ESTIMATIONS OF ACCELERATED GENERALIZED HALF-LOGISTIC DISTRIBUTION IN PRESENCE OF GENERALIZED TYPE-II HYBRID CENSORING SAMPLES

by

Hanaa ABU-ZINADAH [a*] ***and Gamal A. ABD-ELMOUGOD*** [b]

[a] Department of Statistics, College of Science, University of Jeddah, Jeddah, Saudi Arabia
[b] Mathematics Department, Faculty of Science, Damanhour University, Damanhour, Egypt

Original scientific paper
https://doi.org/10.2298/TSCI22S1075A

Generally, the analysis of a lifetime data to quantify the life characteristics of the product done under normal conditions. The engineers for some reasons may be needed to obtain the reliability results more quickly then, accelerated life tests are applied. In this paper, we adopted partially step-stress accelerated life tests model of product has the generalized half-logistic lifetime distribution. This model is applied on a solar lighting device and the stress factor consider to be temperature. Also, to save the minimum and maximum ideal test time, we applied the generalized type-II hybrid censoring scheme. The parameters of the proposed model are estimated by maximum likelihood and Bayes methods for point and corresponding interval estimators. The validity of results is tested under formulation Monte-Carlo simulation study. The proposed model is applied on analysis data obtained from solar lighting device as real data set for illustrative purposes.

Key words: *generalized half-logistic distribution, maximum likelihood estimation, generalized type-II hybrid censoring scheme, Bayesian estimation*

Introduction

Half logistic distribution was introduced by Balakrishnan [1] to describe the distribution of the absolute standard logistic random variable. The generalized version of half logistic distribution with its properties formulated by Balakrishnan and Hossain [2]. The random variable T is called generalized half-logistic (GHL) random variable if it has the cumulative distribution function (CDF):

$$F(t)=1-\left(\frac{2e^{-t}}{1+e^{-t}}\right)^{\alpha},\ t>0,\ \alpha>0 \tag{1}$$

Also, the corresponding probability density function (PDF), reliability function $R(t)$ and hazared rate function $H(t)$ are given:

$$f(t)=\frac{\alpha}{1+e^{-t}}\left(\frac{2e^{-t}}{1+e^{-t}}\right)^{\alpha} \tag{2}$$

$$R(t)=\left(\frac{2e^{-t}}{1+e^{-t}}\right)^{\alpha} \tag{3}$$

* Corresponding author, e-mail: hhabuznadah@uj.edu.sa

and

$$H(t) = \frac{\alpha}{1+e^{-t}} \tag{4}$$

Different authors has discussed the GHL distribution for example, the parameter of stress-strength reliability is estimated by Ramakrishnan [3], for the type-I progressive censoring scheme, the shape parameter is estimated by Arora *et al*. [4], parameters estimation under Bayes method by Kim *et al*. [5], the reliability functions are discussed by Chaturvedi *et al.* [6] and the parameters are estimated under constant-stress by Almarashi [7].

The failure times are collected from a real population in complete or censored data. The word complete data is used when time-to-failure is observed for all units under the test. But, the word censored data is used when time-to-failure is observed for some not all units under test. In literature, several types of censoring schemes are available and the simply and commonly ones in life testing experiments are called type-I and type-II censoring schemes (CS). In type-I CS, the ideal test time is prior proposed to present the experiment terminate time and the number of failure is randomly. In type-II CS, a suitable number of failure needing for statistical inference is prior proposed and the test terminated time is randomly. Hence, we can say that two censoring schemes type-I or type-II have the luck of memory that, a small number of failure may be zero or large test time may be infinity, respectively. The survival units cannot be removed from the test in type-I and type-II CS other than the final point. If, we need to remove any units at any step of the experiment then, we refer to progressive censoring scheme, see Balakrishnan and Aggarwala [8]. In different area of life testing experiment, more conventional to propose the ideal test time τ and the number of failure need for statistical inference m at prior of the experiment which is known by hybrid censoring scheme (HCS). The experiment is terminated at $\min(T_m, \tau)$ in type-I HCS or $\max(T_m, \tau)$ in type-II HCS. Two schemes type-I HCS and type-II HCS have also, the luck of memory that, a small number of failure or large test time, respectively. Therefore, the statistical inference can be done with a low precision results, see Childs *et al*. [9], Gupta and Kundu [10], Kundu and Pradhan [11], and Algarn *et al*. [12]. The luck of memory that, a small number of failure or large test time appeared in the last schemes handled by generalized hybrid censoring scheme (GHCS), see Chandrasekar *et al*. [13]. In GHCS, the experiments has guarantees not only controls within a fixed number of failures but present at least the proper testing period in testing procedure which make more efficiency in statistical inference. The GHCS can be combined with type-I censoring scheme to present type-I GHCS described as follows.

Suppose, a random sample of size n is random selected from a population put under test and two integer numbers (κ, m) with test time τ are prior proposed, where $1 < \kappa < m \le n$. When the experiment is running, the unit failure time T_i is recorded until the κ^{th} failure time is observed. If, κ^{th} failure time less than τ, the experiment terminated at the $\min(T_m, \tau)$. But, if κ^{th} failure time larger than τ then, the experiment is terminated at T_κ. Therefore, the observed data under type-I GHCS define by $\underline{t} = (t_1, t_1, ..., t_r)$, where:

$$r = \begin{cases} \kappa, \text{ if } T_\kappa > \tau \\ \kappa < r < m, \text{ if } T_\kappa < \tau < T_m \\ m, \text{ if } T_\kappa < T_m < \tau \end{cases} \tag{5}$$

and the test terminated time ξ is defined:

$$\zeta = \begin{cases} T_\kappa, \text{ if } T_\kappa > \tau \\ \tau, \text{ if } T_\kappa < \tau < T_m \\ T_m, \text{ if } T_\kappa < T_m < \tau \end{cases} \tag{6}$$

Also, the GHCS can be combined with type-II censoring scheme to present type-II GHCS described as follows.

Suppose, the random sample of size n units put under the life testing experiment and two independent times $0 < \tau_1 < \tau_2 < \infty$ with integer number m are prior proposed. After running the experiment the unit failure time T_i is recorded until the τ_1 is observed. If, T_{m} is observed before the time τ_1, the experiment terminated at τ_1. But, if T_m is observed after the time τ_1 then, the experiment is terminated at T_m if $\tau_1 < T_m < \tau_2$ and the experiment is terminated at τ_2 if $\tau_1 < \tau_2 < T_m$. The observed data under type-II GHCS define by $\underline{t} = (t_1, t_1,..., t_r)$ where:

$$r = \begin{cases} r < m, \text{ if } \tau_1 < \tau_2 < T_m \\ m, \text{ if } \tau_1 < T_m < \tau_2 \\ r > m, \text{ if } T_m < \tau_1 \end{cases} \tag{7}$$

and the test terminated time ξ is defined:

$$\zeta = \begin{cases} \tau_2, \text{ if } \tau_1 < \tau_2 < T_m \\ T_m, \text{ if } \tau_1 < T_m < \tau_2 \\ \tau_1, \text{ if } T_m < \tau_1 \end{cases} \tag{8}$$

