EFFECT OF STARK SHIFT ON THREE-LEVEL ATOM INTERACTING WITH A CORRELATED TWO-MODE OF NON-LINEAR COHERENT STATE

by

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In this manuscript, we study a new non-linear system which represents a Ξ-type five level atom interacting with a quantized four-mode electromagnetic field. A non-linear Stark shift is introduced, through the elimination of intermediate levels (two and four) using the adiabatic elimination technique. By using the Schrodinger equation we obtain the analytic solution this model. Some statistical aspects through the effective Hamiltonian are presented such as the collapses-revivals phenomenon, degree of purity, concurrence, and the squeezing phenomenon with respect to study the effect of Stark shift parameters on these statistical aspects. For small values of the Stark shift parameter, the collapse times increase and the atomic inversion symmetry axis shifts upward, while the entanglement decreases significantly. It has been noted that the system is affected by the Stark shift parameter.

Key words: *Stark shift, pair coherent state, collapses-revivals phenomenon, purity, squeezing phenomenon*

Introduction

The Stark shift (SS) effect results in the separation of energy levels in constant and a homogeneous external electric field that has been considered a classic problem for a long time. The entanglement behavior between the parts of a cavity quantum system containing a bi-level atom interacting with a two-photon field was studied using the von Neumann entropy formula [1]. However, the interaction was studied and then results were obtained neglecting the effect of the Stark shift (ESS), [2]. It is known that a field with two photons is approximate to experimental achievement, so one has to taken into account the ESS dynamic on the overall evaluation of the field. The effect of the Kerr-like medium on the von Neumann entropy formula was studied in [3, 4]. A 2-level atom can be practically realized in a quantum electrodynamic cavity as an effective interaction as in the case of cold trapped laser ions. During previous studies, the study of the two-photon field interacting with atoms inside a cavity has been widely admired by scholars, especially those interested in theoretical and experimental physics, which was achieved by the experimental investigation of the two-photon cascading microwave maser [5]. The two-photon field is instrumental in forming the squeezed states of the electromagnetic field.

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It has also been demonstrated that the interaction of identical atoms with a two-photon field within a Kerr-like medium can create a squeezed amplification state [6].

The ESS on a quantum system is considered one of the most important phenomena in atomic physics, because of its importance in the process of measuring the amount of entanglement between different quantum systems [7]. It is the split and shift of a spectral line into many components under the influence of an electric field. The amount of splitting is called SS [8]. It is considered to be an important role in the dynamics of the reaction. The effect of the dynamics of the ESS on a physical model governed by a laser beam can be observed. Multi-photon transition is more realistic and accurate. The importance of SS is clear in describing a 3-level atom with a 2-level one after using the adiabatic elimination (AE) method [9]. In this situation the SS formalism is explained by adiabatically remove the middle levels of a multi-level atom (the second level) [10]. The behavior of the resulting effective model was studied after deleting the second level of the three-level atom, which becomes containing the SS terms [11]. Its effect has been studied on some phenomena in different quantum systems. The problem of two-photon atomic transitions in the presence of a squeezed cavity field and SS has been studied where ESS on some quantities has been discussed [12].

The ESS on atomic-dipole squeezing in two-photon for an atom started the interaction from a superposition coherent states has been studied [13]. Also, the influence of this shift on the amount of the correlation between the quantum system has been investigated in the two-photon [14]. The ESS was examined for the interaction between two 2-level atoms and a single-mode radiation field [15]. The behavior of the correlation between a bi-level atom and a bi-modal entangled field is studied, in which the effects of a Kerr-like medium and a SS on Wehrl entropy and field purity are discussed [16]. Fisher quantum information dynamics and entanglement for multi-level atomic models interacting with a single-mode for the cavity field under the ESS and Kerr medium are considered in [17]. Influence of SS and Kerr-like medium have been studied for determine the amount of entanglement between the field and the two three-level atomic systems [18]. Also, the effect of both the SS and Kerr-like mean on the entanglement between two three-level atoms was studied. Continuing the development process, the entanglement behavior between a multi-level atomic system (N-levels) was studied, considering the effect of star-shift and Kier-like medium [19].

The aim of this paper is to deduce ESS on five-level atomic system, which was neglected in previous studies, therefore, we study this effect on some statistical aspects.

Description of the physical system and its solution

The model consists of a five-level atom in the Ξ construction levels with $\omega_1 > \omega_2 > \omega_3 > \omega_4 > \omega_5$ interacting with four modes of radiation electromagnetic field. The quantum model is proposed ($\hbar = 1$):

$$\hat{H} = \sum_{i=1}^{4}\Omega_i \hat{n}_i + \sum_{j=1}^{5}\omega_j \hat{\sigma}_{jj} + \lambda_1\left(\hat{a}_1\hat{\sigma}_{12} + \hat{a}_1^{\dagger}\hat{\sigma}_{21}\right) +$$
$$+\lambda_2\left(\hat{a}_2\hat{\sigma}_{23} + \hat{a}_2^{\dagger}\hat{\sigma}_{32}\right) + \lambda_3\left(\hat{a}_3\hat{\sigma}_{34} + \hat{a}_3^{\dagger}\hat{\sigma}_{43}\right) + \lambda_4\left(\hat{a}_4\hat{\sigma}_{45} + \hat{a}_4^{\dagger}\hat{\sigma}_{54}\right) \tag{1}$$

where Ω_i ($i = 1,..., 4$) is the frequency of the i mode and ω_j($j = 1,..., 5$) – the frequencies of the atomic levels. The creation (annihilation) operator $\hat{a}_j^{\dagger}$ ($\hat{a}_j$) governed by the Boson commutation relations $[\hat{a}_i, \hat{a}_j^{\dagger}] = \delta_{ij}$, and the operators $\hat{\sigma}_{ij}$ satisfy the commutation equation:

$$\left[\hat{\sigma}_{ij}, \hat{\sigma}_{kl}\right] = \hat{\sigma}_{il}\delta_{kj} - \hat{\sigma}_{kj}\delta_{il} \tag{2}$$

where λ_i, ($i = 1,..., 4$) is the dipole coupling constant.

