ROUGH FUZZY-TOPOLOGICAL APPROXIMATION SPACE WITH TOOTH DECAY IN DECISION MAKING

by

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Tooth decay is a common problem among people due to many factors, including neglect to clean teeth. Dental caries is becoming more common in a number of countries worldwide. This study aims to determine the prevalence of dental caries and how to treat this problem. We also implement fuzzy soft multifunction between fuzzy soft topological spaces which is Smarandache's generation of the notion. In this paper, we generated additional topologies for an information system based on a similarity relation. This paper discussed two methods for determining accuracy: our method and Pawlak's qualitative data method. Both ideas show that because of the uncertainty and ambiguity of qualitative data, we get a lot of topologies based on one or two attributes. The new method was used to determine the accuracies. This method revealed the difference between one or two attributes. In fact, the most important factors affecting tooth decay have been identified. Furthermore, we are developing a new algorithm to treat dental caries problems. Tooth decay which includes the conclusions of some well-known results in the corresponding literature is highlighted and discussed. In addition, a comparative application that dwells on the generality of our obtained results is constructed. Our proposed approach is reasonable and effective.

Key words*: maximum simply open set, rough set, similarity, tooth decay, fuzzy soft multifunction, intelligence discovery*

Introduction

Caries of the teeth is a worldwide epidemic. It affects roughly 60-90% of schoolchildren and nearly 100% of adults worldwide [1]. In contrast to past studies, the prevalence of dental caries has recently increased in a number of countries around the world [2]. Infants, toddlers, adults, and the elderly are all susceptible to dental caries. Caries causes inflammation of the dental pulp and surrounding tissues, which can lead to tooth loss, cellulitis, and, in rare cases, a brain abscess [3, 4]. Mashour *et al.* [5] introduce pre-open sets, also we introduce modification of simply open sets in [6]. Simply open sets were used by Dontchev and Ganster [7] to develop the concept of strongly simple continuity. The number of research articles published has exploded at a quick pace, particularly in mathematics. Several proposals were given for solving real-world problems with mathematical methodologies and relevant formulas to assist decision-makers in making the best decisions possible. To deal with challenges that are uncertain, see [8-10]. Pawlak [11] established the rough set theory in 1982 as a new mathematical technique or simple tools for dealing with ambiguity in knowledge-based systems and data dissection. This theory has numerous applications in process control, economics, medical diag-

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nosis, chemistry, psychology, finance, marketing, biochemistry, environmental science, intelligent agents, image analysis, biology, conflict analysis, telecommunications, and other domains, [12-15]. The 2016 Nobel Prize in Physics was awarded to international schools in Germany and the United States, for the theory of matter transformation that used topological applications of science and engineering [16].

Tooth decay may also lead to digestive problems such as trouble of colon and stomach acid disorder due to poor digestion. We include in this research symptoms of tooth decay, some of which may arise on people without consciousness of that, until examination by dentist. Also, we will show which of these symptoms will be more indicative of tooth decay that makes the injured person pay attention these symptoms and immediately visits the dentist so that he does not increase his decay and loss the health of many of his teeth.

Since fuzzy set theory was first proposed in 1965, it has grown in numerous diverse ways and in many different fields. This concept can be used in image processing, machine learning, information technology, medical industry, electrical engineering, decision theory, robotics, and so on, as well as in many other fields. Mathematical progress has come a long way and is still going onday. *Soft computing* or *computational intelligence* has been a term for fuzzy set theory, neural nets, and evolutionary programming since 1992, when it was first used. They have always been very close to each other. Authors the first fuzzy set theory papers, like [17-20] show that they wanted to change the way people think about things, like how they judge, evaluate, and make decisions.

Materials and methods literature review and motivations and main contributions of this paper

This section briefly summarizes some recent advances on new classes of simply continuous functions as generalizations of continuous functions, as well as their basic features and linkages with other types of generalized continuous functions in topological spaces [7]. Afterwards, the research motivations and main contributions of this paper are sketched.

Definition 2.1. [11] Let $H \in R$ and R be a family of equivalence relation. If IND(R) = IND(R-$\{H\}$), then H is dispensable in R. If IND (R) ≠ IND(R-$\{H\}$), we say that H is indispensable in R.

Definition 2.2. [12] If (U, H, V, f) is an information system defined as; f: $U \to V$, where H is the set of attributes, V is the domain of the particular attributes in which the values V are real numbers. We define a relation R_i for each objects $i(x)$: pR_iq iff $|i(p) - i(q)| < \sigma$.