In this paper, we adopted type-II GHCS which has the advantage that, experiment will be completed by time τ_2. Hence, the time τ_2 is the allow time that the researcher is willing to complete the experiment. Statistical inference of a life product under modern technology became more difficultly specially, for a high reliable product. Also, the problem of obtaining the sufficient number of information in a small period of time about a life product may need to stress higher than normal stress level which is known by accelerated life test (ALT) model, see Nelson [14]. The ALT model is widely used in several populations, see Bagdonavicius and Nikulin [15]. This model defined in different schemes such as, constant-stress, step-stress and progressive-stress ALT models. The units in a lifetime experiment has loaded under constant stress until the final point of the experiment say, constant-stress ALT model, Abd-Elmougod and Mahmoud [16]. But, if the stress is changed through determined period of time or constant number of failure then, step-stress ALT model is applied, Algarni *et al*. [17] and Ganguly and Kundu [18]. Finally, if the stress is kept continuously increasing at all steps of the experiment then, progressive stress ALT model is applied, Wang and Fei [19], Abdel-Hamid and Al-Hussaini [20], and Abu-Zinadah *et al.* [21]. In several cases, the experimenter may be need to test some units under normal stress level and other units under stress level which is known by partially ALT model. Therefore, in partially constant-stress ALT model some units put under normal stress level and other units put under stress level in the same time. But, in partially step-stress ALT model all units put under normal stress level until constant period of time or number of failure then, the survival units put under stress level.

The reliability analysis under given lifetime data for some population units are often studied based on population characteristics and censoring methodologies for formulating inference of the unknown quantities in the population. The generalized half-logistic lifetime population have wide application in reliability analysis field of material or engineering products. Also, the problem of reliability analysis under modern technology for some life products under normal conditions is more serious. In this paper, we aim to adopt partially step-stress ALT model when the life of units under test have generalized half-logistic lifetime population. The observed lifetime sample are collected under type-II GHCS. The parameters of proposed model and the system reliability and hazard rate functions also are estimated by classical ML and Bayes methods for point and interval estimators. The quality of proposed model and method of estimation are assessed and compared throu gh Monte-Carlo simulation study. The proposed model is used to analyze lifetime data obtained from solar lighting device.

The model and likelihood function

Suppose that, n units are randomly selected from a life product to put under a life testing experiment at normal conditions. The stress change time τ^* and the ideal test times τ_1 and τ_2 are prior proposed to satisfies, $\tau^* \le \tau_1 < \tau_2$. Also, the prior integer number m which define the suitable number needing for statistical inference are suggested. The experiment is running under normal stress conditions until the time τ^* is observed then, it running under stress conditions. The unit failure time T_i is recorded until the time τ_1 is observed. If, T_m is observed before the time τ_1, the experiment terminated at τ_1. But, If T_m is observed after the time τ_1 then, the experiment is terminated at T_m if $\tau_1 < T_m < \tau_2$ and the experiment is terminated at τ_2 if $\tau_1 < \tau_2 < T_m$. The observed data under type-II GHCS define by $\underline{\mathbf{t}} = (t_1, t_2,..., t_J, t_{J+1},..., t_r)$ where J is the number of failure under normal conditions and r is defined by eq. (7). Therefore, the experiment formulated to obtain the failure times for some fixed time, and hence the stress level is switched to the higher stress level until a suitable number of units fails. This switch aims to shorted the life of test units by inverse of the acceleration factor. Also, the total lifetime W under partially step-stress type-II GHCS passes through the normal and accelerated steps:

$$W = \begin{cases} T, & T < \tau^* \\ \theta^{-1}(T - \tau^*) + \tau^*, & T > \tau^* \end{cases} \tag{9}$$

where the lifetime of the unit is denoted at normal conditions by T and the parameter θ denote to the acceleration factor, generally $\theta > 1$. Under consideration that, the lifetime of units distributed with generalized half-logistic given by eq. (1). Then, the random variable W of total lifetime has the distribution given:

$$g(w) = \begin{cases} f_1(w), & 0 < w \le \tau^* \\ f_2(w) & w > \tau^* \\ 0 & w < 0 \end{cases} \tag{10}$$

where the density function $f_1(w)$, defined by eq. (2) and the density $f_2(w)$ is obtained under transformation technique using eqs. (2) and (10) to define:

$$f_2(w) = \frac{\alpha\theta}{1 + e^{-\left[\tau^* + \theta(w-\tau^*)\right]}} \left[\frac{2e^{-\left(\tau^* + \theta(w-\tau^*)\right)}}{1 + e^{-\left(\tau^* + \theta(w-\tau^*)\right)}} \right]^{\alpha}, \quad w > 0, \ \alpha, \theta > 0 \tag{11}$$

Also, the CDF, $S_2(w)$, and $H_2(w)$ of eq. (11) given:

$$F_2(w) = 1 - \left[\frac{2e^{-\left(\tau^* + \theta(w-\tau^*)\right)}}{1 + e^{-\left(\tau^* + \theta(y-\tau^*)\right)}} \right]^{\alpha} \tag{12}$$

$$S_2(w) = \left[\frac{2e^{-\left(\tau^* + \theta(w-\tau^*)\right)}}{1 + e^{-\left(\tau^* + \theta(w-\tau^*)\right)}} \right]^{\alpha} \tag{13}$$

and

$$H_2(w)=\frac{\alpha}{1+e^{-\left[\tau^*+\theta(w-\tau^*)\right]}} \tag{14}$$

For type-II GHCS under partially step-stress model $\underline{\mathbf{t}}=(t_1, t_2,..., t_J, t_{J+1},..., t_r)$ the joint likelihood function is given:

$$f\left(\underline{\mathbf{t}}\,|\,\alpha,\theta\right)=\frac{n!}{(n-r)!}\left(\prod_{i=1}^{J} f_1(t_i)\right)\left(\prod_{i=J+1}^{r} f_2(t_i)\right)\left[S_2(\zeta)\right]^{(n-r)} \tag{15}$$

where ξ is defined by eq. (8) and $0 < t_1 < t_2 < ... < t_J < \tau^* < t_{J+1} < ...< t_r < \infty$.

