ANALYTICAL SOLUTIONS OF THE REFRACTIVE INDEX IN THE IONOSPHERE Different Approximations

by

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From the square of the refractive index, this research presents many solutions for the refractive index of the ionosphere, which has a complex structure. It analyzes the square of the refractive index at a distance of 240 km in the ionosphere using the most often used ordinary wave. When the refractive index is required, the second and first scenarios of the refractive index were found to be the most suited.

Key words: *ionosphere, group and phase velocity, refractive index*

Introduction

The ionosphere is a natural plasma medium that extends from 50 km above the ground to roughly 2000 km [1-5]. The refractive index of the medium, in general, is the identity card of the physical and electrical structure of any medium. because it includes variables like refractive index, medium conductivity, dielectric structure, and resistance. When a wave enters this medium, it can happen in one of two ways. The wave either delivers energy to or absorbs energy from the medium [6-9]. The wave's velocity in the medium changes in both circumstances. As a result, the medium's refractive index, which is affected by phase and group velocity, changes [10-12].

The ionosphere has a birefringent structure, and refraction is direction-dependent and vectorial because the medium is anisotropic and collisional. The magnitudes of reflection, refraction, attenuation, and transmission of an electromagnetic wave are determined by the medium's refractive index [5, 12]. As a result, depending on the conditions to be considered and the operation, the square of the refractive index or its application will change. It is incorrect to work with the square of the refractive index when studying the phase and group velocities of an electromagnetic wave in a material like an ionosphere. The usage of the refractive index is required. Much research on the refractive index has been published in the past [13-18]. However, obtaining the square of the ionosphere medium's refractive index is insufficient for phase and group velocity. Researchers are interested in studying phase and group velocity to fill this study's gap. We calculated the refractive index using several methods, and in order

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to do so, we set the ionosphere to 240 km and determined which method would be best for using the refractive index.

Scenarios for refractive index solution in the ionosphere

When collision frequencies in the ionosphere are taken into consideration, the refractive index, n, of the medium, which discloses the identification information of a medium, has a complex structure [1-9, 13, 15-19]. However, the square of the refractive index, n^2, has been investigated analytically in previous research. Our goal is to overcome the refractive index problem by introducing various refractive index methods. For example, the ionosphere for the ordinary wave is given by:

$$n_o^2 = 1 - \frac{\omega_p^2}{\omega^2 + \nu^2} + i\frac{\nu\omega_p^2}{\omega^2 + \nu^2} \tag{1}$$

where ω_p:(electron plasma frequency), $\nu = (\nu_{\mathrm{ei}} + \nu_{\mathrm{en}})$ electron collision frequency ω:wave frequency. The group and phase velocity of any wave is given, respectively:

$$V_{\mathrm{g}} = \frac{c}{n + \omega \dfrac{\partial n}{\partial \omega}} \tag{2}$$

$$V_{\mathrm{p}} = \frac{c}{n} \tag{3}$$

where V_{g} is the group and V_{p} is the phase velocity. The wave's phase and group velocities are affected by the medium's refractive index, as shown in eqs. (2) and (3). As a result, the refractive index, n, must be researched to study these velocities in any medium.

The refractive index, which governs the physical and electrical characteristics of a medium, can be studied in a variety of ways. The following are the three situations that we have considered.

The first scenario

Three waves emerge from the earth's magnetic field when a wave with frequency moving in the z-direction is delivered into the ionosphere. Waves that are polarized in the magnetic field's direction, as well as ordinary and extraordinary waves that are perpendicular to the magnetic field. When collisions in the media are taken into account, the medium's refractive index gets complicated. We used the ordinary wave as the collisional medium in the ionosphere to test these scenarios. An Independent wave of the magnetic field is an ordinary wave:

$$n_o^2 = 1 - \frac{\omega_{\mathrm{p}}^2}{\omega^2 + \nu^2} + i\frac{\nu\ \omega_{\mathrm{p}}^2}{\omega^2 + \nu^2} = M + iN, \quad n_0 = \sqrt{M + iN} = a + ib \Rightarrow n_0 = (z)^{1/2} \tag{4}$$

