A NOVEL FRACTIONAL STUDY ON FREE CONVECTION FLOW OF BRINKMANN HYBRID NANOFLUID OVER AN INCLINED PLATE

by

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In this paper a free convection unsteady Brinkmann hybrid nanofluids including two or more nanoadditives to the host liquid is investigated. The physical flow phenomena are illustrated using PDE and thermophysical nanoparticle properties, and this paper addresses the Brinkmann fractional fluid along with chemical reaction and heat generation with ramped conditions over an inclined vertical plate. The heat and molecular fluxes are generalized using the novel fractional derivative. The present flow model are solved semi-analytically using the Laplace transform. The effects of different parameters specially fractional parameter are deliberated and plotted graphically. The acquired results reveal that fractional parameters have dual behavior in velocity profiles and temperature profile. Velocity and temperature are also compared to previous studies. Compared to the other fractional derivatives results, field variables and proposed hybrid fractional derivatives showed a more decaying trend.

Key words: *Brinkmann fluid, hybrid nanofluid, heat generation, chemical reaction, CPC fractional derivative*

Introduction

Recently, many researchers have been engrossed to study nanotechnology due to its wide applications in industries. The addition of two or more distinct nanoadditives to the base fluid causes the formation of hybrid nanofluid [1, 2]. Ali *et al.* [3] discussed the Maxwell hybrid nanofluid with pressure gradient in a vertical channel. Khalid *et al.* [4] solved a problem related to nanofluid with ramped conditions. Baleanu *et al.* [5] described the different types of properties of fractional-calculus operators. Using a Cattaneo constitutive equation along with a Caputo-Fabrizio time-fractional derivative, Hristov [6] started working on transient heat. Baleanu *et al.* [7] described a fractional operator that combines proportional Caputo and solved various types of examples using CPC derivatives. Asjad *et al.* [8] investigated the flow of a Maxwell fluid usually contains clay nanoparticles along with constant proportional Caputo types frac-

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tional derivatives. Ahmad *et al.* [9] used novel fractional derivatives (CPC) to obtain analytical solutions of the Casson nanofluid across a vertical plate. Chu *et al.* [10] used Fourier's and Fick's laws to present a model of the differential-type fluid by fractionalized thermal and mass fluxes with CPC derivative.

The aim of this paper to show the analysis of hybrid nanofluid by using the CPC fractional derivative to explore the Brinkmann fluid in an inclined plate along with chemical reaction and heat source using generalized thermal and molecular fluxes. The solution of dimensionless differential equations with boundary conditions is semi-analytically solved by utilizing the Laplace transform. The temperature, concentration and velocity distribution results are attained and graphically discussed.

Formulation of problem

Let us consider the hybrid nanofluid-flow in an inclined vertical plate. The plates are taken along the $x^{\cdot}$-axis and the $z^{\cdot}$-axis is chosen perpendicular to it. When $t^{\cdot} < 0$, the plates as well as fluid are at rest with ambient temperature $T^{\cdot}_{\infty}$. When $0 < t^{\cdot} < t^{\cdot}_0$ the plate's temperature is raised or lowered to $T^{\cdot}_{\infty} + (T^{\cdot}_W - T^{\cdot}_{\infty})t/t^{\cdot}_0$. After $t^{\cdot} > 0$, the plate's temperature rises or lowers to $T^{\cdot}_W$. At this time, the fluid initiate its motion in the x-direction because of the temperature gradient. The value of the magnetic field is insignificant due to very low Reynolds number.

According to the Boussinesq's approximation, the governing equations for an unsteady Brinkmann hybrid nanofluid-flow in an inclined plate are given [11, 12]:

$$\rho_{\text{hnf}}\left[\frac{\partial u_2\left(z^{\cdot},t^{\cdot}\right)}{\partial t^{\cdot}}+\beta_m u_2\left(z^{\cdot},t^{\cdot}\right)\right]=\mu_{\text{hnf}}\frac{\partial^2 u_2\left(z^{\cdot},t^{\cdot}\right)}{\partial z^2}-\sigma_{\text{hnf}}B_0^2 u_2\left(z^{\cdot},t^{\cdot}\right)+ \\ +\text{g}\left(\rho\beta_{T^{\cdot}}\right)_{\text{hnf}}\left(T^{\cdot}-T_0^{\cdot}\right)\cos\theta+g\left(\rho\beta_{C^{\cdot}}\right)_{\text{hnf}}\left(C^{\cdot}-C_0^{\cdot}\right)\cos\theta \tag{1}$$