Maximum likelihood estimation

The likelihood eq. (15) under type-II GHCS and partially step-stress model $\underline{t}=(t_1, t_2,..., t_J, t_{J+1},..., t_r)$ can be:

$$L\left(\alpha,\theta\,|\,\underline{\mathbf{t}}\right)\propto \alpha^r\theta^{(n-r)}\left[\frac{2e^{-\left(\tau^*+\theta(\zeta-\tau^*)\right)}}{1+e^{-\left(\tau^*+\theta(\zeta-\tau^*)\right)}}\right]^{\alpha(n-r)}\prod_{i=1}^{J}\frac{1}{1+e^{-t_i}}\left(\frac{2e^{-t_i}}{1+e^{-t_i}}\right)^{\alpha}\cdot$$

$$\cdot\prod_{i=J+1}^{r}\frac{1}{1+e^{-\left(\tau^*+\theta(t_i-\tau^*)\right)}}\left(\frac{2e^{-\left(\tau^*+\theta(t_i-\tau^*)\right)}}{1+e^{-\left(\tau^*+\theta(t_i-\tau^*)\right)}}\right)^{\alpha} \tag{16}$$

Therefore, the natural logarithm of the likelihood eq. (16) is reduced:

$$\ell\left(\alpha,\theta\,|\,\underline{\mathbf{t}}\right)\propto r\log\alpha+(n-r)\log\theta+\alpha(n-r)\log\left[\frac{2e^{-\left(\tau^*+\theta(\zeta-\tau^*)\right)}}{1+e^{-\left(\tau^*+\theta(\zeta-\tau^*)\right)}}\right]+\alpha\sum_{i=1}^{J}\log\left[\frac{2e^{-t_i}}{1+e^{-t_i}}\right]+$$

$$+\sum_{i=1}^{J}\log\left[\frac{1}{1+e^{-t_i}}\right]+\alpha\sum_{i=J+1}^{r}\log\left[\frac{2e^{-\left(\tau^*+\theta(t_i-\tau^*)\right)}}{1+e^{-\left(\tau^*+\theta(t_i-\tau^*)\right)}}\right]+\sum_{i=J+1}^{r}\log\left[\frac{1}{1+e^{-\left(\tau^*+\theta(t_i-\tau^*)\right)}}\right] \tag{17}$$

Maximum likelihood equations

The likelihood equations are obtained from the log-likelihood function by zero-value for the first partially derivatives respected to the parameters values, see [22, 23]:

$$\frac{\partial\ell\left(\alpha,\theta\,|\,\underline{\mathbf{t}}\right)}{\partial\alpha}=0$$

$$\frac{r}{\alpha}+(n-r)\log\left[\frac{2e^{-\left(\tau^*+\theta(\zeta-\tau^*)\right)}}{1+e^{-\left(\tau^*+\theta(\zeta-\tau^*)\right)}}\right]+\sum_{i=1}^{J}\log\left[\frac{2e^{-t_i}}{1+e^{-t_i}}\right]+\sum_{i=J+1}^{r}\log\left[\frac{2e^{-\left(\tau^*+\theta(t_i-\tau^*)\right)}}{1+e^{-\left(\tau^*+\theta(t_i-\tau^*)\right)}}\right]=0 \tag{18}$$

and

$$\frac{\partial \ell(\alpha,\theta\,|\,\underline{\mathbf{t}})}{\partial\theta}=0$$

$$\frac{(n-r)}{\theta}-\frac{\alpha(n-r)(\zeta-\tau^*)}{1+e^{-\left[\tau^*+\theta(\zeta-\tau^*)\right]}}-\alpha\sum_{i=J+1}^{r}\frac{(t_i-\tau^*)}{1+e^{-\left[\tau^*+\theta(t_i-\tau^*)\right]}}+\sum_{i=J+1}^{r}\frac{(\zeta-\tau^*)e^{-\left[\tau^*+\theta(t_i-\tau^*)\right]}}{\left\{1+e^{-\left[\tau^*+\theta(\zeta-\tau^*)\right]}\right\}^2}=0 \tag{19}$$

The likelihood equations is reduced:

$$\alpha(\theta)=-rD^{-1}(\theta) \tag{20}$$

where

$$D(\theta)=(n-r)\log\left\{\frac{2e^{-\left[\tau^*+\theta(\zeta-\tau^*)\right]}}{1+e^{-\left[\tau^*+\theta(\zeta-\tau^*)\right]}}\right\}+\sum_{i=1}^{J}\log\left[\frac{2e^{-t_i}}{1+e^{-t_i}}\right]+\sum_{i=J+1}^{r}\log\left\{\frac{2e^{-\left[\tau^*+\theta(t_i-\tau^*)\right]}}{1+e^{-\left[\tau^*+\theta(t_i-\tau^*)\right]}}\right\} \tag{21}$$

and

$$\theta=(n-r)h^{-1}(\theta) \tag{22}$$

where

$$h(\theta)=\frac{\alpha(n-r)(\zeta-\tau^*)}{1+e^{-\left[\tau^*+\theta(\zeta-\tau^*)\right]}}+\alpha\sum_{i=J+1}^{r}\frac{(t_i-\tau^*)}{1+e^{-\left[\tau^*+\theta(t_i-\tau^*)\right]}}-\sum_{i=J+1}^{r}\frac{(\zeta-\tau^*)e^{-\left[\tau^*+\theta(t_i-\tau^*)\right]}}{\left\{1+e^{-\left[\tau^*+\theta(\zeta-\tau^*)\right]}\right\}^2} \tag{23}$$

From eqs. (20) and (22), the maximum likelihood estimation (MLE) of the model parameters is reduced to one non-linear eq. (22) of θ can be solve with any iteration methods such as Newton Raphson or fixed point iteration. Hence, the MLE of the parameter α is obtained immediately from eq. (20). The following theorem present the MLE of the parameter θ and the value of initial point of the iteration.

Theorem: The conditional maximum likelihood estimator of θ given accelerated type-II GHCS $\underline{\mathbf{t}}=(t_1, t_2,..., t_J, t_{J+1},..., t_r)$ cab be obtained by fixed point iteration:

$$\theta^{(i+1)}=(n-r)h^{-1}(\theta^{(i)}) \tag{24}$$

where $h(\theta)$ given by eq. (23) after replacing α by eq. (20) and the initial value can be determine by the profile log-likelihood function defined:

$$g(\theta\,|\,\underline{\mathbf{t}})=r\log\left[\frac{-r}{D(\theta)}\right]+(n-r)\log\theta+\sum_{i=1}^{J}\log\left[\frac{1}{1+e^{-t_i}}\right]+\sum_{i=J+1}^{r}\log\left[\frac{1}{1+e^{-\left(\tau^*+\theta(t_i-\tau^*)\right)}}\right]-r \tag{25}$$

Proof: The proof can be obtained immediately from eqs. (20), (22), and (23)

Remark 1:

Equation (20) has shown that, the conditional estimators of the parameters θ dependent on $(r-J)$ but, if $(r-J)=0$ then, the estimate $\hat{\theta}$ does not exist. therefore, the model reduced to normal condition.