The SS terms are derived in this quantum system. By using the AE technique [9, 20], the second and fourth levels are eliminated and the atomic system reduced to three levels as can be seen in the following.

We introduce the following slowly varying operators in order to eliminate the intermediate levels adiabatically [9, 20]:

$$\hat{Q}_{kl} = \hat{\sigma}_{kl}\left\{\exp\left[i(\omega_l - \omega_k)t\right]\right\}, \quad k,l = 1,....,5 \tag{3}$$

and

$$\hat{A}_j = \hat{a}_j\{\exp[i\Omega_j t]\}, \ (j = 1,...,4) \tag{4}$$

The time-dependent differential equations that derive from the quantum system are deduced from the Heisenberg relation:

$$i\frac{\mathrm{d}\hat{Q}_{12}}{\mathrm{d}t} = \lambda_1 \hat{A}_1^\dagger \mathrm{e}^{i\Delta_1 t}\left(\hat{Q}_{11} - \hat{Q}_{22}\right) + \lambda_2 \hat{A}_2 \mathrm{e}^{-i\Delta_2 t}\hat{Q}_{13} \tag{5}$$

$$i\frac{\mathrm{d}\hat{Q}_{22}}{\mathrm{d}t} = \lambda_1\left[-\hat{A}_1 \mathrm{e}^{-i\Delta_1 t}\hat{Q}_{12} + \hat{A}_1^\dagger \mathrm{e}^{i\Delta_1 t}\hat{Q}_{21}\right] + \lambda_2\left[\hat{A}_2^\dagger \mathrm{e}^{i\Delta_2 t}\hat{Q}_{23} - \hat{A}_2^\dagger \mathrm{e}^{i\Delta_2 t}\hat{Q}_{32}\right] = i\frac{\mathrm{d}\hat{S}_{22}}{\mathrm{d}t} \tag{6}$$

$$\begin{aligned} i\frac{\mathrm{d}\hat{Q}_{23}}{\mathrm{d}t} &= \lambda_2 \hat{A}_2^\dagger \mathrm{e}^{i\Delta_2 t}\left(\hat{Q}_{22} - \hat{Q}_{33}\right) - \lambda_1 \hat{A}_1 \mathrm{e}^{-i\Delta_1 t}\hat{Q}_{13} + \lambda_3 \hat{A}_3 \mathrm{e}^{-i\Delta_3 t}\hat{Q}_{24} \\ i\frac{\mathrm{d}\hat{Q}_{33}}{\mathrm{d}t} &= \lambda_2\left[-\hat{A}_2 \mathrm{e}^{-i\Delta_2 t}\hat{Q}_{23} + \hat{A}_2^\dagger \mathrm{e}^{i\Delta_2 t}\hat{Q}_{32}\right] + \lambda_3\left[\hat{A}_3 \mathrm{e}^{-i\Delta_3 t}\hat{Q}_{34} - \hat{A}_3^\dagger \mathrm{e}^{i\Delta_3 t}\hat{Q}_{43}\right] = i\frac{\mathrm{d}\hat{S}_{33}}{\mathrm{d}t} \end{aligned} \tag{7}$$

$$\begin{aligned} i\frac{\mathrm{d}\hat{Q}_{34}}{\mathrm{d}t} &= -\lambda_2 \hat{A}_2 \mathrm{e}^{-i\Delta_2 t}\hat{Q}_{24} + \lambda_3 \hat{A}_3^\dagger \mathrm{e}^{i\Delta_3 t}(\hat{Q}_{33} - \hat{Q}_{44}) + \lambda_4 \hat{A}_4 \mathrm{e}^{-i\Delta_4 t}\hat{Q}_{35} \\ i\frac{\mathrm{d}\hat{Q}_{44}}{\mathrm{d}t} &= \lambda_3\left[-\hat{A}_3 \mathrm{e}^{-i\Delta_3 t}\hat{Q}_{34} + \hat{A}_3^\dagger \mathrm{e}^{i\Delta_3 t}\hat{Q}_{43}\right] + \lambda_4\left[\hat{A}_4 \mathrm{e}^{-i\Delta_4 t}\hat{Q}_{45} - \hat{A}_4^\dagger \mathrm{e}^{i\Delta_4 t}\hat{Q}_{54}\right] = i\frac{\mathrm{d}\hat{S}_{44}}{\mathrm{d}t} \end{aligned} \tag{8}$$

$$i\frac{\mathrm{d}\hat{Q}_{45}}{\mathrm{d}t} = \lambda_4 \hat{A}_4^\dagger e^{i\Delta_4 t}\left(\hat{Q}_{44} - \hat{Q}_{55}\right) - \lambda_3 \hat{A}_3 e^{-i\Delta_3 t}\hat{Q}_{35} \tag{9}$$

with

$$\Delta_i = (\omega_i - \omega_{i+1}) - \Omega_i, \quad (i = 1,..,4) \tag{10}$$

Next, we adiabatically elimination [21] the states $|2\rangle$ and $|4\rangle$ by integrating the equations $\hat{Q}_{12}$, $\hat{Q}_{23}$, $\hat{Q}_{34}$, and $\hat{Q}_{45}$ formally. The problem is addressed by assuming that the actuators in a quantum system change very slowly with respect to time:

$$\begin{aligned} i\frac{\mathrm{d}\hat{S}_{22}}{\mathrm{d}t} &= \lambda_1\lambda_2 \hat{A}_1^\dagger \hat{A}_2^\dagger \mathrm{e}^{i(\Delta_1+\Delta_2)t}\hat{Q}_{31}\left(\frac{1}{\Delta_1} + \frac{1}{\Delta_2}\right) - h.c + \lambda_2\lambda_3 \hat{A}_2^\dagger \hat{A}_3^\dagger \mathrm{e}^{i(\Delta_2+\Delta_3)t}\hat{Q}_{42}\left(\frac{-1}{\Delta_1}\right) - h.c \\ i\frac{\mathrm{d}\hat{S}_{33}}{\mathrm{d}t} &= \lambda_1\lambda_2 \hat{A}_1^\dagger \hat{A}_2^\dagger \mathrm{e}^{i(\Delta_1+\Delta_2)t}\hat{Q}_{31}\left(\frac{-1}{\Delta_1}\right) - h.c + \lambda_2\lambda_3 \hat{A}_2^\dagger \hat{A}_3^\dagger \mathrm{e}^{i(\Delta_2+\Delta_3)t}\hat{Q}_{42}\left(\frac{1}{\Delta_2} + \frac{1}{\Delta_3}\right) - h.c + \\ &\quad + \lambda_3\lambda_4 \hat{A}_3^\dagger \hat{A}_4^\dagger \mathrm{e}^{i(\Delta_3+\Delta_4)t}\hat{Q}_{53}\left(\frac{-1}{\Delta_4}\right) - h.c \end{aligned} \tag{11}$$