Thermal equation:

$$\left(\rho C_P\right)_{\text{hnf}}\frac{\partial T^{\cdot}\left(z^{\cdot},t^{\cdot}\right)}{\partial t^{\cdot}}=-\frac{\partial h_1^{\cdot}\left(z^{\cdot},t^{\cdot}\right)}{\partial z^{\cdot}}+Q_0\left(T^{\cdot}-T_0^{\cdot}\right) \tag{2}$$

The generalized Fourier's Law states [10, 13]:

$$h_1^{\cdot}\left(z^{\cdot},t^{\cdot}\right)=-K_{\text{hnf}}{}^{CPC}D_t^{\gamma}\frac{\partial T^{\cdot}\left(z^{\cdot},t^{\cdot}\right)}{\partial z^{\cdot}},\quad 1\geq\gamma>0 \tag{3}$$

Diffusion equation:

$$\frac{\partial C^{\cdot}\left(z^{\cdot},t^{\cdot}\right)}{\partial t^{\cdot}}=-\frac{\partial J_1^{\cdot}\left(z^{\cdot},t^{\cdot}\right)}{\partial z^{\cdot}}-K_1\left(C^{\cdot}-C_0^{\cdot}\right) \tag{4}$$

The generalized Fick's Law states [10]:

$$J_1^{\cdot}\left(z^{\cdot},t^{\cdot}\right)=-D^{CPC}D_t^{\gamma}\frac{\partial C^{\cdot}\left(z^{\cdot},t^{\cdot}\right)}{\partial z^{\cdot}},\ 1\geq\alpha>0 \tag{5}$$

The initial as well as boundary conditions:

$$u_2\left(z^{\cdot},0\right)=0,\quad T^{\cdot}\left(z^{\cdot},0\right)=T_{\infty}^{\cdot},\quad C^{\cdot}\left(z^{\cdot},0\right)=C_{\infty}^{\cdot},\quad t^{\cdot}\leq 0 \\ u_2\left(0,t^{\cdot}\right)=0, T\left(0,t\right)=\begin{cases} T_{\infty}^{\cdot}+\left(T_W^{\cdot}-T_{\infty}^{\cdot}\right)\dfrac{t^{\cdot}}{t_0^{\cdot}}, & t^{\cdot}\leq t_0^{\cdot} \\ T_W^{\cdot}, & t^{\cdot}>t_0^{\cdot}\end{cases} \tag{6}$$

$$C(0,t)=\begin{cases} C_\infty^{\cdot}+\left(C_W^{\cdot}-C_\infty^{\cdot}\right)\dfrac{t^{\cdot}}{t_0^{\cdot}}, & t^{\cdot}\le t_0^{\cdot} \\ C_W^{\cdot}, & t^{\cdot}>t_0^{\cdot} \end{cases} \tag{7}$$

$$u_2\left(z^{\cdot},t^{\cdot}\right)\to 0,\ T^{\cdot}\left(z^{\cdot},t^{\cdot}\right)\to 0,\ C^{\cdot}\left(z^{\cdot},t^{\cdot}\right)\to 0,\ z^{\cdot}\to\infty,\ t^{\cdot}>0 \tag{8}$$

