ENERGY AND MASS TRANSFER ANALYSIS OF 3-D BOUNDARY-LAYER FLOW OVER A ROTATING DISK WITH BROWNIAN MOTION AND THERMO-PHORETIC EFFECTS

by

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The advancement of nanofluid technology has become an essential tool for investigating thermal conductivity enhancement, which is highly valuable for industrial and engineering applications in many fields including mathematics, physics, engineering, and materials science. This analysis focuses on 3-D boundary-layer flow on nanofluid over a rotating disk by incorporating chemical reaction and thermal radiations effects. One aim of this article is to analyze the energy and mass transport rates for nanofluids. In this study, the Brownian motion and thermophoretic impacts are considered. The governing flow equations are converted to ODE via suitable similarity transformations. The resulting equations were solved via well know technique Keller box method. This analysis revealed that the azimuthal and axial velocities show an inverse pattern against the various values of index factor, n, although the radial velocity has the highest value and decreases significantly. The behavior of the von Karman flow is also recovered for setting the index factor (n = 1). Moreover, it is found that the temperature of nano liquid increases by increasing the Brownian motion and thermophoretic factors.

Key words: *power law rotating disk, chemical reaction, nanofluid, Brownian motion, thermal radiations*

Introduction

In the last few decades, investigation of flow over a rotating disk has become an active area of research due to its potential to provide considerable progress in many industrial and engineering applications, such as gadgets, rotational viscometers, advanced plane design, and chemical engineering. Flow for a rotating disk was addressed first by von Karman [1]. This kind of flow includes the term radial pressure gradient at the outer layer of the disk which settles the centrifugal powers. Thus, the liquid over the surface twists outwards and it is being exchanged by an axial flow in the direction of the disk. Recently, we have seen explosive growth of activities investigating the flow over a rotating disk in-corporating the examination of rotational flow all through the funeral-shaped diffuser, rotation of the shrouded disk, and radiating siphons. The energy transportation phenom-enon has been discussed by the

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following researchers [2, 3]. Ramzan *et al*. [4] investigated rotating disk behavior for nanofluid flow via bvp4c MATLAB program. They considered variable viscosity along with variable thermal conductivity. The investigation of nanoliq-uid for power law stretchable disk was done by Chen *et al*. [5]. Ramzan *et al*. [6] reported the numerical analysis of rotating disks for nanoliquid flow by incorporating the chemical species.

Exploring the thermal conductivity of traditional fluids has become a crucial part of energy exchange investigation. Recently, nanoliquids have turned into the focal point of consideration due to their superb heat exchange performance. Choi and Eastman [7] dis-covered nanofluid in the late 1990's to improve heat conductivity by combining a base flu-id such as water, ethylene glycol, or oil. The existence of the nanoparticles in the fluid can improve the rate of change of the heat and working fluid properties due to their spe-cial properties. Buongiorno [8] investigated that the thermal conductivity of nanoliquid jumped due to the involvement of Brownian movement and thermophoretic phenomenon. Recently, different researchers studied nanoliquids by considering various geometries for reference see [9-15].

Thermal radiations have gained the attention of active researchers because of its key role in the heat transfer phenomenon which is very important in industry and engineering field. In order to design energy conversion systems the impacts of thermal radiations on the flow phenomenon and energy transportation are very significant [16]. Furthermore, with large variations in temperature thermal radiations are not suitable for construction of thermal apparatuses. Ghadikolaei *et al* [17] explored the imapacts of thermal radiations for a stretchy surface by considering nanoliquid. In addition, the investigation of nanoliquid for a stretchy surface with the effects of thermal radiations was done by Sheikholeslami *et al*. [18].

