STOCHASTIC SOLUTIONS TO THE *NON-LINEAR SCHRODINGER EQUATION* IN OPTICAL FIBER

by

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Original scientific paper
https://doi.org/10.2298/TSCI22S1185A

The non-linear random Schrodinger equation via geometric distribution and exponential distribution is considered. We carry out the unified solver technique to obtain some new random solutions. The statistical distributions are utilized to show the dispersion random input. The reported random solutions are so important in fiber optics and their applications. The expectation for the random solutions are drawn to show the behaviour of solutions.

Key words: *Schrodinger problem, unified solver, optical fiber, geometric distribution and exponential distribution*

Introduction

There are numerous non-linear phenomena in optical fibre, physics, biology, economics, and engineering problems [1-5]. These phenomena can be represented by deterministic or stochastic non-linear partial differential equations (NPDE). The importance of non-linear stochastic partial differential equation (NSPDE) cannot be overstated. Researchers have long sought new and efficient approaches to solving NSPDE. Many new methods for obtaining exact solutions of NSPDE have recently been successfully presented, including homotopy perturbation [6], first integral and variational iteration techniques [7, 8], linear superposition principle [9], sub-equation procedure [10], and exponential function method [11].

One of the important model in applied science is the non-linear Schrodinger equation (NLSE). This equation describes the propagation of waves in non-linear and dispersive media. The optical soliton solutions for the NLSE plays an important role in various fields in applied science, such as superfluid, optical fiber communications, quantum mechanics, plasma physics, electro magnetic wave propagation and deep water and superfluid [12-15]. In this work we consider the non-linear random Schrodinger equation (NLRSE) via geometric distribution and exponential distribution.

The statistical properties of many physical systems play the key role in the fundamental studies of integrable turbulence and extreme sea wave formation. If the distribution function is defined and the associated parameters are stated, the probabilistic characteristics of random variables can be fully represented. Nevertheless, in the non-attendance of any parametric distribution, survey statistics are used to determine an approximate representation of the population. The mean, median and mode are three quantities that can be used to express the central tendency of a random variable. A random variable's dispersion refers to how closely or widely the values of a random variable are clustered around the central value. The random effect on soliton

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solution propagation has recently received a lot of attention. This effect is critical in explaining many complex phenomena in a wide range of applied sciences fields for examples, see [16-18].

In this article, we consider the following NRLSE:

$$i\Theta_t + \alpha\Theta_{xx} - \beta|\Theta|^2\Theta + \gamma\Theta = 0, \ , i = \sqrt{-1} \tag{1}$$

where $\Theta = \Theta(x, t)$ are the slowly pulse amplitude, α, β, and γ – the fiber dispersion, non-linear, and fiber loss effects. We introduce some new stochastic solutions through geometric distribution and exponential distribution, utilising the unified solver procedure. Indeed, this solver presents many influential solutions in describing physical phenomena.

Stochastic solutions

Statistical wave transformation may be an important parameter to include in environmental assessments or studies that attempt to understand the role of disturbances in structuring biological systems. However, disruptions have been found to increase, decrease, or have no effect on the population's physics properties (or community heterogeneity). In this paper, we present some new solutions of randomness case for the NRLSE:

$$\Theta(x,t) = \mathrm{e}^{i(cx+\nu)}u(\eta), \ \eta = kx + \rho t \tag{2}$$

where c, k, ν, ρ are constants. The eq. (1) gives the non-linear ODE:

$$\alpha k^2 u'' - \beta u^3 + \left(\gamma - \alpha c^2 - \nu\right)u = 0, \quad \rho = -2\alpha ck \tag{3}$$

In view of the unified solver technique introduced in [19], the random behaviour for eq. (1):

– Rational function solutions

The first class of solutions for eq. (3):

$$u_1(x,t) = \left[\mp\sqrt{\frac{\beta}{2\alpha k^2}}\left(kx + \rho t + \iota\right)\right]^{-1} \tag{4}$$

Hence, the stochastic solutions for eq. (1):

$$\Theta_1(x,t) = \mathrm{e}^{i(cx+\nu t)}\left[\mp\sqrt{\frac{\beta}{2\alpha k^2}}\left(kx + \rho t + \iota\right)\right]^{-1} \tag{5}$$

where p, k, ν, γ, μ are constants with $\rho = -2\alpha ck$.