Definition 2.3. [14] For the relation $R_B \subseteq U \times U$ is defined:

$$pR_Bq = \frac{\sum_{i \in B} |i(p) - i(q)|}{|B|} < \varsigma$$

where |B| is the cardinality of B and ς is determined by an expert of the field.

Definition 2.4. [11] If R is a binary relationship on the universal set U, then *After set* is defined as ${}_xR = \{y: {}_xR_y\}$.

Definition 2.5. [10] Let $K \subseteq U$ and (U, R, τ_R) be a topological space, then the lower (resp. upper) approximation of K is defined by $\underline{R}K = K^O$(resp. $\overline{R}K = \overline{K}$) and K-lower: $\overline{K}Z = \cup\{Y \in \text{IND}(K){:}Y \subseteq Z\}$, K-upper: $\overline{K}Z = \cup\{Y \in \text{IND}(K){:}Y \cap Z \neq \phi\}$.

Remark 2.1. In general $\overline{R}(\underline{R}S) \neq \underline{R}S$ and $\underline{R}(\overline{R}S) \neq \overline{R}S$. This principle is shown in the following example, which also makes use of topological approximations.

Example 2.1. Let $C = \{c_5, c_4, c_3, c_2, c_1\}$and R be a relation defined:

$$R=\{(c_1,c_2),(c_1,c_4),(c_2,c_5),(c_2,c_1),(c_3, c_3),(c_3,c_2),(c_4,c_5),(c_5,c_5),(c_5,c_1)\}$$

$$S = \{\{c_2,c_4\},\{c_1,c_5\},\{c_2,c_3\},\{c_5\}\}$$

$$V = \{\phi,\{c_2\},\{c_5\},\{c_1,c_5\},\{c_2,c_3\},\{c_2,c_4\}\}$$

$$\tau = \{C,\phi,\{c_2\},\{c_5\},\{c_1,c_5\},\{c_2,c_3\},\{c_2,c_4\},\{c_2,c_5\},\{c_1,c_2,c_5\},\{c_3,c_2,c_5\},\{c_4,c_2,c_5\},$$
$$\{c_3,c_2,c_4\},\{c_1,c_2,c_3,c_5\},\{c_1,c_2,c_4,c_5\},\{c_2,c_3,c_4,c_5\}\}$$

$$\tau^c = \{C,\phi,\{c_1\},\{c_3\},\{c_4\},\{c_1,c_4\},\{c_4,c_3\},\{c_1,c_3\},\{c_1,c_5\},\{c_1,c_3,c_4\},\{c_1,c_4,c_5\},$$
$$\{c_2,c_3,c_4\},\{c_1,c_3,c_5\},\{c_1,c_2,c_4,c_3\},\{c_1,c_4,c_3,c_5\}\}$$

Let $D = \{c_5, c_4\}$ then

$$\underline{R}D = \{c_5\},\ \overline{R}(\underline{R}D) = \{c_1,c_5\} \neq \underline{R}D$$
$$\overline{R}D = \{c_1,c_2,c_4,c_5\},\ \underline{R}(\overline{R}D) = \{c_1,c_2,c_3,c_4,c_5\} \neq \overline{R}D$$

Definition 2.6 [6] A subset M of topological space (W, λ). It indicated to be $\overset{M^*}{S}$-open if $M \in \{W, \varphi, M = G^* \cup N, G^*$ is the proper-open set, whereas N is the nowhere dense set$\}$.

Definition 2.7 [5] A mapping F: $(W, \lambda) \to (V, \vartheta)$ is said to be $\overset{M^*}{S}$-continuous if $F^{-1}(M) \in SO(W)$ for all G^* is proper open set in V.

Theorem 2.1. Let F: $(W, \lambda) \to (V, \vartheta)$ be mapping. Then the next statements are identical.
i. F is $\overset{M^*}{S}$-continuous.
ii. $\overset{M^*}{S}cl(F–1(K)) \subseteq F^{-1}(cl(K))$.
iii. $F((M)) \subseteq cl(F(M))$.
iv. If F is bijective then, $\text{int}(F(M)) \subseteq F(\overset{M^*}{S}\text{int}(M))$.
v. If F is bijective, then $F^{-1}(\text{int}(K)) \subseteq \overset{M^*}{S}\text{int}(F^{-1}(\text{K}))$.