Remark 2:

The MLE of the reliability and hazard rate function for given t compute:

$$\hat{R}(t)=\left(\frac{2e^{-t}}{1+e^{-t}}\right)^{\hat{\alpha}}, \text{ and } \hat{H}(t)=\frac{\hat{\alpha}}{1+e^{-t}} \tag{26}$$

Interval estimation

Fisher information matrix defined by the minus expectation of the second derivative of the log-likelihood function. Generally, the expectation of the second derivative more serous specially in high dimensional case. Hence, the approximate information matrix is the natural alternative of Fisher information matrix. Suppose that, $\Omega_0(\hat{\alpha}, \hat{\theta})$ denote to approximate information matrix defined:

$$\Omega_0\left(\hat{\alpha},\hat{\theta}\right)=-\left[\frac{\partial^2\ell\left(\alpha,\theta\,|\,\underline{\mathbf{t}}\right)}{\partial\alpha\partial\theta}\right]|_{\hat{\alpha},\hat{\theta}} \tag{27}$$

The second partial derivatives of log-likelihood function given:

$$\frac{\partial\ell\left(\alpha,\theta\,|\,\underline{\mathbf{t}}\right)}{\partial\alpha^2}=\frac{-r}{\alpha^2} \tag{28}$$

$$\frac{\partial\ell\left(\alpha,\theta\,|\,\underline{\mathbf{t}}\right)}{\partial\theta^2}=-\frac{\alpha(n-r)(\zeta-\tau^*)^2e^{-\left[\tau^*+\theta(\zeta-\tau^*)\right]}}{\left\{1+e^{-\left[\tau^*+\theta(\zeta-\tau^*)\right]}\right\}^2}-\alpha\sum_{i=J+1}^{r}\frac{(t_i-\tau^*)^2e^{-\left[\tau^*+\theta(t_i-\tau^*)\right]}}{\left[1+e^{-\left[\tau^*+\theta(t_i-\tau^*)\right]}\right]^2}-$$
$$-\sum_{i=J+1}^{r}\frac{(t_i-\tau^*)^2e^{-\left[\tau^*+\theta(t_i-\tau^*)\right]}\left\{-1+e^{-\left[\tau^*+\theta(t_i-\tau^*)\right]}\right\}}{\left\{1+e^{-\left[\tau^*+\theta(t_i-\tau^*)\right]}\right\}^3}-\frac{(n-r)}{\theta^2} \tag{29}$$

and

$$\frac{\partial\ell\left(\alpha,\theta\,|\,\underline{\mathbf{t}}\right)}{\partial\alpha\partial\theta}=\frac{\partial\ell\left(\alpha,\theta\,|\,\underline{\mathbf{t}}\right)}{\partial\theta\partial\alpha}=-\frac{(n-r)(\zeta-\tau^*)}{1+e^{-\left[\tau^*+\theta(\zeta-\tau^*)\right]}}-\sum_{i=J+1}^{r}\frac{(t_i-\tau^*)}{1+e^{-\left[\tau^*+\theta(t_i-\tau^*)\right]}} \tag{30}$$

Hence, under consideration the normal properties of MLE $(\hat{\alpha}, \hat{\theta})$, $(1-2\gamma)\times 100\%$ approximate confidence intervals of the parameters α and θ defined:

$$\hat{\alpha}\mp Z_\gamma\Omega_{11} \text{ and } \hat{\theta}\mp Z_\gamma\Omega_{22} \tag{31}$$

where the value Z_γ is the standard normal probability with right tailed γ and $(\Omega_{11}, \Omega_{12})$ are the elements of diagonal taken from approximate invariance of information matrix $\Omega_0^{-1}(\hat{\alpha}, \hat{\theta})$.

Bayesian approach and MCMC method

The model parameters in this section are estimated under consideration that, gamma prior density function for the parameter of the distribution and non-informative prior information for the accelerated parameter. Hence, Bayesian estimation of the unknown model parameters under squared error loss (SEL) function done for point and symmetric credible intervals as follows.

The proposed prior information formulated:

$$\eta_1^*(\alpha) \propto \alpha^{a-1} \exp(-b\alpha), \alpha > 0; a, b > 0 \tag{32}$$

and

$$\eta_2^*(\theta) \propto \theta^{-1} \tag{33}$$

Hence, the joint prior density:

$$\eta^*(\alpha,\theta) \propto \alpha^{a-1}\theta^{-1} \exp(-b\alpha), \ \ \alpha > 0; \ \ a, b > 0 \tag{34}$$

The corresponding posterior distribution formulate with respected to eq. (16) and eq. (33):

$$\eta(\alpha,\theta \,|\, \underline{\mathbf{t}}) \propto \alpha^{r+a-1}\theta^{(n-r-1)} \exp\left\{-b\alpha + \alpha(n-r)\log\left[\frac{2e^{-\left(\tau^*+\theta(\zeta-\tau^*)\right)}}{1+e^{-\left(\tau^*+\theta(\zeta-\tau^*)\right)}}\right] + \alpha\sum_{i=1}^{J}\cdot\right.$$
$$\left.\cdot\log\left[\frac{2e^{-t_i}}{1+e^{-t_i}}\right] - \log\left[1+e^{-\left(\tau^*+\theta(t_i-\tau^*)\right)}\right] + \alpha\sum_{i=J+1}^{r}\log\left[\frac{2e^{-\left(\tau^*+\theta(t_i-\tau^*)\right)}}{1+e^{-\left(\tau^*+\theta(t_i-\tau^*)\right)}}\right]\right\} \tag{35}$$

From the posterior distribution (35), the full conditional densities:

$$\eta(\alpha \,|\, \theta,\underline{\mathbf{t}}) \propto \alpha^{r+a-1} \exp\left\{-b\alpha + \alpha(n-r)\log\left[\frac{2e^{-\left(\tau^*+\theta(\zeta-\tau^*)\right)}}{1+e^{-\left(\tau^*+\theta(\zeta-\tau^*)\right)}}\right] + \alpha\sum_{i=1}^{J}\log\left[\frac{2e^{-t_i}}{1+e^{-t_i}}\right] + \right.$$
$$\left. + \alpha\sum_{i=J+1}^{r}\log\left[\frac{2e^{-\left(\tau^*+\theta(t_i-\tau^*)\right)}}{1+e^{-\left(\tau^*+\theta(t_i-\tau^*)\right)}}\right]\right\} \tag{36}$$

and

$$\eta(\theta \,|\, \alpha,\underline{\mathbf{t}}) \propto \theta^{(n-r-1)} \exp\left\{\alpha(n-r)\log\left[\frac{2e^{-\left(\tau^*+\theta(\zeta-\tau^*)\right)}}{1+e^{-\left(\tau^*+\theta(\zeta-\tau^*)\right)}}\right] - \log\left[1+e^{-\left(\tau^*+\theta(t_i-\tau^*)\right)}\right] + \right.$$
$$\left. + \alpha\sum_{i=J+1}^{r}\log\left[\frac{2e^{-\left(\tau^*+\theta(t_i-\tau^*)\right)}}{1+e^{-\left(\tau^*+\theta(t_i-\tau^*)\right)}}\right]\right\} \tag{37}$$

The conditional distribution defined by eq. (36) is reduced to gamma distribution. Also, the conditional distribution defined by eq. (37) presented function its plot more similar to the normal distribution. Hence, the generation from two conditional distribution (mean posterior distribution) is simpler. Bayes estimation of the unknown model parameters under SEL function is given:

$$\frac{\iint \varphi(\alpha,\theta)\eta^*(\alpha,\theta)L(\alpha,\theta \,|\, \underline{\mathbf{t}})\mathrm{d}\alpha\mathrm{d}\theta}{\iint \eta^*(\alpha,\theta)L(\alpha,\theta \,|\, \underline{\mathbf{t}})\mathrm{d}\alpha\mathrm{d}\theta}$$

where $\varphi(\alpha, \theta)$ be any function of the parameters may be α aor θ. The ratio of two integral rarely obtained in closed form specially, in a high dimensional case hence, different methods can be used to approximation such as, numerical integration, Lindely approximate and MCMC method. In this section, we are used the MCMC method to obtain the empirical posterior distribution. The full conditional distributions presented by eqs. (36) and (37) have shown that, Gibbs algorithms and more general MH under Gibbs are good choice to solve this problem as the following algorithms [24].