The non-dimensional form of the flow variables:

$$x=\frac{z^{\cdot}}{\sqrt{\nu_f t^{\cdot}}},\ t=\frac{t^{\cdot}}{t_0^{\cdot}},\ T=\frac{T^{\cdot}-T_\infty^{\cdot}}{T_w^{\cdot}-T_\infty^{\cdot}},\ \ v=\frac{u_2}{U_0},\ C=\frac{C^{\cdot}-C_\infty^{\cdot}}{C_w^{\cdot}-C_\infty^{\cdot}},\ J_1=\frac{J_1^{\cdot}}{J}$$

$$\mathrm{Gm}=\frac{\left(\mathrm{g}\beta_{C^{\cdot}}\right)_f\left(C_w^{\cdot}-C_0^{\cdot}\right)t_0^{\cdot}}{\rho_f U_0},\ \mathrm{Gr}=\frac{\left(\mathrm{g}\beta_{T^{\cdot}}\right)_f\left(T_w^{\cdot}-T_0^{\cdot}\right)t_0^{\cdot}}{\rho_f U_0},\ M=\frac{B_0^2 t_0^{\cdot}\sigma_f}{\rho_f},\ \mathrm{Br}=t_0^{\cdot}\beta_m$$

$$h_1=\frac{h_1^{\cdot}}{h},\ Q=\frac{Q_0 t_0^{\cdot}}{(\rho C_P)_f},\ \ R=K_1 t_0^{\cdot},\ \ r_1=\left(1-\phi_2\right)\left[\left(1-\phi_1\right)+\phi_1\frac{\rho_{s1}}{\rho_f}\right]\phi_2\frac{\rho_{s2}}{\rho_f}$$

$$r_2=\frac{1}{(1-\phi_1)^{2.5}(1-\phi_2)^{2.5}},\ r_3=\left(1-\phi_2\right)\left[\left(1-\phi_1\right)+\phi_1\frac{(\rho\beta_{T^{\cdot}})_{s1}}{(\rho\beta_{T^{\cdot}})_f}\right]+\phi_2\frac{\left(\rho\beta_{T^{\cdot}}\right)_{s2}}{\left(\rho\beta_{T^{\cdot}}\right)_f}$$

$$r_4=\left(1-\phi_2\right)\left[\left(1-\phi_1\right)+\phi_1\frac{(\rho\beta_{C^{\cdot}})_{s1}}{(\rho\beta_{C^{\cdot}})_f}\right]+\phi_2\frac{\left(\rho\beta_{C^{\cdot}}\right)_{s2}}{\left(\rho\beta_{C^{\cdot}}\right)_f} \tag{9}$$

$$r_5=\frac{\sigma_{\mathrm{nf}}}{\sigma_f}\left\{\frac{\sigma_{s2}\left(1+2\phi_2\right)+2\sigma_{\mathrm{nf}}\left(1-\phi_2\right)}{\sigma_{s2}\left(1-\phi_2\right)+\sigma_{\mathrm{nf}}\left(2+\phi_2\right)}\right\},\ \ r_7=\frac{k_{\mathrm{nf}}}{k_f}\left\{\frac{k_{s2}+2k_{\mathrm{nf}}-2\phi_2\left(k_{\mathrm{nf}}-k_{s2}\right)}{k_{s2}+2k_{\mathrm{nf}}+2\phi_2\left(k_{\mathrm{nf}}-k_{s2}\right)}\right\}$$

$$r_6=\left(1-\phi_2\right)\left[\left(1-\phi_1\right)+\phi_1\frac{(\rho C_P)_{s1}}{(\rho C_P)_f}\right]+\phi_2\frac{(\rho C_P)_{s2}}{(\rho C_P)_f}$$

$$n_1=\frac{h t_0}{(\rho C_P)_f\left(T_w-T_\infty\right)},\ n_3=\frac{J t_0}{\left(C_w-C_\infty\right)\sqrt{\nu_f t_0}},\ \ n_2=\frac{\left(T_w-T_0\right)}{h\sqrt{\nu_f t_0}},\ n_4=\frac{D\left(C_w-C_\infty\right)}{J\sqrt{\nu_f t_0}}$$