Proof.
i $\to$ *ii*) Let $K \subset V$ proper closed set $F^{-1}(cl(K))$ is $\overset{M^*}{S}$ – the closed set in W then $\overset{M^*}{S}cl(F^{-1}(K)) \subset \overset{M^*}{S}cl(F^{-1}cl(K)) = F^{-1}cl(K)$.
ii $\to$ *iii*) Let $M \subset W$ by (*ii*) and let $(K) = F(M)$ then $F^{-1}F(M) \supseteq \overset{M^*}{S}cl(F^{-1}F(M)) \supseteq \overset{M^*}{S}cl(M)$ and therefore $cl(F(M) \supseteq FF^{-1}(M) \supseteq F(\overset{M^*}{S}cl(M))$.
iii $\to$ *iv*) Following by complementation of relation (*iii*) we have $W\backslash F^{-1}(cl(K)) \subseteq W \backslash \overset{M^*}{S}cl(F^{-1}(\text{K}))$, *i.e.*, $F^{-1}(V\backslash cl(K)) \subset \overset{M^*}{S}\text{int}(F^{-1}(V \backslash cl(K))$ put $V\backslash K = F(M)$.Then $F^{-1}(\text{int}F(M))) \subseteq \overset{M^*}{S}\text{int}(F^{-1}(F(M)))$. Since F is bijection. Then $\text{int}F(M)) \subseteq F(\overset{M^*}{S}\text{int}(\text{M}))$.
iv $\to$ *v*)Substitute $F^{-1}(K)$ instead of M in (*iv*) and using bijective map we have $\text{int}(K) = \text{int}(FF^{-1}(K)) \subseteq F(\overset{M^*}{S}\text{int}(F^{-1}(K))$ and therefore $F^{-1}(\text{int}(K)) \subseteq \overset{M^*}{S}\text{int}F^{-1}(K))$.
v $\to$ *i*) Let H is proper open set in V then $F^{-1}(H) = F^{-1}(\text{int}(H)) \subseteq \overset{M^*}{S}\text{int}F^{-1}(H))$ and therefore $F^{-1}(H) \in \overset{M^*}{S}O(W)$ hence F is $\overset{M^*}{S}$-continuous.

Remark 2.2. The composing of two $\overset{M^*}{S}$-continuous mappings not necessarily be $\overset{M^*}{S}$-continuous. The next example shows this remark.

Example 2.2. Let $W = V = \{a_1, b_1, c_1\}$, and $N = \{u_1, v_1, w_1, r_1\}$ with topology

$$\lambda = \{W,\varphi,\{c_1\},\{a_1,b_1\}\}, \vartheta = \{V,\varphi,\{a_1\}\},\ \text{and}\ \xi = \{N,\varphi,\{u_1,w_1\}\}$$

let I be the identity mapping i: $(W, \lambda) \to (V, \vartheta)$ and F: $(V, \vartheta) \to (N, \xi)$ defined as $F(a_1) = u_1$, $F(b_1) = v_1$, $F(c_1) = w_1$, $i(M) = b_1$, $i(b_1) = c_1$, and $i(c_1) = M$. Consequently we have each i and F is $\overset{M^*}{S}$-continuous but $F \circ i$ is not $\overset{M^*}{S}$-continuous since there exist a proper open set $\{u_1, w_1\}$ in N. But

$$F^{-1}(\{u_1,w_1\}) = \{b_1,c_1\} \notin \overset{M^*}{S}O(W)$$

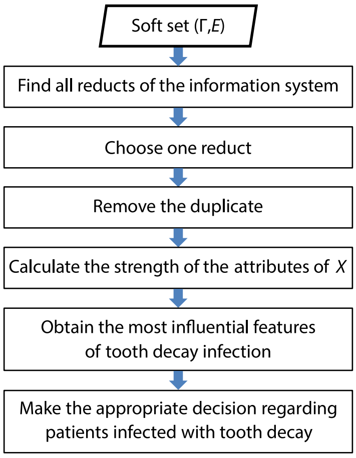

Figure 1. Framework the result of the proposed method

Algorithms and frameworks

This study concludes the method for dealing with tooth decay information. The algorithms (*Algorithm i*) and frameworks, fig. 1, is shown to demonstrate the suggested method's logic and organization structure.

Algorithm-i

Step 1. Input the soft set (Γ, E).
Step 2. Find all reducts of the soft set (Γ, E).
Step 3. Choose one reduct say (Γ, E).
Step 4. We remove the duplicate rows.
Step 5: We find the strength of the attributes with respect to the decision for association rule $x \rightarrow D$ where χ is the inserted values in the table and D is the decision.
Step 6. Find the most influential features of tooth decay infection.
Step 7. We make the appropriate decision regarding patients infected with tooth decay.