Algorithms 1:

Step 1. Begin with initial values $\alpha^{(0)} = \hat{\alpha}$, $\theta^{(0)} = \hat{\theta}$.

Step 2. Set the indicator $\kappa = 1$.

Step 3. From the full conditional gamma density eq. (36) generate $\alpha^{(\kappa)}$.

Step 4. From the conditional density eq. (37) generate $\theta^{(\kappa)}$ by MH algorithms with known normal proposal distribution, with mean $\theta^{(\kappa-1)}$ and variance Ω_{22}.

– Generate the candidate sample points $\theta^{(*)}$, from normal distribution as proposal distributions.
– Compute the acceptance probability:

$$P = \min\left\{1, \frac{\eta\left[\alpha^{(\kappa)}, \theta^{(*)}\right]}{\eta\left[\alpha^{(\kappa)}, \theta^{(\kappa-1)}\right]}\right\}$$

– Generate random uniform value U from uniform (0, 1).
– If $P > U$, we accept $\theta^{(*)}$ as $\theta^{(\kappa)}$. Otherwise, the values reject and replaced by $\theta^{(\kappa-1)}$.

Steps 5. Set the indicator $\kappa = \kappa + 1$.

Steps 6. From Steps 3-5 are repeated N times and for each step report $\alpha^{(\kappa)}$ and $\theta^{(\kappa)}$.

The MCMC Bayes estimate of the model parameters as well as reliability and hazard failure rate function for given time t need to reaching to stationary distribution.

Point estimator: The point estimate after deleting the first burn-in iteration $\mathbf{N}^*$:

$$\hat{\varphi}_B = \mathrm{E}_\eta(\varphi \mid \underline{\mathbf{t}}) = \frac{1}{\mathbf{N} - \mathbf{N}^*} \sum_{i=\mathbf{N}^*+1}^{\mathbf{N}} \varphi\left[\alpha^{(i)}, \theta^{(i)}\right] \tag{36}$$

where $\varphi(\alpha, \theta)$ be any function of the parameters may be α or θ.

Bayes variance: The Bayes variance of $\varphi(\alpha, \theta)$:

$$\hat{V}_B(\varphi \mid \underline{\mathbf{t}}) = \frac{1}{\mathbf{N} - \mathbf{N}^*} \sum_{i=\mathbf{N}^*+1}^{\mathbf{N}} \left(\varphi(\alpha^{(i)}, \theta^{(i)}) - \hat{\varphi}_B\right)^2. \tag{37}$$

Credible interval: The Bayes $(1 - 2\gamma)100\%$ credible interval of the function $\varphi(\alpha, \theta)$:

$$\left[\varphi_{\gamma(\mathbf{N}-\mathbf{N}^*)}, \varphi_{(1-\gamma)(\mathbf{N}-\mathbf{N}^*)}\right] \tag{38}$$

Simulation studies

In this section, we constructed Monte-Carlo simulation study to assess and compare the proposed estimation methods. Also, testing the numerical methods which is used to approximate the estimators. Therefore, we study the effect of change of population parameters, censoring scheme (m, τ_1, τ_2), affect sample size and stress change time τ^*. From partially accelerated model, we generate 1000 random sample and for each sample, we compute the average of the parameters estimate (denoted by AE), the mean squared error (denoted by MSE), average interval length (denoted by AIL) and coverage percentile (denoted by CP). The true parameter value are selected to satisfies that, the prior expectation for given a , b almost equal a/b (mean $E(a) = a/b$) which denoted to informative prior information and non-informative prior information are taken to be $a = b = 0.0001$. Therefore, we are adopted two set of prior information $(a, b) = (1, 4)$ and $(a, b) = (3, 2)$ and the corresponding true parameter values are considered to be $(\alpha, \theta) = (0.2, 2.5)$ and $(1.5, 1.5)$, respectively. The times vector $I = (\tau, \tau_1, \tau_1)$ is selected to be ($I_1 = (2, 3, 5)$, $I_2 = (2, 4, 5)$, and $I_3 = (3, 3, 5)$ for the first choose of the parameter values and $I_1 = (0.7, 0.8, 2)$, $I_2 = (0.7, 1, 2)$, and $I_3 = (0.8, 0.8, 2)$ for the second choose). The numerical results of simulation study reported in tabs. 1-4 according the following algorithms.

Table 1. The (AE and MSE) of MLE and Bayes estimate when $(\alpha, \theta) = \{0.2, 2.5\}$

		MLE				Bayes⁰				Bayes¹			
		α		θ		α		θ		α		θ	
(n, m)	I	AE	MSE	AE	MSE	AE	MSE	AE	MSE	AE	MSE	AE	MSE
(40, 25)	I_1	0.305	0.115	2.705	0.437	0.288	0.117	2.672	0.400	0.242	0.085	2.525	0.366
	I_2	0.277	0.101	2.685	0.419	0.261	0.104	2.640	0.381	0.219	0.081	2.519	0.349
	I_3	0.254	0.089	2.666	0.409	0.249	0.090	2.623	0.377	0.208	0.072	2.482	0.318
(40, 35)	I_1	0.281	0.106	2.692	0.431	0.269	0.108	2.655	0.389	0.228	0.081	2.504	0.357
	I_2	0.261	0.094	2.671	0.408	0.248	0.093	2.638	0.366	0.204	0.079	2.503	0.341
	I_3	0.238	0.078	2.645	0.402	0.237	0.082	2.609	0.371	0.201	0.066	2.465	0.309
(60, 40)	I_1	0.259	0.095	2.669	0.417	0.247	0.100	2.624	0.377	0.214	0.078	2.485	0.346
	I_2	0.243	0.082	2.657	0.392	0.229	0.088	2.619	0.354	0.189	0.071	2.513	0.335
	I_3	0.217	0.069	2.635	0.388	0.225	0.075	2.589	0.351	0.195	0.054	2.479	0.314
(60, 50)	I_1	0.244	0.087	2.648	0.409	0.233	0.094	2.611	0.370	0.200	0.071	2.492	0.331
	I_2	0.231	0.079	2.641	0.381	0.215	0.083	2.603	0.345	0.171	0.0666	2.507	0.318
	I_3	0.214	0.061	2.629	0.380	0.211	0.071	2.565	0.341	0.173	0.051	2.488	0.300
(80, 50)	I_1	0.223	0.081	2.628	0.399	0.221	0.090	2.589	0.362	0.205	0.067	2.477	0.317
	I_2	0.215	0.074	2.624	0.378	0.211	0.075	2.581	0.336	0.182	0.059	2.492	0.309
	I_3	0.202	0.053	2.613	0.375	0.202	0.066	2.560	0.329	0.196	0.049	2.489	0.297
(80, 65)	I_1	0.217	0.073	2.611	0.387	0.217	0.084	2.579	0.344	0.195	0.059	2.464	0.308
	I_2	0.211	0.066	2.602	0.371	0.210	0.069	2.577	0.332	0.191	0.051	2.488	0.301
	I_3	0.189	0.050	2.600	0.367	0.197	0.057	2.554	0.327	0.203	0.039	2.491	0.284

Algorithm 2:

Step 1. Generate random sample of size n from GHL with parameter α.