– Trigonometric function solution

The second class of solutions for eq. (3):

$$u_{2,3}(x,t) = \pm\sqrt{\frac{\alpha c^2 + \nu - \gamma}{\beta}}\tan\left[\sqrt{\frac{\alpha c^2 + \nu - \gamma}{2\alpha k^2}}\left(kx + \rho t + \iota\right)\right] \tag{6}$$

and

$$u_{4,5}(x,t) = u_{2,3}(x,t) = \pm\sqrt{\frac{\alpha c^2 + \nu - \gamma}{\beta}}\cot\left[\sqrt{\frac{\alpha c^2 + \nu - \gamma}{2\alpha k^2}}\left(kx + \rho t + \iota\right)\right] \tag{7}$$

Hence, the exact solutions for eq. (1):

$$\Theta_{2,3}(x,t) = \pm\sqrt{\frac{\alpha c^2 + \nu - \gamma}{\beta}}\,\mathrm{e}^{i(cx+\nu t)}\tan\left[\sqrt{\frac{\alpha c^2 + \nu - \gamma}{2\alpha k^2}}\left(kx + \rho t + \iota\right)\right] \tag{8}$$

and

$$\Theta_{4,5}(x,t)=\pm\sqrt{\frac{\alpha c^2+\nu-\gamma}{\beta}}\,\mathrm{e}^{i(cx+\nu t)}\cot\left[\sqrt{\frac{\alpha c^2+\nu-\gamma}{2\alpha k^2}}\left(kx+\rho t+\iota\right)\right] \tag{9}$$

– Hyperbolic function solution

The third class of solutions for eq. (3):

$$u_{6,7}(x,t)=\pm\sqrt{\frac{\gamma-\alpha c^2-\nu}{\beta}}\tanh\left[\sqrt{\frac{\gamma-\alpha c^2-\nu}{2\alpha k^2}}\left(kx+\rho t+\iota\right)\right] \tag{10}$$

and

$$u_{8,9}(x,t)=\pm\sqrt{\frac{\gamma-\alpha p^2-\nu}{\beta}}\coth\left[\sqrt{\frac{\gamma-\alpha c^2-\nu}{2\alpha k^2}}\left(kx+\rho t+\iota\right)\right] \tag{11}$$

Therefore, the exact solutions for eq. (1):

$$\Theta_{6,7}(x,t)=\pm\sqrt{\frac{\gamma-\alpha c^2-\nu}{\beta}}\,\mathrm{e}^{i(cx+\nu t)}\tanh\left[\sqrt{\frac{\gamma-\alpha c^2-\nu}{2\alpha k^2}}\left(kx+\rho t+\iota\right)\right] \tag{12}$$

and

$$\Theta_{8,9}(x,t)=\pm\sqrt{\frac{\gamma-\alpha c^2-\nu}{\beta}}\,\mathrm{e}^{i(cx+\nu t)}\coth\left[\sqrt{\frac{\gamma-\alpha c^2-\nu}{2\alpha k^2}}\left(kx+\rho t+\iota\right)\right] \tag{13}$$

where γ, α, c, ν, μ, k are arbitrary constants with $\rho = -2\alpha ck$.

Results and discussion

Some new random treatments of NRLSE are obtained in the improved forms, using the unified solver method. The obtained solutions can be used in verify the telecommunications experiments. The proposed solver presents a powerful new types of explicit solutions, such as rational, trigonometric, hyperbolic, explosive and periodic solutions. Our results clarifies that the used solver is reliable in dealing with stochastic NPDE to get new families of stochastic solutions with vital applications.