$$i\frac{\mathrm{d}\hat{S}_{44}}{\mathrm{d}t}=\lambda_2\lambda_3\hat{A}_2^{\dagger}\hat{A}_3^{\dagger}\mathrm{e}^{i(\Delta_2+\Delta_3)t}\hat{Q}_{42}\left(\frac{-1}{\Delta_2}\right)-h.c+$$

$$+\frac{\lambda_3\lambda_4}{\Delta_4}\hat{A}_3^{\dagger}\hat{A}_4^{\dagger}\mathrm{e}^{i(\Delta_3+\Delta_4)t}\hat{Q}_{53}\left(\frac{1}{\Delta_4}+\frac{1}{\Delta_3}\right)-h.c \tag{12}$$

Through the previous equations, we find that AE condition is deduced

$$\Delta_1=-\Delta,\Delta_2=\Delta,\ \Delta_3=-\Delta\ \text{ and }\ \Delta_4=\Delta$$

we note that:

$$i\left(\frac{\mathrm{d}\hat{S}_{22}}{\mathrm{d}t}+\frac{\mathrm{d}\hat{S}_{44}}{\mathrm{d}t}\right)=0 \tag{13}$$

This mean that the system of a five-level atom can be described by an effective system of a three-level atom. Thus, the effective system of the whole Hamiltonian takes the form:

$$H_{\text{eff}}=\sum_{i=1}^{4}\Omega_i\hat{n}_i+\sum_{j}\omega_j\hat{\sigma}_{jj}+\beta_1\hat{n}_1\hat{\sigma}_{11}+\left(\beta_2\hat{n}_2+\beta_3\hat{n}_3\right)\hat{\sigma}_{33}+$$

$$+\beta_4\hat{n}_4\hat{\sigma}_{55}+\sqrt{\beta_1\beta_2}\left(\hat{a}_1\hat{a}_2\hat{\sigma}_{13}+\hat{a}_1^{\dagger}\hat{a}_2^{\dagger}\hat{\sigma}_{31}\right)+\sqrt{\beta_3\beta_4}\left(\hat{a}_3\hat{a}_4\hat{\sigma}_{35}+\hat{a}_3^{\dagger}\hat{a}_4^{\dagger}\hat{\sigma}_{53}\right) \tag{14}$$

where

$$\beta_j=\frac{\lambda_j^2}{\left(\frac{\Delta}{2}\right)},\ (j=1,...,4),\ g_1=\sqrt{\beta_1\beta_2},\ \text{and}\ g_2=\sqrt{\beta_3\beta_4}$$

Now, we use this effective Hamiltonian with the new SS terms to study the dynamics of the system. For simplicity, we put $\hat{n}_1=\hat{n}_3$ and $\hat{n}_2=\hat{n}_4$, thus the effective Hamiltonian (14) can be cast in the form:

$$H_{\text{eff}}=\sum_{i=1}^{2}\Omega_i\hat{n}_i+\sum_{e,i,g}\omega_j\hat{\sigma}_{jj}+\beta_1\hat{n}_1\hat{\sigma}_{ee}+(\beta_2\hat{n}_2+\beta_3\hat{n}_1)\hat{\sigma}_{ii}+$$

$$+\beta_4\hat{n}_2\hat{\sigma}_{gg}+g_1\left(\hat{a}_1\hat{a}_2\hat{\sigma}_{ei}+\hat{a}_1^{\dagger}\hat{a}_2^{\dagger}\hat{\sigma}_{ie}\right)+g_2\left(\hat{a}_1\hat{a}_2\hat{\sigma}_{ig}+\hat{a}_1^{\dagger}\hat{a}_2^{\dagger}\hat{\sigma}_{gi}\right) \tag{15}$$

The initial conditions of a quantum system are taken:

$$|\varphi(0)\rangle=|\varphi_{\text{atom}}(0)\rangle\otimes|\varphi_{\text{field}}(0)\rangle \tag{16}$$

where, the atom begins from the ex-cited state $|\varphi_{\text{fatom}}(0)\rangle=|e\rangle$ and the field begins from the state $|\varphi_{\text{field}}(0)\rangle=|\varepsilon,q\rangle$. The $|\varepsilon,q\rangle$ represents the pair entangled state, it gives [22]:

$$|\varepsilon,q\rangle=N_q\sum_{l=0}^{\infty}\frac{\varepsilon^l}{\sqrt{l!(q+l)!}}|l+q,l\rangle \tag{17}$$

with

$$N_q=\left[\sum_{l=0}^{\infty}\frac{|\varepsilon|^{2l}}{l!(l+q)!}\right]^{-1/2} \tag{18}$$

where q is the integer positive number and ε – the may be a complex number. The wave function in the general case that describes the proposed system can be formulated:

$$|\varphi(t)\rangle = \sum_{l=0}^{\infty}\left[U_1(l,t)|e,q+l,l\rangle + U_2(l,t)|i,q+l+1,l+1\rangle + U_3(l,t)|g,q+l+2,l+2\rangle\right]$$

where $U_i(l, t)$, $(i = 1, 2, 3)$ denote statistical expressions that verify the relation

$$\sum_{i=1}^{3}|U_i(l,t)|^2 = 1$$

By using the Schrodinger equation, the following system of differential equations is obtained:

$$i\frac{\mathrm{d}}{\mathrm{d}t}U_1(l,t) = \alpha_1 U_1(l,t) + \kappa_1 U_2(l,t) \tag{19}$$