Medical decision-making

Medical application by rough fuzzy in tooth decay

We would like to point out that the information gathered in this tooth decay study came from 100 patients. Due to the similarity of the variables in the rows, this large sample of 100 patients was limited to ten patients describing the most severe symptoms of tooth decay (objects), where things were categorized as follows; $U = \{X_1, X_2, ..., X_{10}\}$ denotes 10 listed patients, the attributes as $\{a_1, a_2, ..., a_5\}$ = {pain, initial demineralization, cavity, headache and earache, hyper sensitivity} and decisionoth decay $\{d\}$. The following data was gathered by a dental consultant as well as medical groups that specialize in tooth decay.

Take into account the following information system in tab. 1.

Table 1. Decision-making data set based on information

Patients	Pain	Initial demineralization	Cavity	Headache and earache	Hyper sensitivity	Dental caries decision
X_1	No	Yes	Small	No	Low	No
X_2	Simple	Yes	Fracture	No	No	Yes
X_3	Strong	Yes	Small	No	No	Yes
X_4	No	No	Small	No	No	No
X_5	No	Yes	Fracture	No	Low	Yes
X_6	Simple	No	Small	No	Yes	No
X_7	Simple	Yes	Small	No	Yes	Yes
X_8	Strong	Yes	Small	Yes	Yes	Yes
X_9	No	Yes	Small	Yes	Low	Yes
X_{10}	Simple	Yes	Small	Yes	No	No

By the next tab. 2, we are redrawing the consistent section of tab. 1.

Table 2. Redrawing the consistent section of tab. 1

Patients	Pain (a_1)	Initial demineralization (a_2)	Cavity (a_3)	Headache and earache (a_4)	Hyper sensitivity (a_5)	Dental caries decision (d)
X_1	1	2	2	1	2	1
X_2	2	2	4	1	1	2
X_3	3	2	2	1	1	2
X_4	1	1	2	1	1	1
X_5	1	2	4	1	2	2
X_6	2	1	2	1	3	1
X_7	2	2	2	1	3	2
X_8	3	2	2	2	3	2
X_9	1	2	2	2	2	2
X_{10}	2	2	2	2	1	1

Then, in tab. 3, by omitting the a_1 attribute as shown.

Table 3. Removes the attribute a_1 from tab. 2

$U/A - \{a_1\}$	Attributes $(A - \{a_1\})$			
	(a_2)	(a_3)	(a_4)	(a_5)
$W_1 = \{X_1\}$	2	2	1	2
$W_2 = \{X_2\}$	2	4	1	1
$W_3 = \{X_3\}$	2	2	1	1
$W_4 = \{X_4\}$	1	2	1	1
$W_5 = \{X_5\}$	2	4	1	2
$W_6 = \{X_6\}$	1	2	1	3
$W_7 = \{X_7\}$	2	2	1	3
$W_8 = \{X_8\}$	2	2	2	3
$W_9 = \{X_9\}$	2	2	2	2
$W_{10} = \{X_{10}\}$	2	2	2	1

We note that, IND (A) = IND ($A - \{a_1\}$), …., then a1 is superfluous.
Next, by leaving out the a_2 attribute in tab. 4 as follows.

Table 4. Removes from tab. 2 attribute a_2

$U/A - \{a_2\}$	Attributes $(A - \{a_2\})$			
	(a_1)	(a_3)	(a_4)	(a_5)
$W_1 = \{X_1, X_7\}$	1	2	1	2
$W_2 = \{X_2\}$	2	4	1	1
$W_3 = \{X_3\}$	3	2	1	1
$W_4 = \{X_4\}$	1	2	1	1
$W_5 = \{X_5\}$	1	4	1	2
$W_6 = \{X_6\}$	2	2	1	3
$W_7 = \{X_8\}$	3	2	2	3
$W_8 = \{X_9\}$	1	2	2	2
$W_9 = \{X_{10}\}$	2	2	2	1

We note that, IND (A) ≠ IND ($A - \{a2\}$), …., then a_2 is indispensable.

After that, in tab. 5, remove the a_3 attribute as shown.

Table 5. Deletes the attribute a_3 from tab. 2

$U/A - \{a_3\}$	Attributes $(A - \{a_3\})$			
	(a_1)	(a_2)	(a_4)	(a_5)
$W_1 = \{X_1, X_5\}$	1	2	1	2
$W_2 = \{X_2\}$	2	2	1	1
$W_3 = \{X_3\}$	3	2	1	1
$W_4 = \{X_4\}$	1	1	1	1
$W_5 = \{X_6\}$	2	1	2	2
$W_6 = \{X_7\}$	2	2	1	3
$W_7 = \{X_8\}$	3	2	1	3
$W_8 = \{X_9\}$	1	2	2	3
$W_9 = \{X_{10}\}$	2	2	2	2

It is worth noting that, IND $(A) \neq$ IND $(A - \{a_3\})$, …., then a_3 is required. Following that, in tab. 6, leave off the a_4 attribute as follows.