To obtain the roots of general the refractive index, the roots of the complex number are given:

$$(z)^{1/n} = (r)^{1/n} \cos\left(\frac{\theta + 2k\,\Pi}{n}\right) + \mathrm{i}(r)^{1/n} \sin\left(\frac{\theta + 2k\,\Pi}{n}\right) \tag{5}$$

wher $k = 0,1,2,\ldots,n-1$

$$\theta = \mathrm{Arctan}\left(\frac{b}{a}\right) \text{ and } \mathrm{r} = \sqrt{a^2 + b^2} \tag{6}$$

The second scenario

The second scenario for refractive index solution of ordinary wave:

$$n_0^2 = 1 - \frac{\omega_p^2}{\omega^2 + \nu^2} + i\frac{\nu\ \omega_p^2}{\omega^2 + \nu^2} = M + iN, \quad n_0 = \sqrt{M + iN} = \mu + i\chi \tag{7}$$

$$\mu^2 - \chi^2 + i2\mu\chi = M + iN \tag{8}$$

$$\mu_{1,2}^2 = \frac{1}{2}\left[M \pm \sqrt{\left(M^2 + N^2\right)}\right] \tag{9}$$

$$\chi_{1,2}^2 = \frac{1}{2}\left[-M \pm \sqrt{\left(M^2 + N^2\right)}\right] \tag{10}$$

From here, the general solution:

$$n = \mu + i\chi = \mu_{1,2} + i\chi_{1,2} \tag{11}$$

The third scenario

The third scenario for the refractive index solution of the ordinary wave is:

$$n_0 = \sqrt{M + iN} = \mu + i\chi \tag{12}$$

If we expand this expression into a series, we have:

$$n = M^2\left[1 + i\frac{1}{2}\left(\frac{N}{M}\right) + \cdots\cdots\right] \tag{13}$$

The study used data from the region of the ionosphere called the plasma near F (240 km) Elazig. Data from the international reference ionosphere for 1990, the year with the most sunspots, was used to compute the values. The refractive index was not done using the refractive index itself in most ionosphere investigations, this is not a correct approximation for studies of the phase and group velocity of the ionosphere. However, in experiments and theoretical studies that require the refractive index, this is not a realistic approximation. The refractive index, for example, is required to calculate coefficients such as the reflection and propagation of a wave in a medium. Because the square of the refractive index has a complex structure when collisions are taken into consideration, we established three scenarios for the analytical solution and to acquire the refractive index itself. We tested for ordinary waves in the iono-

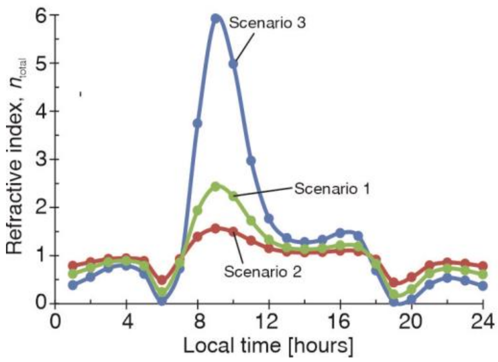

Figure 1. For many scenarios, the total refractive index changes with local time (March 21)

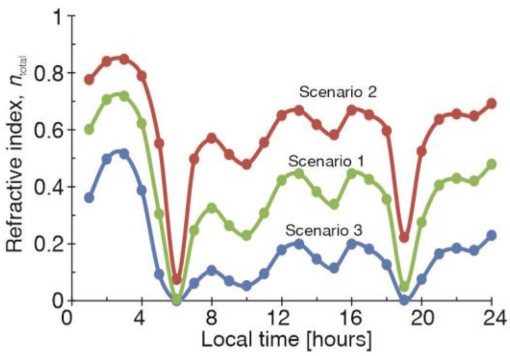

Figure 2. For many scenarios, the total refractive index changes with local time (June 21)