Using dimensionaless variables of eq. (9) in the aforementioned equations, we obtain:

$$r_1\left(\frac{\partial v(x,t)}{\partial t}+\mathrm{Br}\right)=r_2\frac{\partial^2 v(x,t)}{\partial x^2}-Mr_5 v(x,t)+r_3\mathrm{Gr}T(x,t)\cos\theta+r_4\mathrm{Gm}C(x,t)\theta \tag{10}$$

$$r_6\frac{\partial T(x,t)}{\partial t}+n_1\frac{\partial h_1(x,t)}{\partial x}-QT(x,t)=0 \tag{11}$$

$$-n_2 r_7\frac{\partial T(x,t)}{\partial x}=h_1(x,t),\ \ 1\ge\gamma>0 \tag{12}$$

$$\frac{\partial C(x,t)}{\partial t}+n_3\frac{\partial J_1(x,t)}{\partial x}+RC(x,t)=0 \tag{13}$$

$$-n_4\frac{\partial C(x,t)}{\partial x}=J_1(x,t),\ \ 1\geq\alpha>0 \tag{14}$$

$$(x,0)=T(x,0)=C(x,0)=0,\ \ x>0 \tag{15}$$

$$v(0,t)=0,\ \ T(0,t)=\begin{cases}t, & t\leq 1\\ 1, & t>1\end{cases},\ \ C(0,t)=\begin{cases}t, & t\leq 1\\ 1, & t>1\end{cases} \tag{16}$$

$$v(x,t)=T(x,t)=C(x,t)\to 0,\ \ x\to\infty \tag{17}$$

Thermophysical properties of hybrid nanofluid

Thermophysical properties be defined in [11]:

$$\begin{gathered}\rho_{\text{hnf}}=\left[(1-\phi_2)\{(1-\phi_1)+\phi_1\rho_{s1}\}\right]+\phi_2\rho_{s2},\ \ \mu_{\text{hnf}}=\frac{\mu_f}{(1-\phi_1)^{2.5}(1-\phi_2)^{2.5}}\\ (\rho C_P)_{\text{hnf}}=(1-\phi_2)\left[(1-\phi_1)+\phi_1(\rho C_P)_{s1}\right]+\phi_2(\rho C_P)_{s2}\\ (\rho\beta)_{\text{hnf}}=(1-\phi_2)\left[(1-\phi_1)+\phi_1(\rho\beta)_{s1}\right]+\phi_2(\rho\beta)_{s2}\\ k_{\text{hnf}}=k_{nf}\frac{k_{s2}+2k_{\text{nf}}-2\phi_2(k_{\text{nf}}-k_{s2})}{k_{s2}+2k_{\text{nf}}+2\phi_2(k_{\text{nf}}-k_{s2})},\ \ k_{\text{nf}}=k_f\frac{k_{s1}+2k_f-2\phi_1(k_f-k_{s1})}{k_{s1}+2k_f+2\phi_1(k_f-k_{s1})}\\ \sigma_{\text{hnf}}=\sigma_{\text{nf}}\left\{\frac{\sigma_{s2}(1+2\phi_2)+2\sigma_{nf}(1-\phi_2)}{\sigma_{s2}(1-\phi_2)+\sigma_{nf}(2+\phi_2)}\right\},\ \sigma_{\text{nf}}=\sigma_f\left\{\frac{\sigma_{s1}(1+2\phi_1)+2\sigma_f(1-\phi_1)}{\sigma_{s1}(1-\phi_1)+\sigma_f(2+\phi_1)}\right\}\end{gathered} \tag{18}$$

where μ_{hnf}, $(\rho C_P)_{\text{hnf}}$, ρ_{hnf}, κ_{hnf}, σ_{hnf}, and ϕ are the effective dynamic viscosity, heat capacitance, effective density, effective thermal conductivity, effective electrical conductivity, and volume fraction of the hybrid nanoparticles, respectively. The thermophysical properties of the hybrid nanomaterials are defined [8] in tab. 1.