Table 6. The a_4 attribute is removed from tab. 2

$U/A - \{a_4\}$	Attributes $(A - \{a_4\})$			
	(a_1)	(a_2)	(a_3)	(a_5)
$W_1 = \{X_1, X_9\}$	1	2	2	2
$W_2 = \{X_2\}$	2	2	4	1
$W_3 = \{X_3\}$	3	2	2	1
$W_4 = \{X_4\}$	1	1	2	1
$W_5 = \{X_5\}$	1	2	4	2
$W_6 = \{X_6\}$	2	1	2	3
$W_7 = \{X_7\}$	2	2	2	3
$W_8 = \{X_8\}$	3	2	2	3
$W_9 = \{X_{10}\}$	2	2	2	1

We observe that, IND $(A) \neq$ IND $(A - \{a_4\})$, …., then a_4 *is* indispensable. After that, in tab. 7, remove the a_5 attribute as shown.

Table 7. Removes from tab. 2 attribute a_5

$U/\text{A} - \{a_5\}$	Attributes $(A - \{a_5\})$			
	(a_1)	(a_2)	(a_3)	(a_4)
$W_1 = \{X_1\}$	1	2	2	1
$W_2 = \{X_2\}$	2	2	4	1
$W_3 = \{X_3\}$	3	2	2	1
$W_4 = \{X_4\}$	1	1	2	1
$W_5 = \{X_5\}$	1	2	4	1
$W_6 = \{X_6\}$	2	1	2	1
$W_7 = \{X_7\}$	2	2	2	1
$W_8 = \{X_8\}$	3	2	2	2
$W_9 = \{X_9\}$	1	2	2	2
$W_{10} = \{X_{10}\}$	2	2	2	2

As a result, IND (A) = IND $(\text{A} - \{a_5\})$, …., then a_5 is unnecessary.

The elimination of attributes is the following tab. 8.

Table 8. Attribute removal

	Attribute deletion					
Fundamental sets numbers	None	a_1	a_2	a_3	a_4	a_5
	10	10	9	9	9	10

As a result of this reduction, tab. 9 displays a revised information table.

Table 9. Reduced information

U/A'	Attributes, A'		
	a_2	a_3	a_4
X_1	2	2	1
X_2	2	4	1
X_3	2	2	1
X_4	1	2	1
X_5	2	4	1
X_6	1	2	1
X_7	2	2	1
X_8	2	2	2
X_9	2	2	2
X_{10}	2	2	2

In our application, the set {initial demineralization, headache and earache, and cavity} is a reduct of attributes original set {pain, initial demineralization, cavity, headache and earache, hyper sensitivity} but {pain, hyper sensitivity} are redundant.

Table 10 presents a new information table based on this partition, as the follows.

Table 10. Partition information

$S = U/A'$	Attributes, A'		
	a_2	a_3	a_4
$s_1 = \{X_1, X_3, X_7\}$	2	2	1
$s_2 = \{X_2, X_5\}$	2	4	1
$s_3 = \{X_4, X_6\}$	1	2	1
$s_4 = \{X_8, X_9, X_{10}\}$	2	2	2

For attribute a_2, *Initial demineralization* we get $|A| = 3$.

$$x_i R y_i \leftrightarrow |x_i - y_i| \geq \partial : \partial = \frac{1}{4}$$

as given in tab. 11

Table 11. Similarity by attribute a_2

S	S			
	s_1	s_2	s_3	s_4
s_1	0	0	1/3	0
s_2	0	0	1/3	0
s_3	1/3	1/3	0	1/3
s_4	0	0	1/3	0

From tab. 11, we get the equivalence relation which is quasi discrete topology and hence the class of simply * open sets is itself the quasi-discrete topology:

$$R = \{(s_1, s_3), (s_2, r_3), (s_4, s_3), (s_3, s_1), (s_3, s_2), (s_3, s_4)\}$$

and the right neighborhood of all elements

$$R_r s_1 = \{s_3\},\ R_r s_2 = \{s_3\},\ R_r s_3 = \{s_1, s_2, s_4\},\ R_r s_4 = \{s_3\}$$

Then the topology deduced:

$$\tau = \{S, \phi, \{s_3\}, \{s_1, s_2, s_4\}\} \text{ and } \tau^c = \{S, \phi, \{s_3\}, \{s_1, s_2, s_4\}\}$$

Hence the class of simply* open set of S is:

$$\overset{M*}{S}\, O(S)_{a_2} = \{S, \varphi, \{s_3\}, \{s_1, s_2, s_4\}\}$$

For Attribute a_3, *Headache and earache* we get $|A| = 3$

$$x_i R y_i \leftrightarrow |x_i - y_i| \geq \partial : \partial = \frac{1}{4}$$

as given in tab. 12.