sphere under various conditions to test these possibilities, and the findings are shown in the figs. 1-4. The fluctuation of the total magnitude, n, of the ionosphere's refractive index with respect to local time is shown in the figures for several seasons. According to these, the trends in each season are comparable in approximation in three different scenarios. In the same season, though, it takes varied values. In general, a medium's refractive index provides its identity card, containing nearly all of the medium's attributes (such as dielectric, conductivity, polarization, and current). The refractive index is calculated by dividing the speed of a wave in a vacuum by the speed of propagation in any medium. As a result, except in unusual circumstances, the speed of propagation of a wave in a medium cannot exceed the speed of light. As a result, the refractive index values that should be taken should be in the range of 0-1. The refractive index supplied by the third scenario in the figures for each season is more than one as a result of this technique, whereas Scenario 1 is not. Examining all of the figures demonstrates this. (n = Total refractive index, n_R; real refractive index, n_I; imaginary refractive index).

Numerical analysis and results

Equations (3), (7), (8), and (11) F (240 km)-a area of ionosphere plasma near Elazig geographic co-ordinates were used in these studies. The values were calculated using IRI data for the year 1990 when the sunspot number was at its highest. The results are: According to the refractive index scenarios obtained as a result of the analytical solutions, the local time variation for March 21 and June 21 is given in figs. 1 and 2. Accordingly, the trend of refractive indices is the same for all three scenarios, but according to the third scenario, the mean value of the refractive index is larger than the other scenarios, and the lowest value in the second scenario. In figs. 3 and 4, the variation of the refractive index scenarios is given according to the accepted conditions. According to these figures, the change trends of the refractive index scenarios are similar to those in figs. 1 and 2. At around 5.00 a. m. and around 7.00 p. m. local time, the minimums reach maximum values between 5.00-19.00 local times. For all seasons, the change is paralleled by the change in electron density.

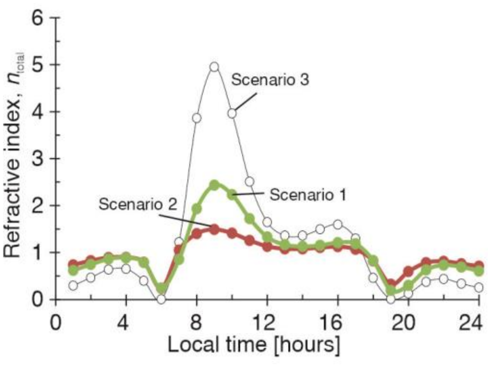

Figure 3. For many scenarios, the total refractive index changes with local time (September 23)

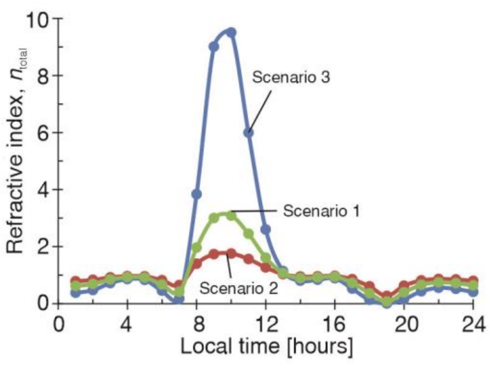

Figure 4. For many scenarios, the total refractive index changes with local time (December 21)

Conclusion

When collisions between particles in the ionosphere are taken into consideration, the square of the refractive index, n^2, becomes complex. Because the ionosphere's refractive index, *n*, contains so many characteristics, obtaining the refractive index, *n*, is challenging, and the square of the refractive index, n^2, is not adequate for several investigations. (For example, group velocity and phase). We looked at the refractive index itself rather than the square of the refractive index in this investigation, *n*,. We used three alternative ways to prepare three scenarios for the refractive index, *n*,. If the refractive index, *n*, itself is to be explored in the ionosphere, the second and first scenarios are more suitable, according to the data obtained.

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