Table 1. Thermophysical properties of hybrid nanofluids

	ρ [kgm^{-3}]	K [Wm^{-1}K^{-1}]	σ [sm^{-1}]	$\beta\cdot 10^{-5}$ [K^{-1}]	C_P [Jkg^{-1}K^{-1}]
H_2O(f)	997.1	0.0613	$5.5\cdot 10^{-6}$	21	4179
Al_2O_3($s1$)	3970	40	$35\cdot 10^{6}$	0.85	765
Cu($s2$)	8933	401	$59.6\cdot 10^{6}$	1.67	385
CuO($s2$)	6320	76.5	$6.9\cdot 10^{-2}$	1.80	531.8
Ag($s2$)	10500	429	$6.30\cdot 10^{7}$	1.80	235

Generalization

Generalization of thermal diffusion

The fractional form of Fourier's law [10, 13] is from eq. (12) and used in eq. (11), we get:

$$\frac{\partial T(x,t)}{\partial t}=\frac{r_7}{r_6 \mathrm{Pr}}\,{}^{CPC}D_t^{\gamma}\frac{\partial^2 T(x,t)}{\partial x^2}+QT(x,t) \tag{19}$$

where ${}^{CPC}D_t^{\gamma} f(x, t)$ indicates the CPC fractional derivative of $f(x, t)$ [7]:

$${}^{CPC}D_t^{\gamma} f(x,t)=\frac{1}{\Gamma(1-\gamma)}\int_0^t\left[k_1(\gamma)f(x,\tau)+k_0(\gamma)f'(x,\tau)\right](t-\tau)^{-\gamma}\,\mathrm{d}\tau,\; k_0,\; k_1\in(0,1) \tag{20}$$

Generalization of molecular diffusion

The fractional form of Fick's Law [10] is used from eq. (14) into eq. (13), we obtain:

$$\frac{\partial C(x,t)}{\partial t}=\frac{1}{\mathrm{Sc}}\,{}^{CPC}D_t^{\alpha}\frac{\partial^2 C(x,t)}{\partial x^2}-RC(x,t) \tag{21}$$

Solution of problem

The equations for energy, diffusion and momentum (10), (19), and (21) via the technique of Laplace transform can be solved numerically by using Stehfest's as well as Tzou's algorithms [14, 15] in the case of a complex expression.

Solution of temperature

The eq. (19) is solved the subject to the conditions stated in eqs. (15)-(17) by the use of Laplace transform method for temperature:

$$\bar{T}(x,q)=\frac{1-\mathrm{e}^{-q}}{q^2}\mathrm{e}^{-x\sqrt{\frac{r_6\mathrm{Pr}(q-Q)}{r_7\left(\frac{K_1(\gamma)}{q}+K_0(\gamma)\right)q^{\gamma}}}} \tag{22}$$

Solution of concentration

The eq. (21) is solved using the conditions given in eqs. (15)-(17) via Laplace transform method for concentration species:

$$\bar{C}(x,q)=\frac{1-e^{-q}}{q^2}e^{-x\sqrt{\frac{\mathrm{Sc}(q+R)}{\left(\frac{K_1(\gamma)}{q}+K_0(\gamma)\right)q^{\gamma}}}} \tag{23}$$

Solution of velocity

The solution of the velocity field of eq. (10) is subject to initial and boundary conditions (15-17), by using Laplace transform, we get:

$$\bar{v}(x,q)=$$

$$=\left[\frac{\dfrac{r_3}{r_2}\dfrac{1-\mathrm{e}^{-q}}{q^2}\mathrm{Gr}\cos\theta}{\dfrac{r_6 Pr(q-Q)}{r_7\left[\dfrac{K_1(\gamma)}{q}+K_0(\gamma)q^{\gamma}\right]}-\dfrac{Mr_5+qr_1+r_1Br}{r_2}}\right]\left[\mathrm{e}^{-x\sqrt{\frac{Mr_5+qr_1+r_1Br}{r_2}}}-\mathrm{e}^{-x\sqrt{\frac{r_6\mathrm{Pr}(q-Q)}{r_7\left[\frac{K_1(\gamma)}{q}+K_0(\gamma)\right]q^{\gamma}}}}\right]+$$

$$+\left[\frac{\dfrac{r_4}{r_2}\dfrac{1-\mathrm{e}^{-q}}{q^2}\mathrm{Gm}\cos\theta}{\dfrac{\mathrm{Sc}(q+R)}{\left[\dfrac{K_1(\gamma)}{q}+K_0(\gamma)q^{\gamma}\right]}-\dfrac{Mr_5+qr_1+r_1Br}{r_2}}\right]\left[\mathrm{e}^{-x\sqrt{\frac{Mr_5+qr_1+r_1Br}{r_2}}}-\mathrm{e}^{-x\sqrt{\frac{\mathrm{Sc}(q+R)}{\left[\frac{K_1(\gamma)}{q}+K_0(\gamma)\right]q^{\gamma}}}}\right] \tag{24}$$