Chemical reactions can be characterized into different classifications like single or multi-stage reactions, catalyst, and non-catalyst reactions, heterogeneous, and homogenous reactions, *etc*. Mostly, the chemical reaction occurs through a synthetic interaction that comprises various phases called primary stages, which make a multifaceted chemical reaction. To diminish the difficulty of complex chemical reactions, we design a mathematical model. Bhandari [19] studied the effect of chemical reactions on the flow of micropolar nanofliquid via the finite element technique. Khan *et al*. [20] employed the homotopy method to investigate the impact of chemical reactions on hybrid nanoliquid flow. Moreover, the energy species transport rate of Casson-type nanoliquid by incorporating chemical impacts was investigated by Panigrahi *et al*. [21]. Anjum *et al*. [22] discussed the chemical reaction effects on nanoliquid flow by the utilization of double stratification. Further, Mondal *et al*. [23] discussed the chemical reaction influence on nanoliquid flow for a stretchable cylinder. In addition, Reddy *et al*. [24] considered Eyring-Powell nanoliquid to investigate the chemical reaction effects on a stretchable cylinder numerically. In recent years, different researchers utilized the chemical reaction impacts for different geometries [25-36].

All the aforementioned studies explored a rotating disk and nanofluid flow in various circumstances. However, there has been no prior research on the chemical reaction and thermal radiations effects on 3-D viscous nanofluid flow over a rotating stretchy disk. This paper aims to address that gap by investigating the influence of the physical factors involved on modeled flow. Brownian motion, and thermophoresis are all used in this innovative study. Using similarity transformations, the governing problems are turned into a series of ODE with proper boundary conditions, which are then numerically resolved using the Keller box approach. Graphs are used to quantitatively analyze and display the impacts of various physical factors on the contours of temperature, velocity, and concentration.

Mathematical formulation

This section analyzes the flow behavior of considering 3-D viscous nanofluid flow with azimuthal velocity over a rotating disk. The disk is rotating in an azimuthal direction with power-law velocity. The Brownian movement and thermophoretic effects are considered in this steady and incompressible flow. In addition, the chemical reaction and thermal radiations impacts are utilized. Under these effects and assumptions, the governing equations for the current study are given as:

$$\frac{\partial(ur)}{\partial r}+\frac{\partial(wr)}{\partial z}=0 \tag{1}$$

$$u\frac{\partial u}{\partial r}+w\frac{\partial u}{\partial z}+\frac{1}{\rho}\frac{\partial p}{\partial r}=\frac{v^2}{r}+\nu\frac{\partial^2 u}{\partial z^2} \tag{2}$$

$$u\frac{\partial v}{\partial r}+w\frac{\partial v}{\partial z}+\frac{uv}{r}=\nu\frac{\partial^2 v}{\partial z^2} \tag{3}$$

$$\frac{\partial w}{\partial r}u+\frac{\partial w}{\partial z}w+\frac{1}{\rho}\frac{\partial p}{\partial z}=\nu\frac{\partial^2 w}{\partial z^2} \tag{4}$$

$$\frac{\partial T}{\partial r}u+\frac{\partial T}{\partial z}w=\alpha\frac{\partial^2 T}{\partial z^2}-\frac{1}{(\rho c)_p}\frac{\partial q_r}{\partial z}+\tau\left[\frac{D_T}{T_\infty}\left(\frac{\partial T}{\partial z}\right)^2+D_B\frac{\partial C}{\partial z}\frac{\partial T}{\partial z}\right] \tag{5}$$

$$\frac{\partial C}{\partial r}u+\frac{\partial C}{\partial z}w=D_B\frac{\partial^2 C}{\partial z^2}+\frac{D_T}{T_\infty}\frac{\partial^2 T}{\partial z^2}+R^*\left(C-C_\infty\right) \tag{6}$$

The boundary conditions for the current flow phenomenon are:

$$v(r,0)=ar^n,\ u(r,0)=0,\ T(r,0)=T_w,\ w(r,0)=0,\ C(r,0)=C_w \tag{7}$$

$$u(r,z)\to 0,\ v(r,z)\to 0,\ T(r,z)\to T_\infty,\ C(r,z)\to C_\infty \text{ as } z\to\infty \tag{8}$$

