ON THE INFLUENCE OF DILATION ANGLE IN SOIL SLOPE STABILITY ANALYSIS AMONG OTHER MODEL PARAMETERS

by

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Associated with slope stability, cohesion, and friction angle which are the most factors that can affect the slope failure, Investigation studies in geotechnical structure analysis establish that the dilatancy angle, ψ, is rarely acceptable in slope stability analysis, and when it considers the process is poorly sophisticated and simplistic. The paper describes two numerical slope stability analyses of a homogeneous soil with a plane strain model, to study the influence of dilation angle through different friction angle ranges (20, 25, and 30). The analysis used an immutable friction angle with variable dilation angle, results show that the safety factor increases gradually with the increase of dilation angle and dramatically decreases when the dilation angle is close to the friction angle, which means the dilation angle, ψ, affect the safety factor through slope analysis. It is very significant to check out the scrupulous value of dilation angle, ψ, when dealing with associated and non-associated soil parameters. In geotechnical and thermal analysis, dilation angle can affect the volume change as well as a safety factor.

Key words: *slope stability, numerical method, dilation angle, safety factor, friction angle*

Introduction

The most recent techniques of slope stability analysis in earth movement control are considering friction angle simply and disregarding the effect of dilation. Rowe [1] one of the earlier academics had examined the impact of soil dilatancy in geotechnical structure analysis and highlighted the impact of the dilatancy angle to slope analysis then set the relationship formula of stress dilatancy in slope stability. The importance of including change volume features of soil in slope stability analysis is the stress dilatancy. It is significant to notice that when the limit equilibrium method (LEM) is employed; the dilation angle is neglected, and friction angle can be used as a critical and that every time gives more reliable outcomes. However, it is difficult to get an intelligible value for the factor of safety (FoS) by using LEM due to it is assumptions [2, 3]. Conversely, in circumstances where a limit plasticity analysis or a finite element method (FEM) is used, unawareness of dilatancy special effects will result in an implied reflection of high dilation angles. Griffiths and Lane [4] first shown the volume alteration of the soil flowed by dilation angle, ψ, through yielding for instance, through shearing of average bushy material, firstly volume decrease ($\psi < 0$), might happen, pursued by a

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dilative phase ($\psi > 0$), leading eventually to yield under steady volume conditions ($\psi = 0$). The question then arises as to what value of ψ to use. If $\psi = ø$, where ø is the material's friction angle, then the plasticity flow rule is related and direct assessments with theorems from classical plasticity can be made. It is similarly the situation that when the flow rule is related, the stress and speediness qualities concurrence, thus closer convention can be predictable between failure mechanisms expected by finite elements and significant failure surfaces generated by LEM. The dilation angle cannot reflect the shearing dilatancy distinctive of soil material because it does not take into account flow rules [5]. Commonly, the failure tonnage for non-associated flow rule material is equal to those gained for the similar material when an associated flow rule is expected [6-9]. However, in the case of $\psi \neq ø$ (non-associated flow rule) the equilibrium of numerical is not easy to achieve, [10, 11].

The numerical analysis appear by FEM has gained acceptance grown in the application of geotechnical engineering since the 1970's, and with the rapped improvement of technology, it has become a powerful analysis tool [12-18]. Oka *et al.* [14] have done great work about the repercussions of soil dilatancy. Depending on the geometry of the difference between dilation angle, ψ, and friction angle, ø, the increasing result of ψ at the steady value of ø might both be impartial or advantageous to the comprehensive stability. So any underestimating values of ψ and ø in the following analysis then will lead to inaccuracy value [19].

Dilatancy angle is an important parameter which has much influence on slope safety factor. The objective of the paper is to explain the ideas of friction and dilatancy relative to the choice of strength parameters for slope design, and the influence of dilation angle among other materials on the geotechnical analysis especially slope stability.

The connections between friction and dilation angles

A lot of consideration has been centered around the connection between friction angle, ø', and dilation angle, ψ, defined the relation between ø' and ψ as shown in eq (1), [19]:

$$ø' = ø'_{crit} + \psi \tag{1}$$

where $ø'_{crit}$ is the angle of shear observed in a simple shear test on soil loose enough to be in critical state, [20] defined the two angles.