Result and iscussion

This paper investigates hybrid nanoparticles in Brinkmann fluid along with CPC fractional derivative. The semi-analytical results of velocity, concentration, and temperature are obtained. Furthermore, some graphs are positioned to represent the physical effect of the involved parameters, particularly the influence of hybrid nanoparticles and fractional parameters.

The graphical behavior of fractional parameter $\alpha = \gamma$ on $v(x, t)$ and $T(x, t)$ are illustrated in figs. 1-4. The figures show the effect of a fractional parameter and reveal the dual nature of the velocity and temperature towards the fractional parameter $\alpha = \gamma$ for longer and shorter times. For a more extended period of time, a velocity and temperature distribution showed an upward trend, and thus the boundary-layers grow with the increase in the values of $\alpha = \gamma$. Its behavior is the opposite for a smaller times.

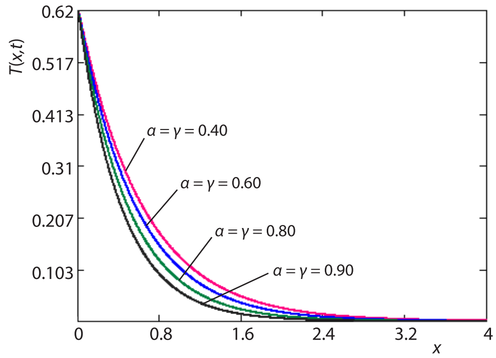

Figure 1. Temperature profile $T(x, y)$ for fractional parameters $\alpha = \gamma$ for small time at Q = 0.3 and Pr = 6.2

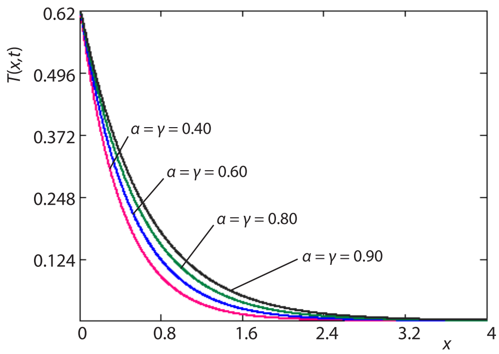

Figure 2. Temperature profile $T(x, y)$ for fractional parameters $\alpha = \gamma$ for large time at Q = 0.3 and Pr = 6.2

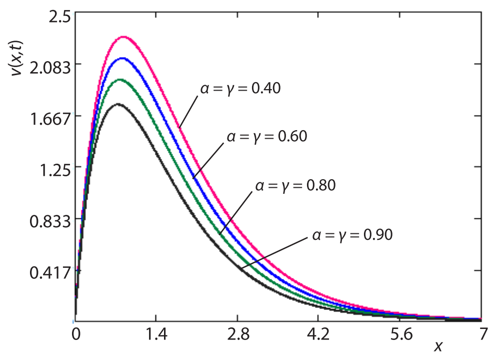

Figure 3. Velocity profile $v(x, y)$ for fractional parameters $\alpha = \gamma$ for small time at Gr = 9, Q = 0.3, Gm = 12, M = 0.5, Pr = 6.2, and R = 1.5

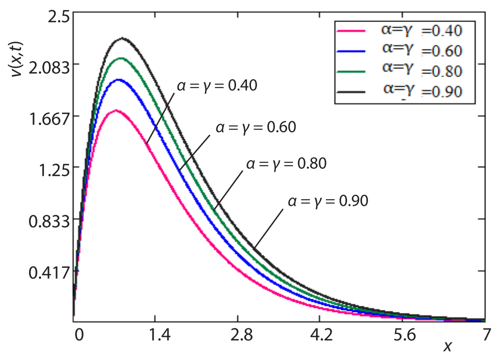

Figure 4. Velocity profile $v(x, y)$ for fractional parameters $\alpha = \gamma$ for large time at Gr = 9, Q = 0.3, Gm = 12, M = 0.5, Pr = 6.2, and R = 1.5