$$i\frac{\mathrm{d}}{\mathrm{d}t}U_2(l,t) = \kappa_1 U_1(l,t) + \alpha_2 U_2(l,t) + \kappa_2 U_3(l,t) \tag{20}$$

$$i\frac{\mathrm{d}}{\mathrm{d}t}U_3(l,t) = \kappa_2 U_2(l,t) + \alpha_3 U_3(l,t) \tag{21}$$

where

$$\kappa_1 = \lambda_1\sqrt{(q+l+1)(l+1)} \tag{22}$$

$$\kappa_2 = \lambda_2\sqrt{(q+l+2)(l+2)} \tag{23}$$

and

$$\alpha_1 = \Omega_1(q+l) + \Omega_2 l + \omega_e + \beta_1(q+l) \tag{24}$$

$$\alpha_2 = \Omega_1(q+l+1) + \Omega_2(l+1) + \omega_i + \beta_2(l+1) + \beta_3(q+l+1) \tag{25}$$

$$\alpha_3 = \Omega_1(q+l+2) + \Omega_2(l+2) + \omega_g + \beta_4(l+2) \tag{26}$$

Now, we obtain the reduced atomic density operator $\hat{x}_{\text{atom}}(t)$:

$$\hat{\chi}_{\text{atom}}(t) = Tr_{\text{field}}\,|\varphi(t)\rangle\langle\varphi(t)|$$

$$\hat{\chi}_{\text{atom}}(t) = \begin{pmatrix} \chi_{ee}(t) & \chi_{ei}(t) & \chi_{eg}(t) \\ \chi_{ie}(t) & \chi_{ii}(t) & \chi_{ig}(t) \\ \chi_{ge}(t) & \chi_{gi}(t) & \chi_{gg}(t) \end{pmatrix} \tag{27}$$

where

$$\chi_{ee}(t) = \sum_{l=0}^{\infty}|U_1(l,t)|^2,\ \chi_{ii}(t) = \sum_{l=0}^{\infty}|U_2(l,t)|^2,\ \chi_{gg}(t) = \sum_{l=0}^{\infty}|U_3(l,t)|^2$$

$$\chi_{ei}(t) = \sum_{l=0}^{\infty}U_1(l+1,t)U_2^*(l,t) = \chi_{ie}^*(t)$$

$$\chi_{eg}(t) = \sum_{l=0}^{\infty}U_1(l+2,t)U_3^*(l,t) = \chi_{ge}^*(t) \tag{28}$$

$$\chi_{ig}(t) = \sum_{l=0}^{\infty}U_2(l+1,t)U_3^*(l,t) = \chi_{gi}^*(t)$$

The probability density operator for an atom is obtained by taking the trace over the field:

$$\hat{\chi}_f(t) = Tr_{\text{atom}} |\varphi(t)\rangle\langle\varphi(t)| \tag{29}$$

$$\hat{\chi}_f(t) = \sum_{m=0}^{\infty}\sum_{l=0}^{\infty}\begin{bmatrix} U_1(m,t)U_1^*(l,t)|q+m,m\rangle\langle q+l,l| + U_2(m,t)U_2^*(l,t)|q+m+1,m+1\rangle \\ \langle q+l+1,l+1| + U_3(m,t)U_3^*(l,t)|q+m+2,m+2\rangle\langle q+l+2,l+2| \end{bmatrix} \tag{30}$$

In what follows, we discuss some non-classical aspects of the present system, such as the collapses-revivals phenomenon, degree of entanglement, squeezing phenomenon.

The collapses-revivals phenomenon

In this section, we study ESS on the collapses-revivals phenomenon by using the definition of atomic population, which is considered to be the difference between occupation in the ex-cited state $x_{ee}(t)$ and the ground state $x_{gg}(t)$ (28) [23]:

$$W(t) = \chi_{ee}(t) - \chi_{gg}(t) \tag{31}$$

Examining the atomic population sheds light on the behavior of the interaction by determining the location of the atom over time. Such as, to determine when the atom in its ground, middle or ex-cited state.

We plot the function $W(t)$ against gt, where $g = (\beta_1\beta_2)^{1/2} = (\beta_3\beta_4)^{1/2}$. We display the evolution of $W(t)$ for the proposed system for some values of SS parameter $r = (\beta_1/\beta_2)^{1/2}$. The initial conditions are defined, the atom begins from the upper most ex-cited state $|e\rangle$ while the field starts from the pair coherent state $|\varepsilon, q\rangle$ (17), with $q = 5$ and $\varepsilon = 10$. Initially, ESS is almost eliminated by specifying the values of the variable $r = 1$ (almost absence of ESS). The popula-