The angle of friction, ø', expresses the ratio of shear stress to normal stress and can be defined in terms of principal stresses, eq. (2) and fig. 1:

$$\sin ø' = \frac{\sigma'_1 - \sigma'_3}{\sigma'_1 + \sigma'_3} \tag{2}$$

where σ'_1 and σ'_3 are the minor and major shear stresses. Similarly, the angle of dilation ψ expresses the ratio between volumetric strain rate and shear strain rate. In the case of the plane strain $\varepsilon_2 = 0$ it can be defined in terms of the principal strain rates eq. (3) and fig. 1:

$$\sin\psi = \frac{-\left(\dot{\varepsilon}_1 + \dot{\varepsilon}_3\right)}{\dot{\varepsilon}_1 - \dot{\varepsilon}_3} \tag{3}$$

where $\dot{\varepsilon}_1$ and $\dot{\varepsilon}_3$ are the minor and major shear strain, it should be noted that dilation will always be important it will control the appropriate angle of friction.

By the growth in the practice of numerical modeling in rock mechanic engineering recently, excavation design has relied partially on numerical studies. Conduct investigations into the same field discovered that dilatancy angle, ψ, is rarely engaged in analysis, and when it is considered, the method needs improvement and facile, comprising of an associated flow rule (ø = ψ) or a non-associated flow rule with $\psi = 0$. Post-failure rock does not point out by an associated flow rule as some researchers mention [21].

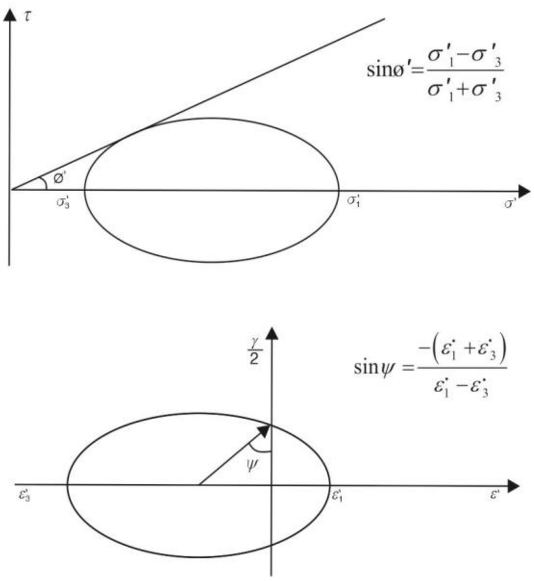

Figure 1. Definition of friction angle and dilation angel

Slope stability examples

Two slope examples of homogenous material are supposed to be elastic-perfectly plastic with a Mohr-Coulomb's criterion. The shear strength reduction (SSR) technique was applied in the finite element.

Software Abaqus 6.10-1 the properties of the materials are shown in tab. 1.

Table 1. Material properties of slope example (a) and (b)

Variables	Cohesion	Friction angle	Yong's modulus	Unit weight	Poison's ratio
	C [kPa]	Ø [°]	E [MPa]	γ [kgm^{-3}]	ν
Slope (a)	15	20,25,30	100	20	0.3
Slope (b)	12.38	20,25,30	100	20	0.3

Strength reduction method

The SSR approach considers prevalent methods for performance of FEM slope analysis, [4]. *The strength reduction technique for slope stability analysis was applied earlier* 1975 by [22], and then later by [4, 23-27]. In this technique they start logic reduction succession for the shear strength factors c' and ø' to allow the slope collapse to get the safety factor. So, this shear strength decreases the amount parameters c'_f and $ø'_f$ are defined in:

$$c'_f = \frac{c'}{\text{SRF}} \tag{4}$$

$$ø_f' = \tan^{-1}\left(\frac{\tan ø'}{\text{FOS}}\right) \tag{5}$$

where SRF is a strength reduction factor, the amount of the SRF is enlarged frequently till the slope collapse, so identifying the FoS of the slope. To recognize the failure point, the maximum displacements against SRF were plotted. The SRF at which we notice a sudden Increment in displacement provides the FoS [6].

Slope example (a)

The Mohr-Coulomb materials of slope example (a), which undergo gravity burden forces created by the soil-weight and applied to the slope once. The FoS is known as the percentage by which tan ø and *c* are decreased to cause failure which the gravity weight keeps constant. A six-trial strength reduction factor ranging (1, 1.2, 1.4, 1.5, 1.55, and 1.6) gradually weakens the soil until the algorithm fails to converge. The soil strength parameters used in the elastoplstic analysis, are obtained from the following tab. 1, and fig. 2, (material properties for slope example (a) and (b), shows the finite elements mesh of homogeneous 2:1 (H: V slope), the slope contains 1141 knots and 350 eight-knot solid elements, material properties. Vertical rollers on both sides were given as boundary conditions and fixed at the bottom, the tolerance and iteration are set to 0.0001 and 500 respectively.