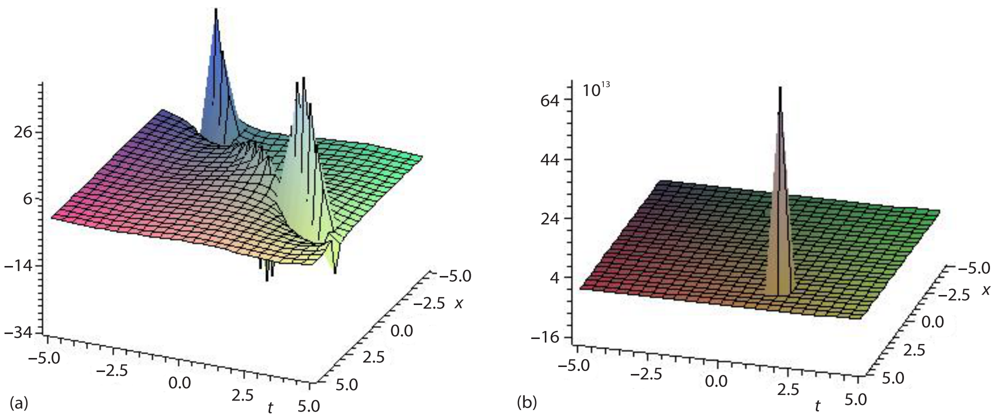

Figure 1. Expectation for the random solution Θ_1 with $k = 0.1$, $\beta = 0.3$, $\nu = 1$, and $\alpha = 2$, $\iota = 0.2$, c has geometric distribution (0.8) and exponential distribution (0.8), the first moment on the (a) and the second moment on the (b)

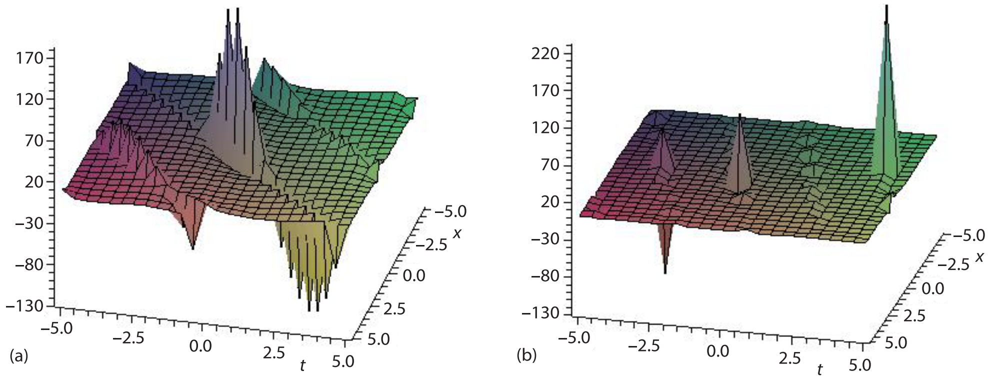

Figure 2. Expectation for the random solution Θ_3 with $k = 0.1$, $\beta = 0.3$, $\nu = 1$, and $\alpha = 2$, $\gamma = 3$, $\iota = 0.2$, c has geometric distribution (0.8) and exponential distribution (0.8), the first moment on the (a) and the second moment on the (b)

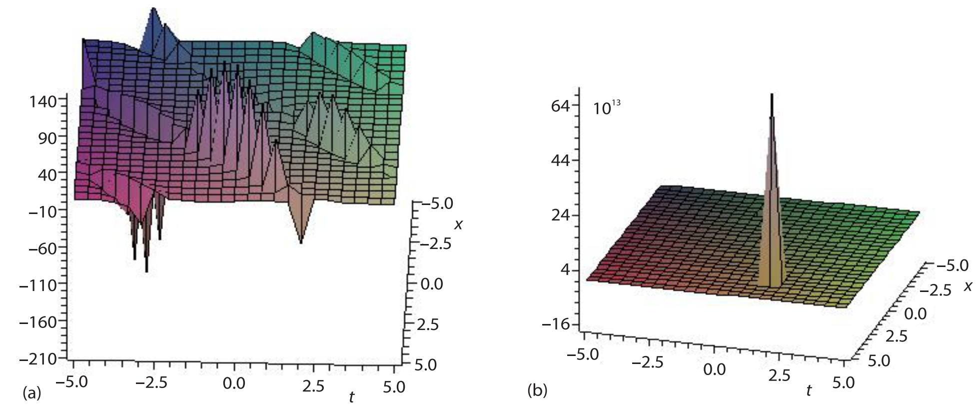

Figure 3. Expectation for the random solution Θ_5 with $k = 0.1$, $\beta = 0.3$, $\nu = 1$, and $\alpha = 2$, $\gamma = 3$, $\iota = 0.2$, c has geometric distribution (0.8) and exponential distribution (0.8), the first moment on the (a) and the second moment on the (b)