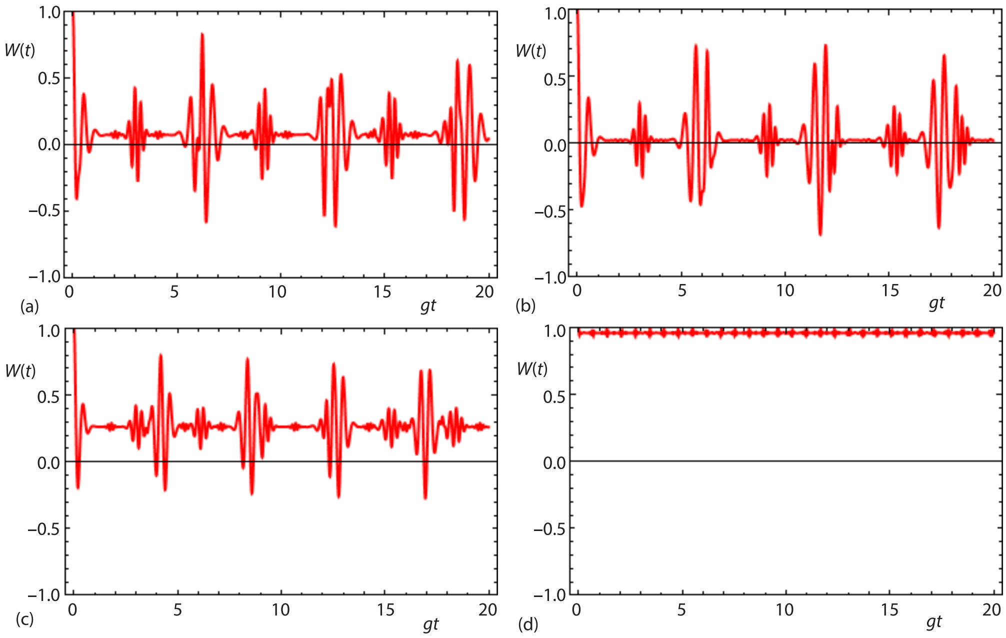

Figure 1. The atomic inversion as a function of the scaled time gt with $\Omega_1 = \Omega_2 = 0.2$, $\omega_e = 0.8$, $\omega_i = 0.6$, $\omega_g = 0.4$, $q = 5$, $\varepsilon = 10$; (a) $r = 1$, (b) $r = 0.75$, (c) $r = 0.50$, and (d) $r = 0.10$

tion oscillates with a small upward shift of the function's axis of symmetry. The function $W(t)$ oscillates periodically and is repeated every 2π. Moreover, the phenomena of revival and collapse that are formed periodically as is clear from fig 1(a). Figure 1(b), shows ESS, by adding a variable r with a value different from one ($r = 0.75$). The amount of the fluctuations of the function $W(t)$ increases and the X-axis is shifted upwards, and the phenomena of collapse and revival gradually disappears. Figure 1(c) shows that with an increase in SS parameter, both the intensity of the oscillations and the upward shift of the axis of symmetry increase. By increasing the SS parameter, the atom gains energy from the field. A noticeable activity is generated in the oscillations, and the atom is in the ex-cited level and never returns to the ground level. As the parameter r approaches zero, ESS increases. By setting $r = 0.1$, the amplitude of the fluctuations is greatly reduced and the atom is almost in the excited state, as shown in fig. 1(d).

Degree of entanglement

In this section, ESS is studied on the degree of entanglement between the field and the atom. It is well known that the amount of entanglement has an important aspect of quantum systems that shows correlations that cannot be treated classically.

Purity

The purity of the system can be treated as a instrument to indicate the amount of the entanglement between the components of the system. The mathematical formula for purity is given by the following relationship [24]:

$$P(t) = Tr\chi_f^2(t) \tag{32}$$

where $P(t) = 1$ corresponding to the pure state, while the maximum entanglement for $P(t) = 1/d$, where d is the dimension of the density matrix from eq. (30). Through the probability density operator, the time-dependent purity function is obtained:

$$P(t) = \chi_{ee}^2(t) + \chi_{ii}^2(t) + \chi_{gg}^2(t) + 2\left(\left|\chi_{ei}(t)\right|^2 + \left|\chi_{eg}(t)\right|^2 + \left|\chi_{ig}(t)\right|^2\right)$$

the effect of x.

This part is devoted to studying ESS on the amount of purity state generated from the interaction of the field with a three-level atom, using the conditions aforementioned in the atomic inversion. Figure 2(a) shows the amount of entanglement after specifying the parameter $r = 1$. The interaction starts from a pure state (a complete separation between the field and the atom), entanglement is generated between the parts of the quantum system, it reaches its peak at the middle of the revival period, while it reduces sharply at the middle of the collapse period. When the effect of the parameter r is inserted with a value different from one ($r = 0.75$), the maximum values of $P(t)$ function are increased. This indicates a weak entanglement between the parties of the proposed system as observed in fig. 2(b). Figure 2(c) confirms that the function $P(t)$ approaches the pure state, $P(t) = 1$, by decreasing the amount of the SS parameter r. Moreover, the amplitude of the oscillations is greatly reduced, therefore, a weak entanglement is generated between the parts of the proposed system. The separation state for field and atom becomes more pronounced with further reduction of the values of parameter r. With $r = 0.1$, the function $P(t)$ is observed to be close to its maximum values (very close to the pure state), see fig. 2(d).

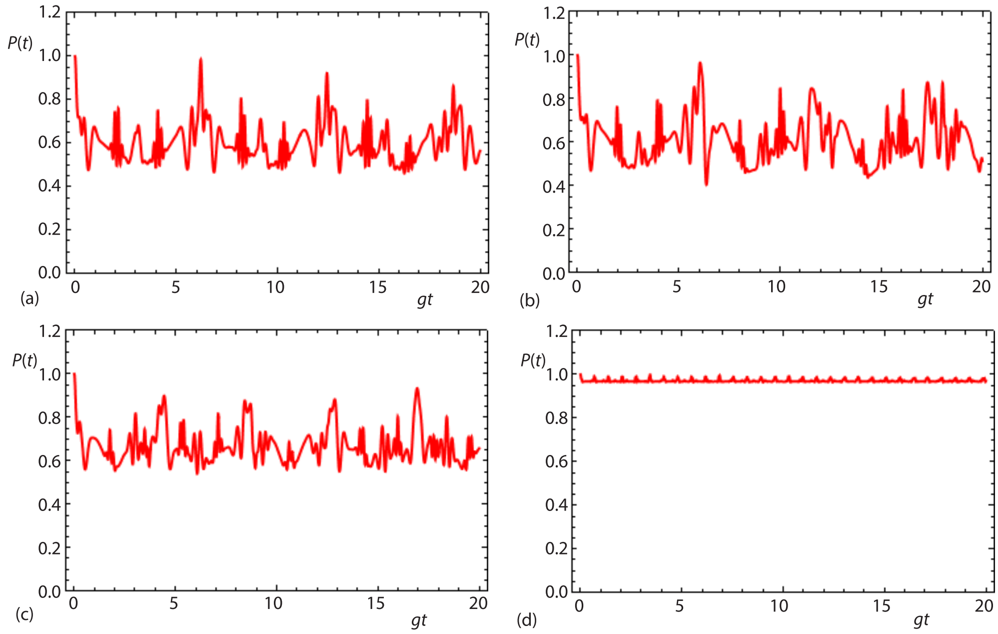

Figure 2. The purity as a function of the scaled time *gt* ; the parameters same as the fig. 1

Concurrence

This subsection is interested to the study of entanglement using concurrence. The concurrence is considered as an indicator to the amount of the entanglement between the subsystem. It ensures the scale between 0 for a disentangled state and

$$\sqrt{\frac{2(d-1)}{d}}$$

for the state of the maximally entangled. It takes the form [25]:

$$C(t)=\sqrt{2\sum_{i,j=1,2,3}\left[\chi_{ii}(t)\chi_{jj}(t)-\chi_{ij}(t)\chi_{ji}(t)\right]},i\neq j \tag{33}$$

where $\chi_{ii}(t)$, $\chi_{jj}(t)$, $\chi_{ji}(t)$,and $\chi_{ij}(t)$ are given by eq. (28).