Table 12. Similarity by Attribute a_2

S	S			
	s_1	s_2	s_3	s_4
s_1	0	2/3	0	0
s_2	2/3	0	2/3	2/3
s_3	0	2/3	0	0
s_4	0	2/3	0	0

From tab. 12, we get the equivalence relation which is quasi discrete topology and hence the class of simply* open sets is itself the quasi-discrete topology:

$$R = \{(s_1, s_2), (s_2, s_1), (s_2, s_3), (s_2, s_4), (s_3, s_2), (s_4, s_2)\}$$

and the right neighborhood of all elements.

$$R_r s_1 = \{s_2\}, R_r s_2 = \{s_1, s_3, s_4\}, R_r s_3 = \{s_2\}, R_r s_4 = \{s_2\}$$

Then the topology deduced:

$$\tau = \{S, \phi, \{s_2\}, \{s_1, s_3, s_4\}\} \text{ and } \tau^c = \{S, \phi, \{s_2\}, \{s_1, s_3, s_4\}\}$$

Hence the class of simply* open set of S is:

$$\overset{M*}{S}\, O(S)_{a_3} = \{S, \varphi, \{s_2\}, \{s_1, s_3, s_4\}\}$$

For attribute a_4, *Cavity* we get $|A| = 3$:

$$x_i R y_i \leftrightarrow |x_i - y_i| \geq \partial : \partial = \frac{1}{4}$$

as given in tab. 13.

Table 13. Similarity by Attribute a_4

S	S			
	s_1	s_2	s_3	s_4
s_1	0	0	0	1/3
s_2	0	0	0	1/3
s_3	0	0	0	1/3
s_4	1/3	1/3	1/3	0

From tab. 13, we get the equivalence relation which is quasi discrete topology and hence the class of simply* open sets is itself the quasi-discrete topology:

$$R = \{(s_1, s_4), (s_2, r_4), (s_3, s_4), (s_4, s_1), (s_4, s_2), (s_4, s_3)\}$$

and the right neighborhood of all elements

$$R_r s_1 = \{s_4\}, R_r s_2 = \{s_4\}, R_r s_3 = \{s_4\}, R_r s_4 = \{s_1, s_2, s_3\}$$

Then the topology deduced:

$$\tau = \{S, \phi, \{s_4\}, \{s_1, s_2, s_3\}\} \text{ and } \tau^c = \{S, \phi, \{s_4\}, \{s_1, s_2, s_3\}\}$$

Hence the class of simply* open set of S:

$$\overset{M*}{S}\, O(S)_{a_4} = \{S, \varphi, \{s_4\}, \{s_1, s_2, s_3\}\}$$

Result 1. From the aforementioned tables, we deduce that: tabs. 6 and 7 observe the class of simply*open sets:

$$\overset{M*}{S}\, O(S)_{a_2} \neq \overset{M*}{S}\, O(S)_{a_3} \neq \overset{M*}{S}\, O(S)_{a_4}$$

and hence the CORE is $\{a_2, a_3, a_4\}$. Initial demineralization, headache and earache, and cavity are important indicators of tooth decay.

Discussion-a_2

Removed Attribute a_1: we get the results shown in tab. 14.

For a_2 [(d = Yes, $X_{\text{Yes}} = \{X_2, X_3, X_5, X_7, X_8, X_9\}$), ($d$ = No, $X_{\text{No}} = \{X_1, X_4, X_6, X_{10}\}$)] $X = X_{\text{Yes}} + X_{\text{No}}$.

Table 14. Indispensable a_2

$S = U/A' - \{a_2\}$	Attributes ($A' - \{a_2\}$)	
	a_3	a_4
$s_{12} = \{X_1, X_3, X_4, X_6, X_7\}$	2	1
$s_{22} = \{X_2, X_5\}$	4	1
$S_{32} = \{X_8, X_9, X_{10}\}$	2	2

For a_2 [(d = Yes, $X_{\text{Yes}} = \{X_2, X_3, X_5, X_7, X_8, X_9\}$)]:

- Lower approximation is donated as $\{X_2, X_5\}$, then $|L_{\text{Yes}}| = 2$.
- Upper approximation is $\{X_1, X_2, X_3, X_4, X_5, X_6, X_7, X_8, X_9, X_{10}\}$, then $|U_{\text{Yes}}| = 10$.
- Accuracy of approximation is

$$\mu(a_2) = |L_{\text{Yes}}| / |U_{\text{Yes}}| = \frac{2}{10} = 20\%$$

For a_2 [(d = No, $X_{Yes} = \{X_1, X_4, X_6, X_{10}\}$)]

- Lower approximation is donated as $\{\phi\}$, then $|L_{No}| = 0$.
- Upper approximation is $\{X_1, X_3, X_4, X_6, X_7, X_8, X_9, X_{10}\}$, then $|U_{No}| = 8$.
- Accuracy of approximation is

$$\mu(a_2) = |L_{No}|/|U_{No}| = 0$$

Discussion-a_3

Removed attribute a_3: we will get the results shown tab. 15.