Considering similarity transformations:

$$u(r,\eta)=af'(\eta)r^n,\ v(r,\eta)=ar^n g(\eta),\ \eta=cr^{\frac{n-1}{2}}z,\ C(r,\eta)=\frac{C-C_\infty}{C_w-C_\infty}$$
$$w(r,\eta)=-\frac{a}{2c}\left[(n+3)f+(n-1)\eta f'\right]r^{\frac{n-1}{2}},\ \theta(r,\eta)=\frac{T-T_\infty}{T_w-T_\infty} \text{ with } C=\sqrt{\frac{a(n+3)}{4\nu}} \tag{9}$$

For Rosseland approximation see reference [37]. With the utilization of similarity transformations eq. (1) identically satisfied, whereas eqs. (3), (5), and (6) are converted into:

$$g''+(2f)g'-\left(\frac{n+1}{n+3}\right)(4g)f'=0 \tag{10}$$

$$\left(1+\frac{4}{3}N\right)\theta''+\Pr\left[N_b\theta'\varphi'+N_t\theta'^2\right]+2\Pr f\theta'=0 \tag{11}$$

$$\varphi'' + Nt_b\theta'' + 2\mathrm{Le}f\varphi' + \mathrm{Le}R\varphi = 0 \tag{12}$$

where the physical parameters are defined as:

$N_b = \dfrac{\tau D_B\left(C_w - C_\infty\right)}{\nu}$ denotes Brownian motion, $N_T = \dfrac{\tau D_T\left(T_w - T_\infty\right)}{\nu T_\infty}$ thermophretic factor

$N = \dfrac{4\sigma^* T_\infty^3}{\alpha k^*}$ shows radiations parameter, $R = \dfrac{R^*}{a}$ denotes chemical reaction

$\mathrm{Pr} = \dfrac{\nu}{\alpha}$ denotes the Prandtl number, and $\mathrm{Le} = \dfrac{\nu}{D_B}$ shows the Lewis number

Now, eq. (2) converted into:

$$f''' + 2ff'' - f'^2\left(\frac{4n}{n+3}\right) + \left(\frac{4}{n+3}\right)g^2 + \frac{P_r}{\rho ac^2}r^{1-2n} = 0 \tag{13}$$

Equation (13) contains P_r (pressure gradient), thus by eq (4) we get:

$$P = \left[\int_\infty^\eta f' H d\eta(n-1) - \frac{1}{2}H'(n+3) - \frac{1}{2}H^2 + \frac{1}{2}H^2(\infty)\right]r^{n-1}\frac{\rho a\nu}{n+3} - \rho p_\infty(r) \tag{14}$$

where

$$H(\eta) = f(n+3) + f'(n-1)\eta$$

From eq. (14), noted that order of pressure is $O(r^{n-1})$. We neglect the pressure gradient term by considering axis of spinning of the disk and the radial position far from eachother in eq. (13). Thus eq. (13) reduces in the form:

$$f''' + 2ff'' - (f'^2)\left(\frac{4n}{n+3}\right) + \left(\frac{4}{n+3}\right)(g^2) = 0 \tag{15}$$

The physical quantities of our interest including skin friction (radial and azimuthal), local Nusselt number and Sherwood number are:

$$\tau_{rz} = \mu\left(\frac{\partial u}{\partial z}\right)_{z=o},\quad \tau_{\theta z} = \mu\left(\frac{\partial v}{\partial z}\right)_{z=o},\quad \mathrm{Sh} = \frac{\left(\dfrac{\partial C}{\partial z}\right)_{z=o}}{C_w - C_\infty},\quad \mathrm{Nu} = \frac{rQ_w}{T_w - T_\infty} \tag{16}$$

where μ stands for liquid viscosity.