The initial conditions mentioned previously are used to examine the amount of the entanglement between the subsystem. When taking the least ESS (r = 1), the process of interaction between the subsystem starts from the pure state (separable state) followed by the generation of entanglement significantly. The entanglement increases in the periods of collapse, while it decreases in the periods of revival, by comparing the figs. 1(a) and 3(a). When considering a value close to one (r = 0.75), the entanglement between the subsystem improves slightly and reaches its peak at the centre of the revival periods as seen in fig. 3(b). This result is fully consistent with purity. At the setting of (r = 0.5), the entanglement between the field and the atom decreases, and thus gradually moves away from the pure state as observed in fig. 3(c). Emphasizing the role of SS by causing an increase in the separation of the field from the atom. The entanglement collapses obviously by increasing the parameter r, and the quantum system approaches the pure state, see fig. 3(d).

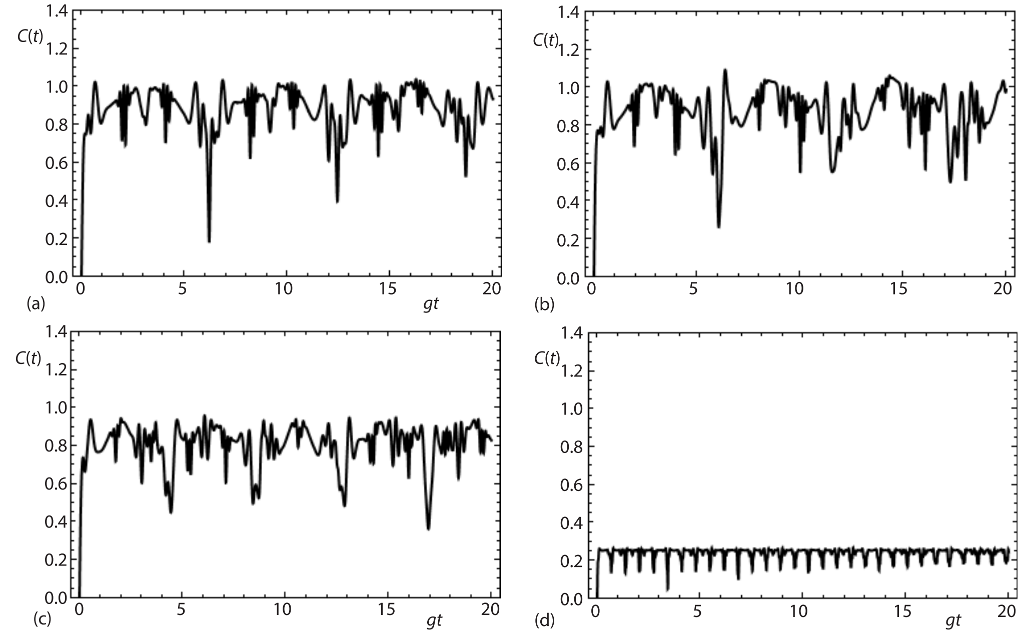

Figure 3. The concurrence as a function of the scaled time *gt*; the parameters same as the fig. 1

The Squeezing phenomenon

The squeezing phenomenon is one of the non-classical phenomena in quantum optics. The atomic variables squeezing and the squeezing in entropy are basic instruments to measure this phenomenon. This method is based on the uncertainty principle as shown in the following.

Squeezing of the atomic variables

As we know, the uncertainty relation for the three-level atom can be defined as the form [26]:

$$\Delta\hat{j}_x\Delta\hat{j}_y \geq \frac{1}{2}\left|\left\langle \hat{j}_z \right\rangle\right| \tag{34}$$

where $\Delta^2\langle\hat{j}_i^2\rangle - \langle\hat{j}_i\rangle^2$, $(i = x, y, z)$ with $\hat{j}_x$, $\hat{j}_y$, and $\hat{j}_z$ satisfy the relation $[\hat{j}_x, \hat{j}_y] = i\hat{j}_z$. Fluctuations in the component $\hat{j}_i$ of the atomic variables are said to be squeezed if $\Delta\hat{j}_i$ satisfies the condition:

$$V(\hat{j}_i) = \left(\triangle\hat{j}_i - \sqrt{\frac{\left|\left\langle \hat{j}_z \right\rangle\right|}{2}}\right) < 0, \quad i = x \text{ or } y \tag{35}$$

where

$$\left\langle \hat{j}_x \right\rangle = \frac{1}{\sqrt{2}}\left[\chi_{ei}(t) + \chi_{ie}(t) + \chi_{ig}(t) + \chi_{gi}(t)\right] \tag{36}$$

$$\left\langle \hat{j}_y \right\rangle = \frac{i}{\sqrt{2}}\left[-\chi_{ei}(t) + \chi_{ie}(t) - \chi_{ig}(t) + \chi_{gi}(t)\right] \tag{37}$$

$$\left\langle \hat{j}_z \right\rangle = \chi_{ee}(t) - \chi_{gg}(t) \tag{38}$$

and

$$\left\langle \hat{j}_x^2 \right\rangle = \frac{1}{2}\left[\chi_{ee}(t) + \chi_{eg}(t) + 2\chi_{ii}(t) + \chi_{ge}(t) + \chi_{gg}(t) \right] \tag{39}$$

$$\left\langle \hat{j}_y^2 \right\rangle = \frac{1}{2}\left[\chi_{ee}(t) - \chi_{eg}(t) + 2\chi_{ii}(t) - \chi_{ge}(t) + \chi_{gg}(t) \right] \tag{40}$$

$$\left\langle \hat{j}_z^2 \right\rangle = \left[\chi_{ee}(t) + \chi_{gg}(t) \right] \tag{41}$$