The impact of magnetic field on fluid velocity distribution is depicted in fig. 5 for taken fixed parameters and various kinds of hybrid nanofluids. The figure shows that when magnetic parameters increase, the kinetic energy of the fluids decreases. The changing behavior of nanoparticle of volume fraction on fluid velocity distribution is presented in fig. 6. It is seen that increasing the volume fraction of hybrid nanoparticles speed up the viscous properties for the nanofluid as it reduces the fluid motion. Figure 8 compares the velocity profile between the CPC fractional derivative and viscous flow [11]. Because the velocity used in [11] refers to a viscous fluid, whereas the velocity in this study refers to a Brinkman fluid.

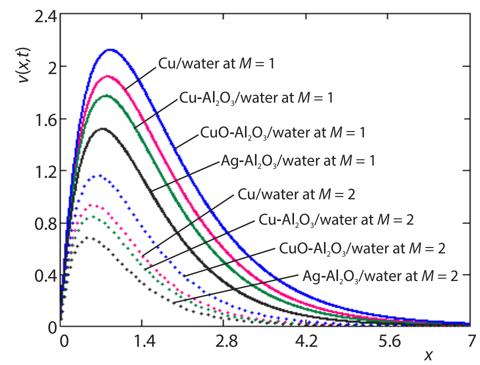

Figure 5. Velocity profile $v(x, t)$ for different value of M at Gr = 9, R = 1.5, Gm = 12, Pr = 6.2, and Q = 0.3

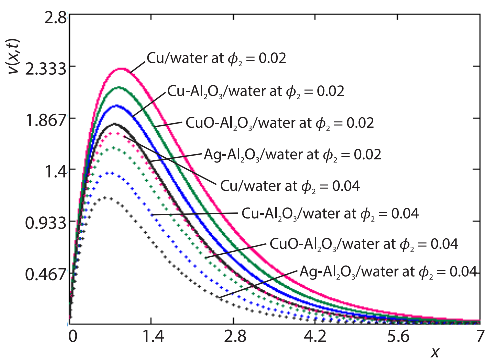

Figure 6. Velocity profile $v(x, t)$ for different value of volume fraction ϕ_2 at Gr = 9, Q = 0.3, Gm = 12, Pr = 6.2, and R = 1.5

Because Brinkman fluids are thicker than viscous fluids. The temperature and velocity comparisons in figs. 7 and 9 demonstrate the current work with the Caputo Fabrizio fractional derivative being used by Ul Haq *et a*l. [11]. When $\beta_m = \alpha_1 = Q_0 = 0$ in [11], velocity and temperature with CPC decrease faster (more decaying nature) than velocity and temperature using Caputo Fabrizio fractional derivative, respectively. Figures 10 and 11 describe the authenticity of inversion algorithms for temperature as well as concentration distributions, respectively. The velocity distributions overlap that shows the authenticity of inversion algorithms as depicted in fig. 12.

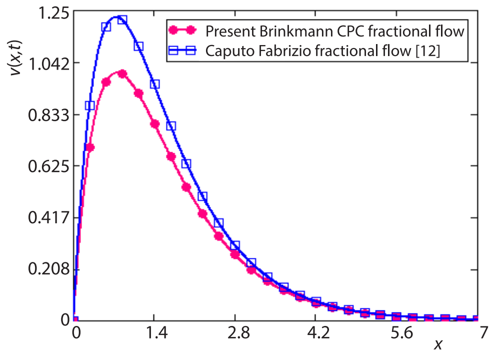

Figure 7. Comparison of velocity profiles between CPC and Caputo Fabrizio fractional derivatives for $\alpha = 0.5$ and $\beta_m = \alpha_1 = 0$