For $m = 0$ our problem reduces to Von Karman flow behavior for spinning disk. Thus, we introduced $a = \Omega$. against gyration rate of disk.

$$u(r,\eta) = r\Omega f'(\eta),\ v(r,\eta) = r\Omega f(\eta),\ w(r,\eta) = -2\sqrt{\nu\Omega}f(\eta) \text{ where } \eta = \sqrt{\frac{\Omega}{\nu}}z$$

As referred by Turner [38] for the numerical results of eqs. (10)-(12) and (15), the value

$\zeta = \sqrt{\dfrac{a}{4\nu}}r^{\frac{n-1}{2}z}$, consider as a new variable.

Thus, we defined new value of c in eq (9), denoted by:

$$c_1 = \sqrt{\frac{1}{n+3}},\; c = \sqrt{\frac{a}{4\nu}}$$

Thus, the converted eqs. (10)-(12) and (15) take the form:

$$f''' + 2(n+3)(ff'') - (4n)f'^2 + 4g^2 = 0 \tag{17}$$

$$g'' + f(2n+6)g' - (4n+4)f'g = 0 \tag{18}$$

$$\left(1+\frac{4}{3}N\right)\theta'' + \mathrm{Pr}\left[N_t\theta'^2 + N_b\varphi'\theta'\right](n+3)^{\frac{1}{2}} + 2(n+3)\mathrm{Pr}f\theta' = 0 \tag{19}$$

$$\varphi'' + Nt_b\theta'' + 2(n+3)\mathrm{Le}f\varphi' + \mathrm{Le}R\varphi = 0 \tag{20}$$

The converted boundary conditions becomes:

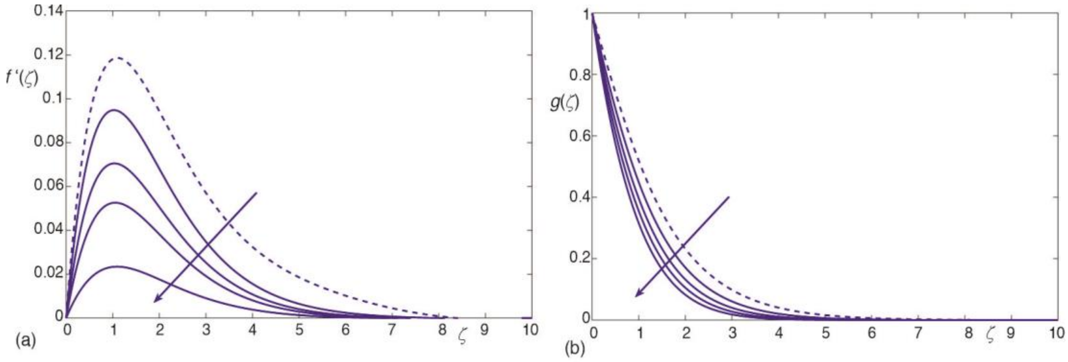
$$\begin{gathered} f(\zeta)=0, f'(\zeta)=0, g(\zeta)=1, \theta(\zeta)=1, \varphi(\zeta)=1 \text{ at } \zeta=0 \\ f'(\zeta)=0, g(\zeta)=0, \theta(\zeta)=0, \varphi(\zeta)=0 \text{ as } \zeta\to\infty \end{gathered} \tag{21}$$

Results and discussion

This section presents numerical simulations of 3-D viscous nanoliquid flow for a spinning disk. The energy equation takes into account the impacts of heat generation or absorption, whereas the mass transport equation includes chemical reaction. The supported similarity transformation is used to translate differential equations, which are then numerically solved using the Keller box technique.