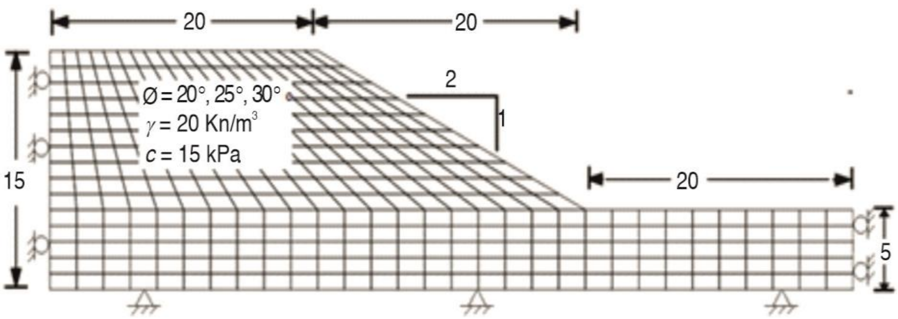

Figure 2. Homogeneous Slope examples (a) with dimensions and mesh

Slope example (b)

The Mohr-Coulomb materials of slope example (b) are also subjected to gravity burden forces. A six-trial strength reduction factor ranging (0.5, 0.75, 1, 1.25, 1.5, 1.75, and 2) was slowly macerated to make the algorithm unsuccessful to joint soil. Fig. 3, illustrates the finite elements mesh of homogeneous slope which contains 780 knots and 390 eight-knot solid elements, boundary conditions, and tolerance same as slope (a).

Analysis procedure

The two slope models were built according to materials properties in tab. 1, figs. 1 and 2, and then the following procedure was performed as follow. Firstly, fixed the friction angle ø and analyzed the different amounts of the dilation angle ψ from zero to the friction angle ø according to eq. (6) to get diverse safety factors:

$$\psi^i = X_\psi ø \tag{6}$$

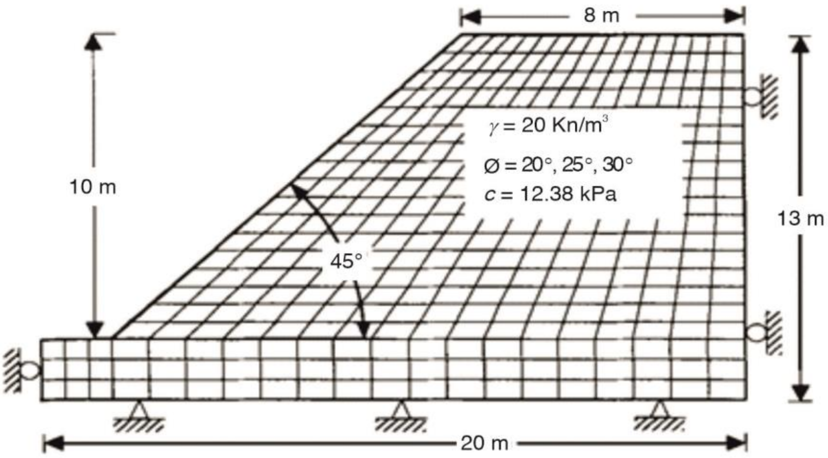

Figure 3. Homogeneous Slope examples (b) with dimensions and mesh

where $X\psi$ is the deviation factor of dilation angle, which is values change from 0 to 1, ψ^i is the dilation angle for *i* step. Second, change the friction angle from (20°, 25° to 30°) and then calculated the safety factor for each case as shown in tabs. 2 and 3, then plot the variation factor of dilation angle, $X\psi$, *vs*. safety factor difference (the difference of safety factor when $\psi = 0$ and the other safety factor with various amount of ψ) as shown in figs. 4 and 5. The results of plot variation FoS *vs*. safety factor difference shown also in figs. 4 and 5.