The initial conditions mentioned in the atomic inversion are used to define the periods of squeezing by the relation eq. (40). When the parameter r approaches one, the squeezing intervals are not formed at all in both variables $V(\hat{j}_x)$ and $V(\hat{j}_y)$. In fact, the variable $V(\hat{j}_x)$ is close to fulfilling the condition of squeezing at the middle of the revival period, as seen in figs. 4(a) and 4(b). Figure 4(c) describes the squeezing intervals for small value of the parameter (r = 0.1). When parameter r approaches zero, the squeezing intervals are formed repeatedly for variable $V(\hat{j}_x)$, while they are never formed for variable $V(\hat{j}_y)$. Squeezing periods are more pronounced when the parameter r is very close to zero, see fig. 4(d). Therefore, the greater ESS, more squeezing periods appear in the variable $V(\hat{j}_x)$.

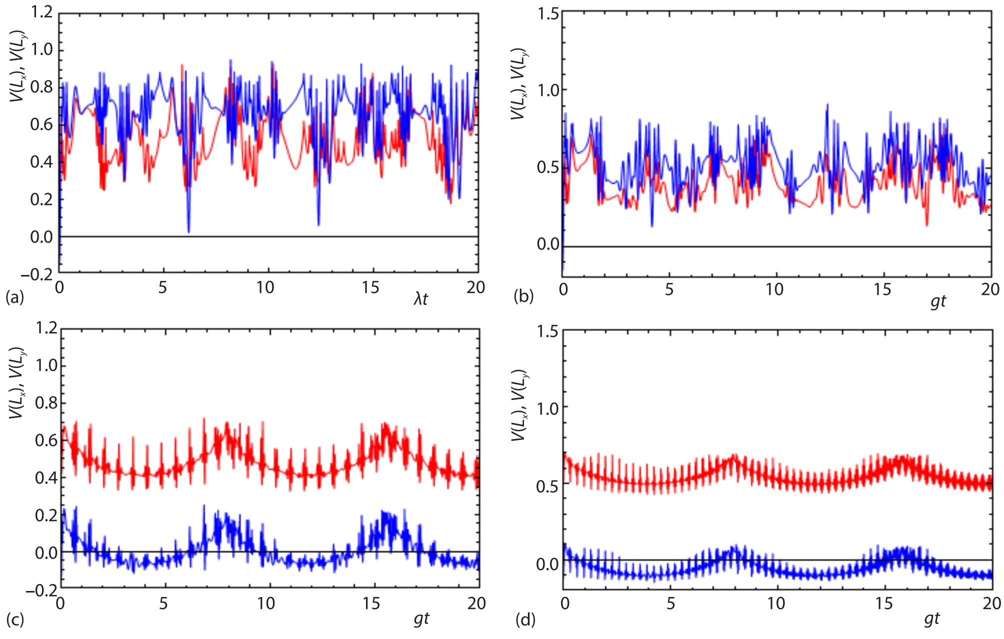

Figure 4. The variance squeezing components $V(\hat{j}_x)$ and $V(\hat{j}_y)$ as functions of the scaled time *gt*, $V(\hat{j}_x)$ is represented by the red curve, and $V(\hat{j}_y)$ is represented by the blue curve; (a) r = 1, (b) r = 0.75, (c) r = 0.10, and (d) r = 0.05; the other parameters same as the fig. 1

The entropy squeezing

The entropic uncertainty relation for d + 1 observables has been defined by the inequality [27, 28]:

$$\sum_{\rho=1}^{d+1} H\left(\hat{j}_\rho\right) \geq d \ln\left[\frac{1}{2}(d+1)\right] \tag{42}$$

where d is a prime dimensional Hilbert space and $H(\hat{j}_\rho)$ – the Shannon-entropy, it is defined:

$$H(\hat{j}_\rho) = -\sum_{\alpha=1}^{d} G_\alpha(\hat{j}_\rho) \ln G_\alpha(\hat{j}_\rho), \quad \rho = x, y, z \tag{43}$$

where $G_\alpha(\hat{j}_\rho)$ are calculated [29]:

$$G_1(\hat{j}_x) = \frac{1}{2}\left[\chi_{ee}(t) - 2\text{Re}[\chi_{eg}(t)] + \chi_{gg}(t)\right] \tag{44}$$

$$G_2(\hat{j}_x) = \frac{1}{4}\chi_{ee}(t) + \frac{1}{\sqrt{2}}\text{Re}\left[\chi_{ei}(t)\right] + \frac{1}{2}\chi_{ii}(t) + \frac{1}{2}\text{Re}\left[\chi_{eg}(t)\right] + \frac{1}{\sqrt{2}}\text{Re}\left[\chi_{ig}(t)\right] + \frac{1}{4}\chi_{gg}(t) \tag{45}$$

$$G_3(\hat{j}_x) = \frac{1}{4}\chi_{ee}(t) - \frac{1}{\sqrt{2}}\text{Re}\left[\chi_{ei}(t)\right] + \frac{1}{2}\chi_{ii}(t) + \frac{1}{2}\text{Re}[\chi_{eg}(t)] - \frac{1}{\sqrt{2}}\text{Re}[\chi_{ig}(t)] + \frac{1}{4}\chi_{gg}(t) \tag{46}$$

$$G_1(\hat{j}_y) = \frac{1}{2}\left[\chi_{ee}(t) + \chi_{gg}(t) + 2\text{Re}[\chi_{eg}(t)]\right] \tag{47}$$

$$G_2(\hat{j}_y) = \frac{1}{4}\chi_{ee}(t) + \frac{1}{2}\chi_{ii}(t) + \frac{1}{4}\chi_{gg}(t) + \frac{1}{\sqrt{2}}Im\left[\chi_{ei}(t)\right] - \frac{1}{2}\text{Re}[\chi_{eg}(t)] + \frac{1}{\sqrt{2}}Im\left[\chi_{ig}(t)\right] \tag{48}$$

$$G_3(\hat{j}_y) = \frac{1}{4}\chi_{ee}(t) + \frac{1}{2}\chi_{ii}(t) - \frac{1}{\sqrt{2}}Im[\chi_{ei}(t)] - \frac{1}{2}\text{Re}\left[\chi_{eg}(t)\right] - \frac{1}{\sqrt{2}}Im\left[\chi_{ig}(t)\right] + \frac{1}{4}\chi_{gg}(t) \tag{49}$$

$$G_1(\hat{j}_z) = \chi_{ee}(t), G_2(\hat{j}_z) = \chi_{ii}(t),\ G_3(\hat{j}_z) = \chi_{gg}(t) \tag{50}$$