Step 2. For given τ^* and θ used transformation (9) to obtain the complete partially step-stress accelerated sample.

Step 3. For given the parameters of type-II GHCS (τ_1, τ_1, m) generate partially accelerated type-II GHC sample ($t_1, t_2, ..., t_J, t_{J+1}, ... t_r$). Hence, the values of J, r, and ξ from a sample are estimated.

Step 4. For given observed J, r, ξ, and $\underline{\mathbf{t}} = (t_1, t_2, ..., t_J, t_{J+1}, ... t_r)$ compute, MLE and Bayes estimate point and interval estimate.

Step 5. Steps from 1-4 are repeated 1000 times.

Step 6. Compute the value of AE, MSE, MIL and CP and the results are reported in tabs. from 1-4.

From the numerical results reported in tabs. 1-4, we observe some points about the quality of the proposed model and the effect of, τ^*, τ_1, τ_2, censoring scheme and parameter choice as follows.

Table 2. The (MIL and CP) of 95% MLE and Bayes interval estimate when (α, θ) = {0.2, 2.5}

		MLE				Bayes⁰				Bayes¹			
		α		θ		α		θ		α		θ	
(*n, m*)	*I*	MIL	CP	MIL	CP	MIL	CP	MIL	CP	MIL	CP	MIL	CP
(40, 25)	I_1	0.549	0.87	4.752	0.88	0.539	0.89	4.745	0.89	0.401	0.89	4.255	0.90
	I_2	0.525	0.89	4.729	0.89	0.518	0.90	4.721	0.90	0.392	0.91	4.237	0.91
	I_3	0.512	0.90	4.711	0.89	0.507	0.91	4.700	0.90	0.378	0.90	4.215	0.92
(40, 35)	I_1	0.532	0.90	4.734	0.89	0.524	0.90	4.718	0.90	0.392	0.91	4.238	0.92
	I_2	0.514	0.89	4.718	0.90	0.504	0.91	4.704	0.91	0.381	0.92	4.224	0.91
	I_3	0.501	0.91	4.702	0.90	0.495	0.92	4.687	0.90	0.369	0.96	4.207	0.93
(60, 40)	I_1	0.517	0.90	4.715	0.90	0.511	0.93	4.703	0.92	0.379	0.92	4.215	0.93
	I_2	0.503	0.92	4.700	0.90	0.492	0.91	4.685	0.91	0.368	0.92	4.204	0.91
	I_3	0.489	0.93	4.792	0.91	0.481	0.91	4.665	0.94	0.351	0.93	4.189	0.92
(60,50)	I_1	0.501	0.92	4.691	0.91	0.491	0.92	4.685	0.93	0.362	0.92	4.177	0.93
	I_2	0.488	0.89	4.675	0.92	0.482	0.91	4.674	0.95	0.348	0.94	4.165	0.93
	I_3	0.471	0.92	4.771	0.91	0.471	0.92	4.652	0.91	0.333	0.91	4.145	0.94
(80, 50)	I_1	0.487	0.91	4.669	0.92	0.473	0.93	4.669	0.92	0.339	0.93	4.155	0.94
	I_2	0.465	0.91	4.654	0.92	0.467	0.91	4.653	0.93	0.328	0.94	4.147	0.93
	I_3	0.459	0.90	4.745	0.96	0.458	0.93	4.641	0.92	0.319	0.94	4.134	0.92
(80, 65)	I_1	0.452	0.92	4.639	0.93	0.461	0.93	4.638	0.94	0.318	0.94	4.127	0.94
	I_2	0.463	0.91	4.634	0.92	0.452	0.92	4.635	0.93	0.311	0.94	4.118	0.92
	I_3	0.451	0.94	4.725	0.90	0.445	0.92	4.619	0.91	0.302	0.95	4.109	0.93

- Partially accelerated type-II GHCS model serve well for statistical inference of the generalized half-logistic lifetime distribution.
- The MLE and non-informative Bayes estimate are more closed for each.
- Informative Bayes estimate serve well than maximum likelihood estimate or non-informative Baye estimate.
- When affect sample size has increase MSE and MIL are decreases and CP more closed to proposed one.
- The large value of τ^* serve well of model parameter.
- The large value of τ_1 and τ_2 serve well than small ones.

Table 3. The (AE and MSE) of MLE and Bayes estimate when (α, θ) = {1.5, 1.5}

		MLE				Bayes[0]				Bayes[1]			
		α		θ		α		θ		α		θ	
(n, m)	I	AE	MSE	AE	MSE	AE	MSE	AE	MSE	AE	MSE	AE	MSE
(40, 25)	I_1	1.803	0.311	1.952	0.362	1.795	0.305	1.943	0.354	1.711	0.232	1.852	0.287
	I_2	1.785	0.301	1.933	0.354	1.777	0.293	1.927	0.345	1.692	0.224	1.831	0.275
	I_3	1.770	0.292	1.917	0.341	1.759	0.281	1.908	0.336	1.677	0.211	1.815	0.267
(40, 35)	I_1	1.771	0.294	1.919	0.348	1.766	0.291	1.917	0.326	1.688	0.218	1.824	0.275
	I_2	1.752	0.282	1.904	0.335	1.751	0.277	1.911	0.319	1.675	0.209	1.814	0.263
	I_3	1.741	0.278	1.900	0.328	1.737	0.271	1.890	0.311	1.669	0.200	1.802	0.258
(60, 40)	I_1	1.766	0.287	1.901	0.340	1.752	0.282	1.903	0.315	1.672	0.211	1.811	0.266
	I_2	1.748	0.275	1.891	0.328	1.745	0.270	1.900	0.311	1.664	0.203	1.800	0.254
	I_3	1.741	0.268	1.884	0.319	1.731	0.264	1.881	0.304	1.655	0.189	1.792	0.249
(60, 50)	I_1	1.747	0.275	1.891	0.328	1.735	0.273	1.887	0.305	1.659	0.204	1.781	0.251
	I_2	1.735	0.264	1.879	0.314	1.728	0.265	1.881	0.300	1.651	0.200	1.777	0.244
	I_3	1.729	0.255	1.871	0.307	1.718	0.255	1.874	0.297	1.644	0.179	1.762	0.237
(80, 50)	I_1	1.728	0.266	1.878	0.317	1.717	0.264	1.869	0.294	1.641	0.189	1.722	0.243
	I_2	1.718	0.251	1.864	0.309	1.714	0.257	1.855	0.288	1.632	0.182	1.718	0.237
	I_3	1.711	0.250	1.853	0.301	1.708	0.250	1.848	0.281	1.621	0.174	1.711	0.229
(80,65)	I_1	1.704	0.251	1.852	0.304	1.695	0.252	1.844	0.285	1.618	0.162	1.701	0.225
	I_2	1.691	0.247	1.841	0.300	1.687	0.247	1.835	0.272	1.611	0.157	1.691	0.219
	I_3	1.687	0.239	1.838	0.295	1.677	0.239	1.819	0.266	1.602	0.151	1.682	0.211