The ODE (17)-(20) are solved numerically by utilizing eq. (21) via Keller box technique, for detail of method see [39]. The results for velocity, temperature and concentration are drawn, in which dotted lines show von Karman flow while remaining show current behavior of the flow. For the physics of the current problem, we assign suitable values to the involved factors including Brownian motion factor, N_b, thermophoretic factor, N_t, Lewis number, Prandtl number, and index factor, n. Figure 1 presents the radial in rectangle fig. 1(a), azimuthal in rectangle fig. 1(b) as well as axial velocity in rectangle fig. 1(c) behavior against index factor, n. The pattern of azimuthal and axial velocities show inverse relation against the various values of n, while radial velocity presents the maximum value which decrease marginally. Additionally, for $n = 1$ we recover the behavior of von Karman flow (dashed lines), whereas against $n = 10,15,20$ solid lines shows current flow. Physically, the increment in index factor, n, boost the nonlinearity as a result the resistive force dimishes to the incoming flow behavior thus the azimuthal, radial and axial velocities slow down.

Figure 2 presents $\theta(\zeta)$ shows direct correspondence with the impact of radiations. Physically, due to the increment in radiations factor more energy transferred to the liquid. Therefore by the improvement of radiations impacts the temperature of the liquid increases in return profile upsurges. The recovered results match with the temperature profile pattern of Shaheen *et al*. [40]. Figure 3 exhibits the behavior of chemical reaction effects on $\phi(\zeta)$ (concentration profile). It is cleared from fig. 3 the concentration distribution portrays inverse relation with chemical reaction effects which is similar results as recovered by Salleh *et al*. [41].



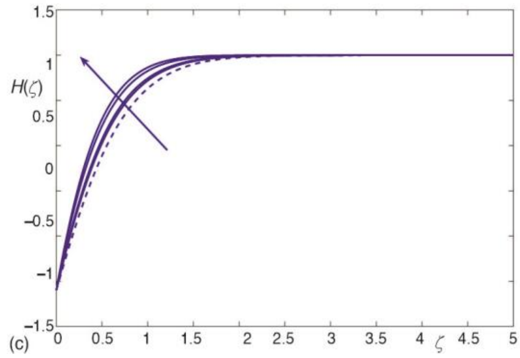


Figure 1. Impact of (n =10, 15, 20, 25 solid lines); presents radial (a), azimuthal (b), and axial (c) velocity distribution while (n =1 for von Karman flow presents via dashed lines)

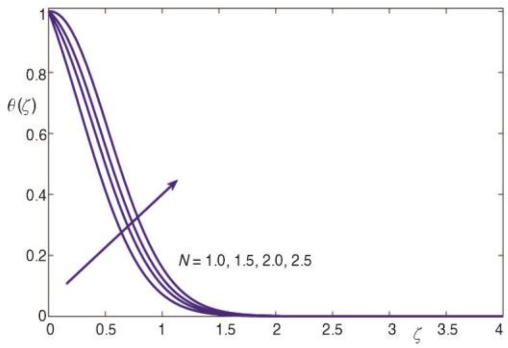


Figure 2. Impact of N on $\theta(\zeta)$

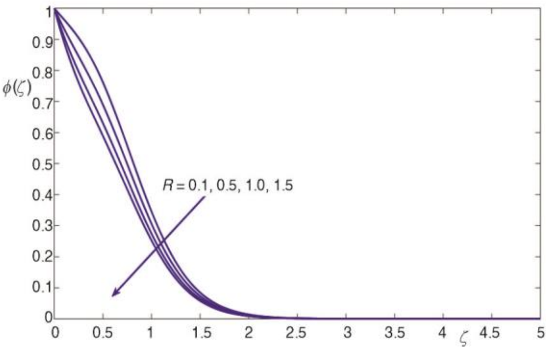


Figure 3. Impact of R on $\phi(\zeta)$

Figure 4 presents the behavior of $\theta(\zeta)$ and $\phi(\zeta)$ against n. Figure 4 clearly exhibits the temperature and concentration profile decreases with the increment in index factor, P_r. Physically the resistive force reduces due to the index factor which accelerates the flow in return solutal and thermal boundary-layer thickness diminishes. Figure 5 portrays the behavior of $\theta(\zeta)$ against Brownian motion factor, N_b, thermophoretic factor, N_t, and Prandtle number, which shows that the temperature of nano liquid increases by strengthen N_b and N_t. Physically by strengthen the thermophoretic factor the liquid particles pulls towards the cooler zone from the hot zone, while by strengthen the Brownian motion factor the kintetic energy of the particles enhance as a result the thermal boundary-layer thickness increases which present in figs. 5(a) and 5(b).