By using the in eq. (42), $H(\hat{j}_\rho)$ must achieve:

$$H(\hat{j}_x) + H(\hat{j}_y) + H(\hat{j}_z) \geq 3\ln 2 \tag{51}$$

Thus the fluctuation in $\hat{j}_\rho$ (ρ = x or y) indicts squeezing if $H(\hat{j}_\rho)$ achieves the condition:

$$E(\hat{j}_\rho) = \left[\delta H(\hat{j}_\rho) - \frac{2\sqrt{2}}{\sqrt{|\delta H(\hat{j}_z)|}}\right] < 0, \ \rho = x, y \tag{52}$$

when $\delta H(\hat{j}_\rho) = \exp[H(\hat{j}_\rho)]$. In this subsection, ESS on entropy squeezing is studied with the same conditions mentioned in the atomic inversion. Figure 5(a) shows the entropy squeezing in the least ESS, the squeezing is achieved for the quadrature y while it is not for the quadrature x. The squeezing is also monitored before and after the centre of the revival intervals, while the function $E(\hat{j}_\rho)$ reaches its minimum values at the centre of the collapse intervals. When considering ESS with a value of the parameter r that is different from one adjust (r = 0.75), the squeezing is satisfied for both variables x and y. At the beginning of the considered interaction period, the squeezing is satisfied with respect to variable x and the end is satisfied with respect to variable y, as shown in fig. 5(b). The squeezing decreases by increasing the influence of a SS (by decreasing the parameter r). The maximum values of the function $E(\hat{j}_\rho)$ decrease while the minimum values increase, and this confirms that the Stark shift causes a decrease in the squeezing intervals, see fig. 5(c). With more ESS, the squeezing intervals decrease significantly. The squeezing is achieved in both variables x and y alternately at different intervals, as seen in fig. 5(d).

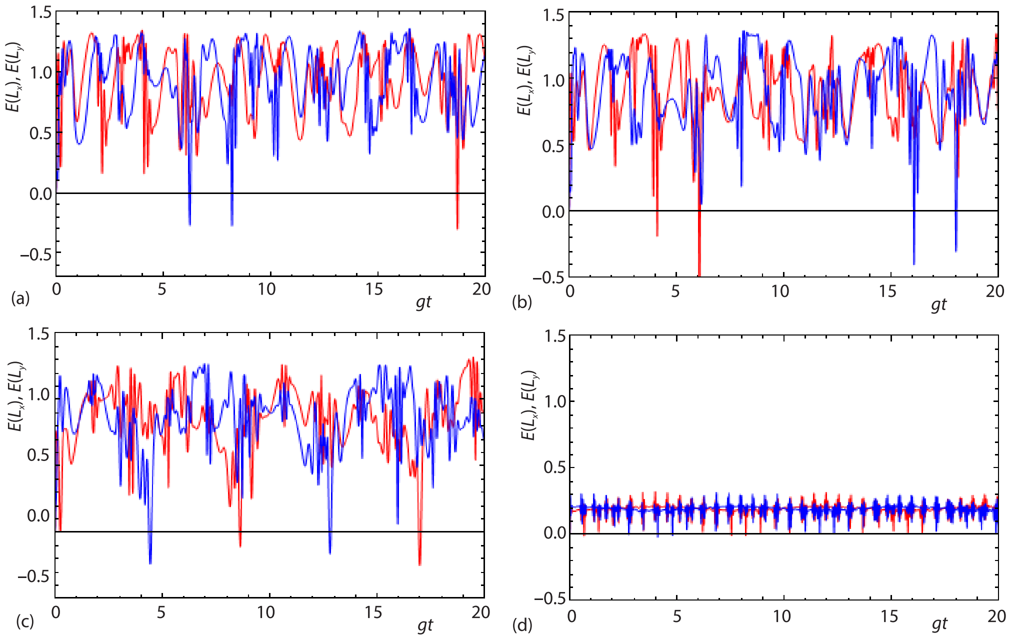

Figure 5. The entropy squeezing components $E(\hat{j}_x)$ and $E(\hat{j}_y)$ of the scaled time gt; $E(\hat{j}_x)$ is represented by the red curve, and $E(\hat{j}_y)$ is represented by the blue curve; the other parameters same as the fig. 1

Conclusion

In this manuscript, the SS terms for a quantum system consisting of the interaction of a three-level atom with a bimodal field are derived. An atom with five levels was considered, by using the AE method, the levels number was reduced to three. The general solution (wave function) was obtained by solving the Schrodinger differential equation. ESS on the atomic inversion, the entanglement between the subsystem by purity and concurrence, the variance and entropy squeezing were studied. The results showed that the SS significantly affected the atomic inversion when the parameter r takes a small amount close to zero. When parameter r approaches one, ESS decreases, and the collapses and revivals phenomena appears repeatedly. When ESS is small, a clear entanglement is generated between the parts of the quantum system, while this entanglement dissipates with an increase in ESS ($r \to 0$). Squeezing periods are more pronounced when the parameter r is very close to zero. Therefore, increasing parameter r leads to more realization of the squeezing periods in the variable $V(\hat{j}_x)$. Entropy squeezing is achieved in the variable x and not in y when ESS parameter is small. Increasing ESS parameter is achieved in both variables x and y interchangeably.

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