Table 4. The (MIL and CP) of 95% MLE and Bayes interval estimate when (α, θ) = {1.5, 1.5}

		MLE				Bayes[0]				Bayes[1]			
		α		θ		α		θ		α		θ	
(n, m)	I	MIL	CP	MIL	CP	MIL	CP	MIL	CP	MIL	CP	MIL	CP
(40, 25)	I_1	3.520	0.89	3.842	0.89	3.518	0.90	3.824	0.89	3.385	0.90	3.722	0.90
	I_2	3.502	0.89	3.819	0.90	3.504	0.90	3.804	0.90	3.361	0.91	3.692	0.91
	I_3	3.487	0.90	3.800	0.90	3.481	0.91	3.775	0.92	3.332	0.90	3.651	0.92
(40, 35)	I_1	3.491	0.91	3.819	0.90	3.491	0.93	3.785	0.93	3.351	0.92	3.700	0.93
	I_2	3.482	0.90	3.811	0.93	3.482	0.90	3.772	0.90	3.342	0.91	3.679	0.91
	I_3	3.475	0.92	3.792	0.90	3.469	0.96	3.759	0.95	3.336	0.91	3.644	0.93
(60, 40)	I_1	3.478	0.92	3.800	0.92	3.470	0.93	3.766	0.93	3.331	0.92	3.679	0.93
	I_2	3.469	0.92	3.789	0.93	3.462	0.92	3.752	0.96	3.319	0.93	3.662	0.92
	I_3	3.457	0.93	3.781	0.91	3.456	0.93	3.741	0.92	3.311	0.93	3.651	0.93
(60, 50)	I_1	3.454	0.93	3.777	0.92	3.441	0.94	3.742	0.92	3.311	0.94	3.651	0.93
	I_2	3.444	0.92	3.765	0.94	3.429	0.90	3.731	0.96	3.301	0.93	3.644	0.94
	I_3	3.439	0.94	3.761	0.92	3.417	0.92	3.724	0.97	3.289	0.95	3.631	0.93
(80, 50)	I_1	3.438	0.90	3.751	0.93	3.419	0.92	3.718	0.93	3.291	0.92	3.629	0.93
	I_2	3.417	0.92	3.741	0.94	3.411	0.92	3.707	0.94	3.284	0.93	3.622	0.93
	I_3	3.412	0.90	3.732	0.90	3.403	0.93	3.702	0.91	3.280	0.94	3.617	0.92
(80, 65)	I_1	3.415	0.94	3.731	0.92	3.400	0.93	3.694	0.94	3.262	0.93	3.601	0.94
	I_2	3.402	0.93	3.724	0.94	3.394	0.91	3.687	0.94	3.255	0.93	3.599	0.93
	I_3	3.388	0.92	3.717	0.92	3.391	0.93	3.679	0.92	3.238	0.92	3.582	0.91

Data analysis

In this section, we adopt a real data set obtained of a solar lighting device sample accelerated under temperature. We are adopted the analysis of partially step-stress ALT sample obtained from partially step-stress model formulated to assess the reliability characteristics of a solar lighting device. Also, the stress factor consider to be the temperature and normal stress considered at temperature 293 K and the stress level was changed during the time $\tau^* = 5.0$ (in hundred hours) to 353 K, see Kundu and Ganguly [25]. The lifetime of eq. (31) devices put under test are recorded to be:

Normal	0.140	0.783	1.324	1.582	1.716	1.794	1.883	2.293	2.660	2.674	2.725	3.085
Conditions	3.924	4.396	4.612	4.892								
Stress	5.002	5.022	5.082	5.112	5.147	5.238	5.244	5.247	5.305	5.337	5.407	5.408
Conditions	5.445	5.483	5.717									

To test whether accelerated generalized half-logistic distribution presents a good fit for the last data. Figure 1 has shown the MLE fit survival functions and the corresponding empirical survival function. The two, fitted distribution functions and observed distribution functions have Kolmogorov-Smirnov (KS) distance to be 0.1496 and the corresponding p-value to be 0.8329. Hence, generalized half-logistic distribution presents a good fit for these sample of data. From the complete data, we observe the MLE $\hat{\alpha}$ and $\hat{\theta}$ of the parameters α and θ is equal to $\hat{\alpha} = 0.1514$ and $\hat{\theta} = 14.503$. Now, we generate type-II GHC sample under consideration $\tau_1 = 5.1$, $\tau_2 = 5.4$, $m = 25$, and, $J = 16$. The accelerated type-II GHC sample is reported to be {0.14, 0.783, 1.324, 1.582, 1.716, 1.794, 1.883, 2.293, 2.66, 2.674, 2.725, 3.085, 3.924, 4.396, 4.612, 4.892, 5.002, 5.022, 5.082, 5.112, 5.147, 5.238, 5.244, 5.247, 5.305}. Also, the integer value of $r = 25$ and the terminated test time $\xi = t_m = 5.305$. From the profile log-likelihood function (22) in fig. 2, we used the initial value of iteration be $\theta = 16.0$. The point estimate of the model parameters and the parameters of life when $t = 3.0$ are computed by ML and Bayes methods under SEL and non-informative prior information ($a = 0.0001$ and $b = 0.0003$) are reported in tab. 5. The corresponding 95% asymptotic confidence interval and credible intervals are reported in tab. 6. The problem generation from the full conditional posterior distribution and its convergence under Bayesian approach is described by figures which present a simulation number of α and θ generated by MCMC method and the corresponding histogram by figs. 3-6 which have shown the quality of the generation from posterior distribution.

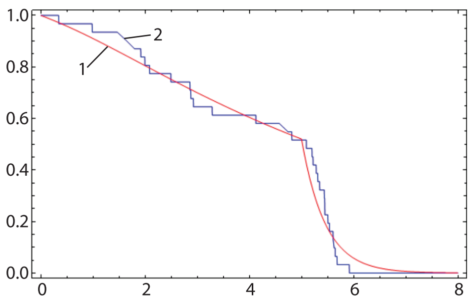

Figure 1. The fitted survival functions – 1 based on MLE and empirical survival function – 2 of the solar lighting device sample

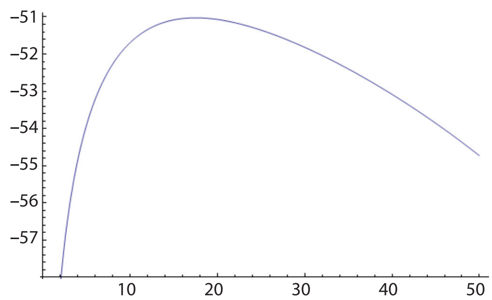

Figure 2. The profile log-likelihood function of θ

Table 5. The Point ML and Bayes estimate

Method	α	θ	$R(3.0)$	$H(3.0)$
$(.)_{ML}$	0.1951	17.4966	0.6315	0.6335
$(.)_{B\text{-}MCMC}$	0.1959	18.0467	0.1859	0.1867