Table 15. Indispensable a_3

$S = U/A' - \{a_3\}$	Attributes $(A' - \{a_3\})$	
	a_2	a_4
$s_{13} = \{X_1, X_2, X_3, X_5, X_7\}$	2	1
$s_{23} = \{X_4, X_6\}$	1	1
$S_{33} = \{X_8, X_9, X_{10}\}$	2	2

For a_3 [(d = yes, $X_{Yes} = \{X_2, X_3, X_5, X_7, X_8, X_9\}$)]:

- Lower estimate is given $\{\phi\}$, then $|L_{Yes}| = 0$.
- Upper approximation is given as $\{X_1, X_2, X_3, X_5, X_7, X_8, X_9, X_{10}\}$, then $|U_{Yes}| = 8$.
- Accuracy is

$$\mu(a_3) = |L_{Yes}|/|U_{Yes}| = 0\%$$

For a_3 [(d = No, $X_{No} = \{X_1, X_4, X_6, X_{10}\}$)]:

- Lower is $\{X_4, X_6\}$, then $|L_{No}| = 2$.
- Upper is $\{X_1, X_2, X_3, X_5, X_7, X_8, X_9, X_{10}\}$, then $|U_{No}| = 10$.
- Accuracy is

$$\mu(a_3) = |L_{Yes}|/|U_{Yes}| = \frac{2}{10} = 20\%$$

Discussion-a_4

Attribute eliminated a_4: we will get the results shown tab. 16.

Table 16. Indispensable a_4

$S = U/A' - \{a_4\}$	Attributes $(A' - \{a_4\})$	
	a_2	a_3
$s_{14} = \{X_1, X_3, X_7, X_8, X_9, X_{10}\}$	2	2
$s_{24} = \{X_2, X_5\}$	2	4
$S_{34} = \{X_4, X_6\}$	1	2

For a_4 [(d = yes, $X_{Yes} = \{X_2, X_3, X_5, X_7, X_8, X_9\}$)]:

- As a lower approximation, $\{X_2, X_5\}$, then $|L_{Yes}| = 0$.
- Upper is $\{X_1, X_2, X_3, X_5, X_7, X_8, X_9, X_{10}\}$, then $|U_{Yes}| = 8$.
- Accuracy is

$$\mu(a_4) = |L_{Yes}|/|U_{Yes}| = \frac{2}{8} = 25\%$$

For a_4 [(d = No, $X_{No} = \{X_1, X_4, X_6, X_{10}\}$)]:

- Lower donated as $\{X_4, X_6\}$, then $|L_{No}| = 2$.

– Upper is $\{X_1, X_3, X_4, X_6, X_7, X_8, X_9, X_{10}\}$, then $|U_{\text{No}}| = 8$.
– Accuracy is

$$\mu(a_4) = |L_{\text{Yes}}|/|U_{\text{Yes}}| = \frac{2}{8} = 25\%$$

Result 2. Therefore, the order in attributes of importance to tooth decay is $\{a_3, a_2. a_4\}$, cavity, initial demineralization, and headache and earache. Next, we shall introduce a new function called fuzzy soft simply* multifunction between two fuzzy soft topologies. Also, we introduce the concepts of fuzzy soft simply* lower and fuzzy soft simply* upper. We will use the new approximation in medical.

Definition 3.1. [17] Let F: $X \rightarrow Y$ be fuzzy soft multifunction from the ordinary into fuzzy soft space (Y, σ, E) for each $x \in X$ a soft set $F(x)$ over (Y, σ, E), F is said to be on if for each fuzzy soft set G_B over Y there exist $x \in X$ such that $F(x) = G_B$.

