors for a variety of systems and systems with a variety of unit types using the survival signature. In addition examining its theoretical properties, this method may be useful for increasing system performance in a variety of application fields, including economics, management, manufacturing, government, service, health care, engineering, and others, at a reasonable cost. In this study, we focus on particular examples to show how widely applicable this strategy and to demonstrate the many benefits and the kinds of inferences that it might provide in real-world situations.

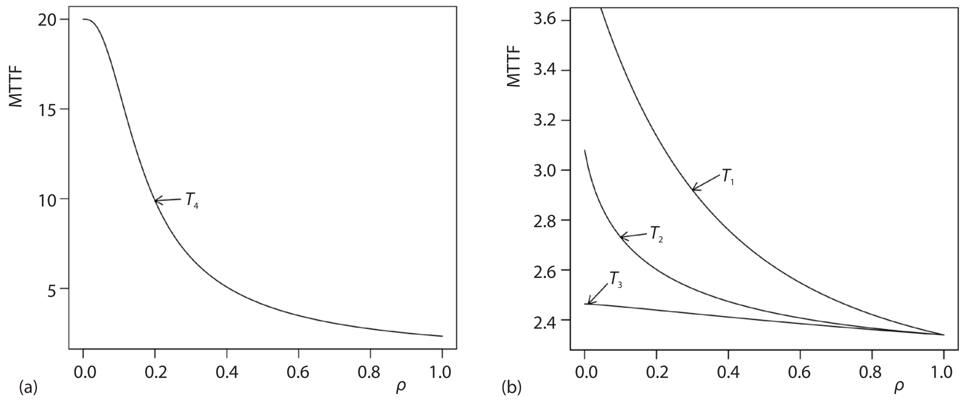

Figure 4. The mean time to failure behavior *vs.* υ, for the units types of the considered system; (a) 4th type and (b) other types

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