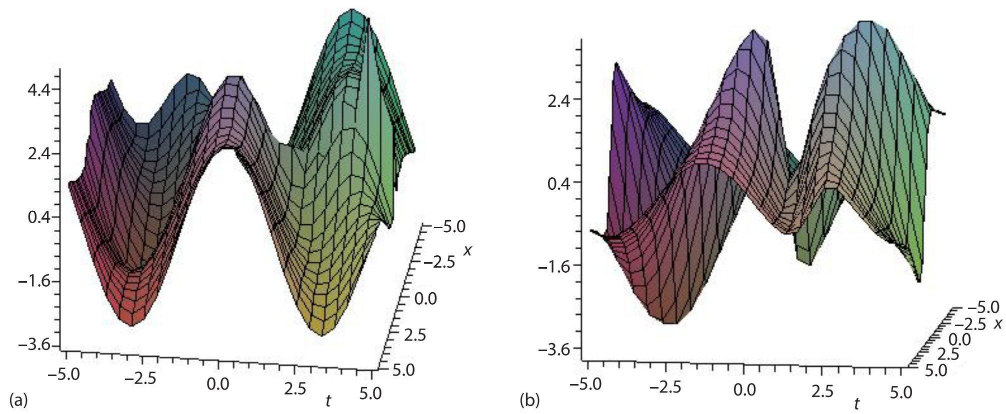

Figure 4. Expectation for the random solution Θ_7 with $k = 0.1$, $\beta = 0.3$, $\nu = 1$, and $\alpha = 2$, $\gamma = 5$, $\iota = 0.2$, c has geometric distribution (0.8) and exponential distribution (0.8), the first moment on the (a) and the second moment on the (b)

To demonstrate the effectiveness of the random variable parameters, we compute the mean graphically in the figs. 1-5 for our obtained stochastic solutions under geometric and exponential distributions. The mean is one of the measures of central tendency that provides information about the data value in the centre of the data set, and the variance is one of the measures of dispersion that provides information about the data scattered throughout the centre. According to our simulation, the geometric distribution is physically greater than the exponential distribution. Furthermore, $\Theta_1(x,t)$ is more stable than the other random solutions.

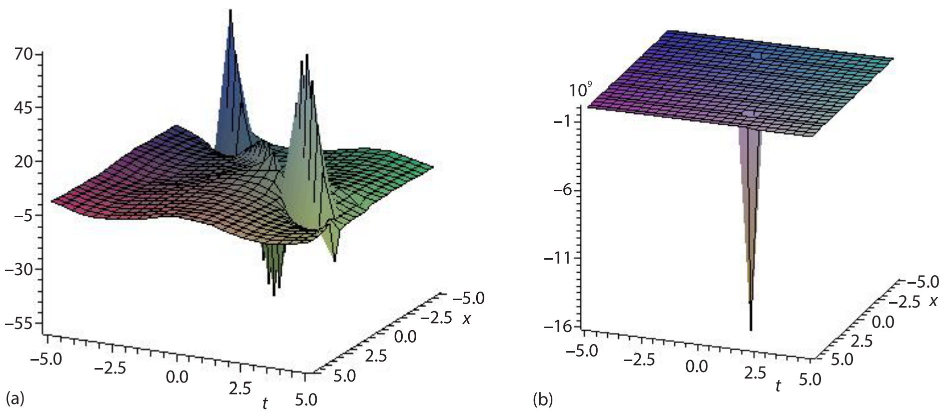

Figure 5. Expectation for the random solution Θ_9 with $k = 0.1$, $\beta = 0.3$, $\nu = 1$, and $\alpha = 2$, $\gamma = 5$, $\iota = 0.2$, c has geometric distribution (0.8) and exponential distribution (0.8), the first moment on the (a) and the second moment on the (b)

Conclusion

We present an accurate stochastic technique with random wave transformation in this paper. This solver provides the closed formula for the solutions. In this regard, a solver is created to precisely resolve and represent the entire wave structure of various types of NPDE with random inputs. The results in this paper, we believe, may be useful in explaining some physical phenomena for some non-linear models arising in mathematical physics. Two random distributions are used to compute statistical properties such as mean.

Acknowledgment

The researcher would be thankful for Qassim University's Deanship of Scientific Research for funding the publication of this research work.

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Paper submitted: August 21, 2022
Paper revised: September 12, 2022
Paper accepted: October 23, 2022