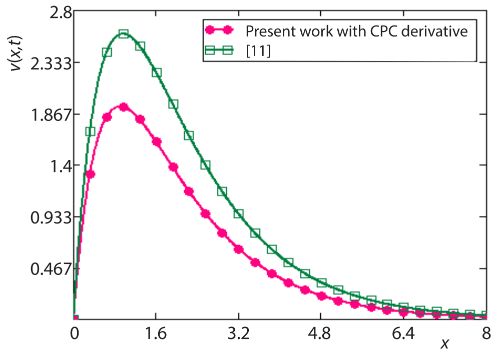

Figure 8. Comparison of velocity profiles of fractional fluid with viscous fluid [11] for $\beta_m = \theta = \mathrm{Gm} = 0$

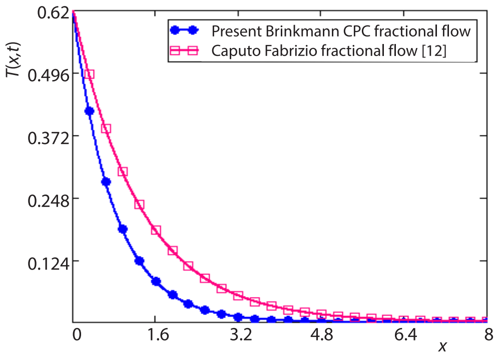

Figure 9. Comparison of temperature profiles between CPC and Caputpo Fabrizio fractional derivative for $\alpha = 0.5$ and $Q_0 = 0$

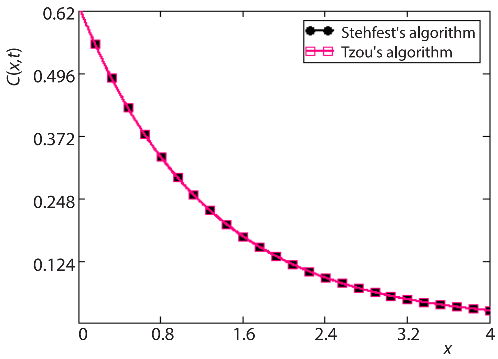

Figure 10. Concentration obtain by Stehfest's and Tzou's algorithms

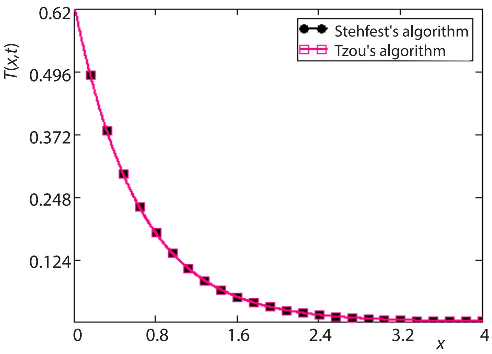

Figure 11. Temperature obtain by Stehfest's and Tzou's algorithms

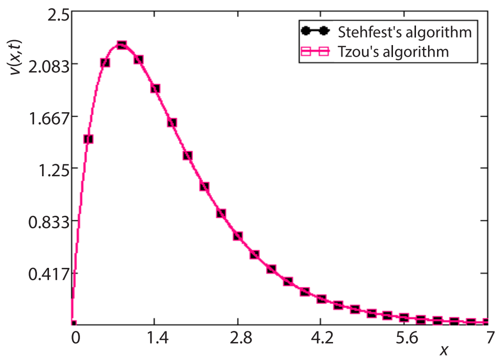

Figure 12. Velocity obtain by Stehfest's and Tzou's algorithms

Conclusions

A hybrid fractional derivative is used in model of Brinkmann nanofluid over an inclined plate. The velocity, concentration, and temperature fields are determined by solving this flow model semi-analytically. To demonstrate the influencing parameters, multiple graphs of optimizing fields are plotted. The following are outcomes of this flow model are as follows.

- The use of hybrid nanofluids produces good results than nanofluid having single nanoparticle. It is noted that the highest value is obtained for Ag-Al_2O_3-water hybrid nanofluids.
- Due to fractional parameters, fluid velocity and energy have dual behavior. Both are increases for a long time by raising the value of the fractional parameter $\alpha = \gamma$ but behaves in the opposite way for a shorter time.
- By applying the ramped conditions on inclined plate, is an efficient method to inaugurate the preferable flow control.

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