Table 2. Safety factor of different ψ slope (a)

Ø = 20°		Ø = 25°		Ø = 30°	
ψ	FoS	ψ	FoS	ψ	FoS
0	1.6	0	1.61	0	1.62
5	1.62	5	1.67	5	1.645
10	1.67	10	1.67	10	1.84
14	1.67	18	1.65	15	1.84
15	1.547	20	1.463	20	1.84
17	1.395	22	1.352	25	1.404
20	1.204	25	1.172	27	1.327

Table 3. Safety factor of different ψ slope (b)

Ø = 20°		Ø = 25°		Ø = 30°	
ψ	FoS	ψ	FoS	ψ	FoS
0	1.057	0	1.124	0	1.269
5	1.132	5	1.294	5	1.464
10	1.144	10	1.311	10	1.485
15	1.147	15	1.319	15	1.495
17	1.135	20	1.319	20	1.483
20	1.068	25	1.156	25	1.348

Results and discussions

Associated to slope stability, cohesion, and friction angle are the most effective parameters to the slope failure, it needs to study the enter action effect among *c*, ø and dilation angle ψ. A lot of researchers reported that the dilation angle ψ have an effect on the volume alteration of the soil during yielding, [4, 19], considerable work has been done to fictional soil model with associated and non-associated flow rules but the significant question was what is the appropriate value of ψ should be used?. As shown in figs. 4 and 5, which is plotting safety factor variation (FoS when $\varnothing = 0$ associated flow rules and FoS of variation of ψ *vs*. variation factors of dilation angle. We can note that the FoS was increased with increasing of dilation angle ψ in the range of ($\psi = 0$ to $\psi = \varnothing$) as in eq. (1) until reaching the peak which is always in the last third part of carve and this is very clear as shown in fig. 4. With both three-friction angle's range $\varnothing$ = (20°, 25°, and 30°), it means the dilation angle ψ affect the safety factor through slope analysis, then it is very important to consider dilation angle when dealing with associated and non-associated soil parameters. On the other hand, the variation factor of dilation angle was dramatically dropped in the last quarter of the curve although in this part it was difficult to get convergence especially when ψ close to ø, so we can have observed that; to get a high FoS when chose dilation angle equal or close to 0.75 of friction angle ($\psi \approx 0.75$ ø), when dilation angle same as friction angle ($\psi = \varnothing$) get the low FoS.

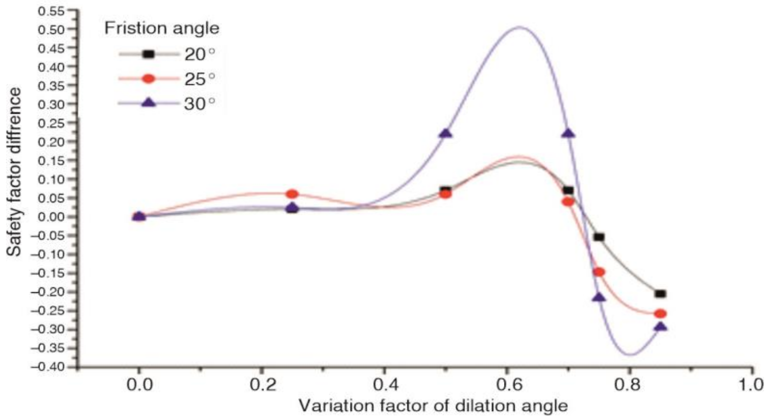

Figure 4. Effect of dilation ψ to the FoS under different friction Ø slope (a)

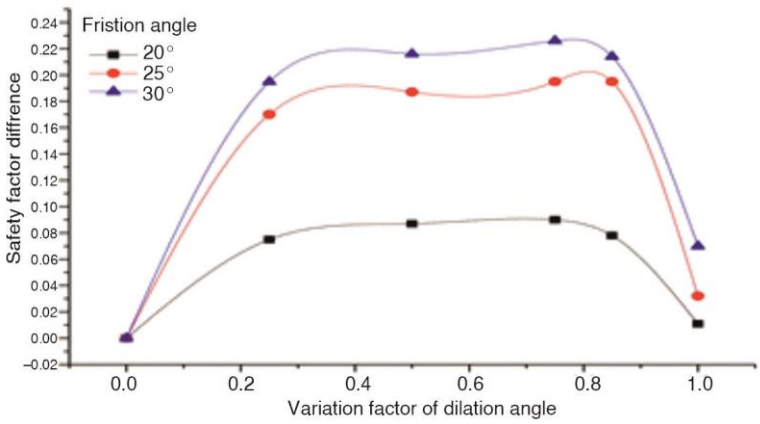

Figure 5. Effect of dilation ψ to the FoS under different friction Ø slope (b)

Conclusion

From the former analysis, we conclude that dilation angle is an important parameter in geotechnical analysis especially slope stability, use different dilation angles led to the difference safety factor, then it is essential to check out the accurate value of dilation angle. In this analysis, the previous notes are essentially significant, which is used finite element analysis strength reduction technique, although safety factor increases gradually utile last third of carve and dramatically decrease to a low value when dilation angle equal friction angle.

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