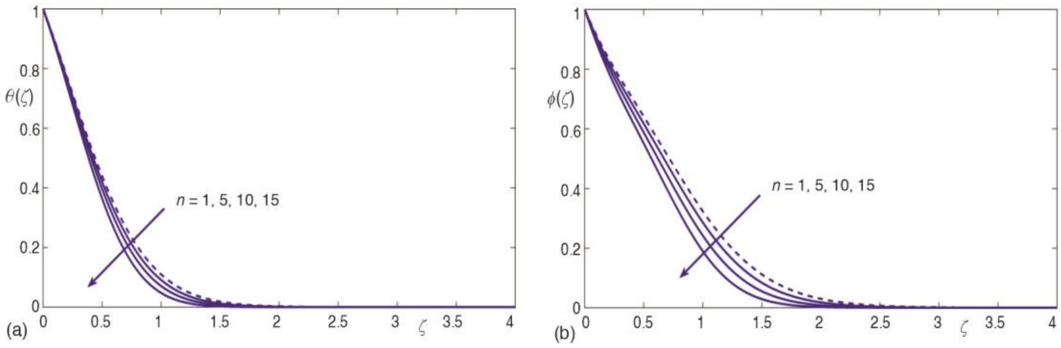


Figure 4. The $\theta(\zeta)$ and $\phi(\zeta)$ against (n = 5, 10, 15 solid lines) while (n = 1 for von Karman flow presents via dashed lines)

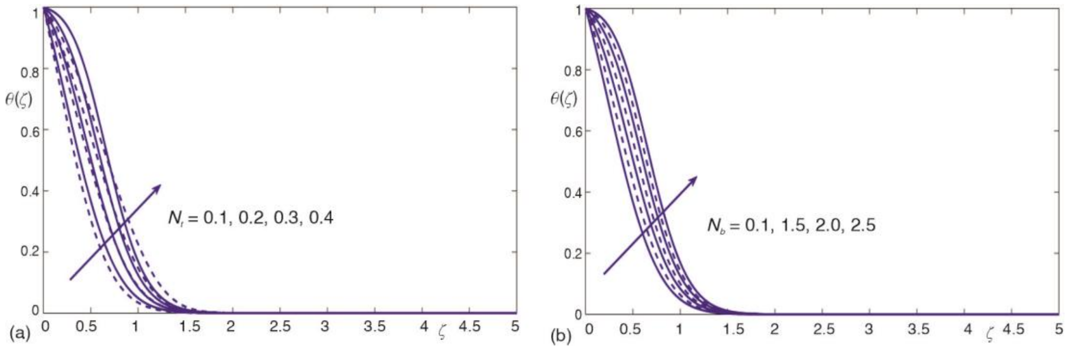


Figure 5. The $\theta(\zeta)$ against N_t, N_b, and Pr (n = 10) solid line while (n = 1 for von Karman flow)

Conclusions

This study presented numerical simulation of thermal transport and mass exchange rate by applying Keller box technique. By considering chemical reaction and thermal radiations along with Brownian motion, this study seeks to further and enhance the examination of the nanofluid flow over a rotating disk. In this study, we recovered the following outcomes.

- The $\phi(\zeta)$ (Concentration profile) portrays an inverse behavior against chemical reaction factor.
- The n (Power law index) shows an inverse correspondence against velocity, temperature, and concentration distribution.
- Mass transport look more efficient as compared to energy transport for Von Karman flow.
- The $\theta(\zeta)$ boosts as the thermal radiations factor increases.

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