Definition 3.2. For fuzzy soft multifunction Γ: $(U, \tau) \rightarrow (Z, \sigma, I)$ the lower inverse $F^{-}(W, I)$ and the upper inverse $\Gamma^{+}(W, I)$ of a soft-set (W, I) upon Z as follows:

$$\Gamma^{-}(W,I) = \left\{n \in U : \Gamma(n) \leq (W,I)\right\},\ \Gamma^{+}(W,I) = \left\{n \in U : \Gamma(n) \wedge (W,I) \neq \varphi\right\}$$

Medical application by fuzzy soft multifunction

Explains the basic tasks performed by the medical expert in a group of patients and by transmitting their complaint into the possible causes of set that are of the cause of their disease. So, we get two soft classes (R, E), where (R, E) is the soft class of symptoms and their importance for the patient, T symbolize the causes and medical advantage for treatment. Take:

$$R = \{r_1, r_2, r_3, r_4, r_5, r_6, r_7, r_8\}, E = \{e_1, e_2, e_3, e_4\}$$

where r_1 is the eczema, r_2 – the migraine, r_3 – the burning stomach, r_4 – the joint paine, r_5 – the sleep massless, r_6 – the headache, r_7 – the herpes, r_8 – the anxiety, e_1 – the high important, e_2 – the medium important, e_3 – the low important, e_4 – the very low important, and $T = \{t_1, t_2, t_3, t_4, t_5, t_6, t_7, t_8\}$ where t_1 – the allergy, t_2 – the blood pressure, t_3 – the acidity, t_4 – the mod-disorder, t_5 – the fatigue, t_6 – the lock of appetite, t_7 – the obesity, and t_8 – the depression. The fuzzy soft set of patients may be given:

$$(G,E) = \{(e_1, \{r_2{}^{0.5}, r_4{}^{0.4}, r_8{}^{0.8}\}),\ (e_2, \{r_2{}^{0.3}, r_4{}^{0.2}, r_5{}^{0.6}, r_6{}^{0.7}\})$$
$$(e_3, \{r_4{}^{0.2}, r_6{}^{0.6}\}),\ (e_4, \{r_2{}^{0.2}, r_3{}^{0.3}, r_8{}^{1}\})\}.$$

The medical expert must first rely on the patient's historical knowledge of the condition of the disease, which is stored in the doctor's files. Map F: $(T, \tau) \rightarrow (R, \tilde{\tau}, E)$, is soft multifunction defined:

$$f(t_1) = \{(e_1, \{r_3, r_6\}), (e_2, \{r_7\}), (e_3, \{r_6\}), (e_4, \{r_6\})\}$$
$$f(t_2) = \{(e_1, \{r_4\}), (e_2, \varphi)\}, (e_3, \varphi), (e_4, \varphi)\}$$
$$f(t_3) = \{(e_1, \{r_1, r_2\}), (e_2, \{r_3, r_4\})\}, (e_3, \varphi), (e_4, \{r_8\})\}$$
$$f(t_4) = \{(e_1, \varphi), (e_2, \{r_1\}), (e_3, \{r_1, r_2\},), (e_4, \varphi)$$
$$f(t_5) = \{(e_1, \{r_1, r_3, r_4\}), (e_2, \{r_2, r_6\}), (e_3, \{r_8\}), (e_4, \{r_6, r_8\})\}$$
$$f(t_6) = \{(e_1, \{r_2, r_8\}), (e_2, \{r_2, r_6, r_7\}), (e_3, \{r_3\}), (e_4, \{r_1, r_7\})\}$$
$$f(t_7) = \{(e_1, \{r_6\}), (e_2, \varphi), (e_3, \{r_3\}), (e_4, \{r_1, r_7\})\}$$
$$f(t_8) = \{(e_1, \{r_2\}), (e_2, \{r_5\}), (e_3, \varphi), (e_4, \{r_8\})\}$$

So, from by computations give us the advantage causes and medical advantage for treatment:

$$F^{-}(G,E)=\{t_2,t_3,t_5,t_6,t_8\}$$

i.e. $F^{-}(G, E)$ = {blood pressure, acidity, fatigue, locks of appetite, depression}.

Conclusion

Safety from decay is important for human health, especially as teeth play a role in the first steps of digestion. Tooth decay is a common problem among people due to many factors, including neglect to clean teeth as well as unhealthy nutrition, which may lead some to take out their teeth. This work is a novel type of topology in which the general topology is derived from the information system after the extraneous data is removed. The current study is significant because it not only introduces new forms of continuous functions based on simply* open sets, but it also identifies a novel notion for identifying the primary symptoms of tooth decay using topological concepts. This research sheds new light on the issue of attribute reduction. It implies that more semantic features maintained by an attribute reduce should be carefully analyzed, so that the creator has a good probability of selecting an acceptable option. We have explained our method with a new algorithm and how to apply it using MATLAB. In reality, our suggestion is helpful in solving any future real-life problems. The proposed methods will be extended to a variety of other concepts in the future, such as the fuzzy set and fuzzy rough set.

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