Table 6. The 95% ML and Bayes interval estimate

Pa	ACI	Lenth	CI	Lenth
α	(0.1082, 0.2822)	0.1740	(0.1212, 0.2883)	0.1671
θ	(1.8116, 33.1816)	31.3700	(6.4719, 35.8470)	29.3752

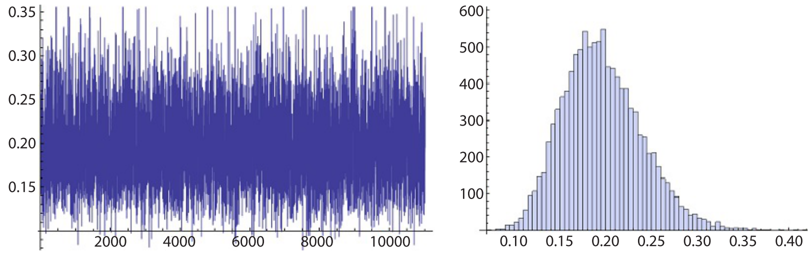

Figure 3. The generated MCMC samples of α

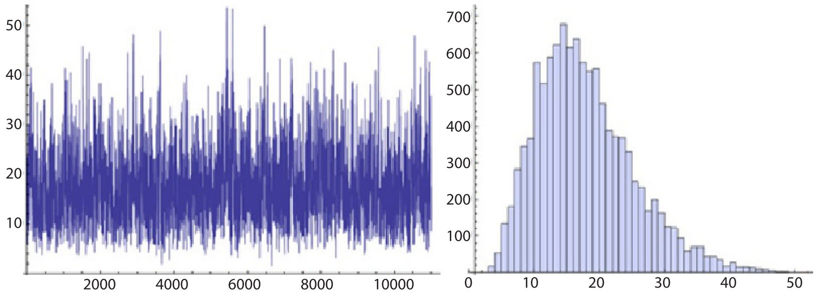

Figure 4. The generated MCMC samples of θ

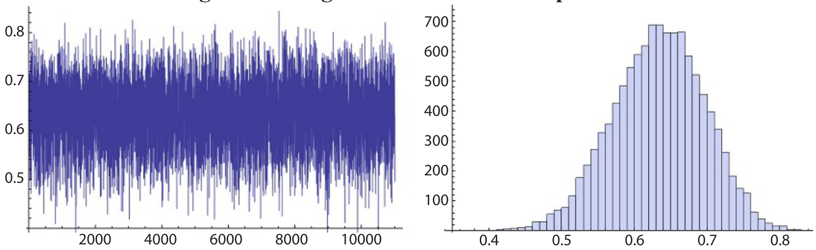

Figure 5. The generated MCMC samples of R

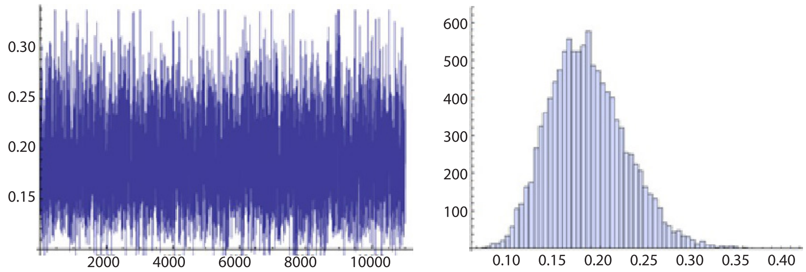

Figure 6. The generated MCMC samples of H

Conclusion

The reliability of products need to collect some information about the life of product. Therefore, needing to the censoring scheme which save the ideal test time and the minimum number of failures needing for statistical inference, we applied the generalized type-II hybrid censoring scheme. Also, needing to obtain the reliability results more quickly for a high reliable product then, we applied the ALT model. In our paper, we consider the products have the life distributed with generalized half-logistic lifetime distribution. The unknown model parameters are estimated by ML and Bayes methods. The numerical results from Monte-Carlo simulation study and illustrative example to assess and discuss the developed results.

Acknowledgment

This work was funded by the University of Jeddah, Jeddah, Saudi Arabia, under grant No. (UJ-20-141-DR). The authors, therefore, acknowledge with thanks the University technical and financial support.

References

[1] Balakrishnan, N., Order Statistics from the Half Logistic Distribution, *Journal of Statistical Computation and Simulation*, *20* (1985), 1, pp. 287-309
[2] Balakrsihnan, N., Hossain, A., Inference for the Type-II Generalized Logistic Distribution under Progressive Type-II Censoring, *Journal of Statistical Computation and Simulation*, *77* (2007), 12, pp. 1013-1031
[3] Ramakrsihnan, V., Generalizations to Half Logistic Distribution and Related Inference, Ph. D. thesis, Acharya Nagarjuna University, Nagarjuna Nagar, India, (AP), 2008
[4] Arora, S. H., *et al*., Some Results on Maximum Likelihood Estimators of Parameters of Generalized Half Logistic Distribution under Type-I Progressive Censoring with Changing, *International Journal of Contemporary Mathematical Sciences*, *5* (2010), Jan., pp. 685-698
[5] Kim, Y., *et al*., Bayesian Estimation in the Generalized Half Logistic Distribution under Progressively Type II Censoring, *Journal of the Korean Data and Information Science Society, 22* (2011), Dec., pp. 977-987
[6] Chaturvedi, A., *et al*., Estimation and Testing Procedures for the Reliability Functions of Generalized Half Logistic Distribution, *Journal of the Korean Statistical Society*, *45* (2016), 2, pp. 314-328
[7] Almarashi, A. M., Parameters Estimation for Constant-Stress Partially Accelerated Life Tests of Generalized Half-Logistic Distribution Based on Progressive Type-II Censoring, *Revstat*, *18* (2020), 4, pp. 437-452
[8] Balakrishnan, N., Aggarwala, R., *Progressive Censoring – Theory, Methods, and Applications*, Birkhauser, Boston, USA, 2000
[9] Childs, A., *et al*., Exact Likelihood Inference Based on Type-I and Type-II hybrid Censored Samples from the Exponential Distribution, *Annals of the Institute of Statistical Mathematics*, *55* (2003), June, pp. 319-330
[10] Gupta, R. D., Kundu, D., Hybrid Censoring Schemes with Exponential Failure Distribution, *Communications in Statistics-Theory and Methods*, *27* (1998), 12, pp. 3065-3083
[11] Kundu, D., Pradhan, B., Estimating the Parameters of the Generalized Exponential Distribution in Presence of Hybrid Censoring, *Communications in Statistics-Theory and Methods*, *38* (2009), 12, pp. 2030-2041
[12] Algarni, A., *et al*., Joint Type-I Generalized Hybrid Censoring for Estimation the two Weibull Distributions, *Journal of Information Science and Engineering*, *36* (2020), 6, pp. 1243-1260
[13] Chandrasekar, B., *et al*., Exact Likelihood Inference for the Exponential Distribution under Generalized Type-I and Type-II Hybrid Censoring, *Naval Research Logistics*, *51* (2004), 7, pp. 994-1004
[14] Nelson, W., *Accelerated Testing Statistical Models, Test, Plans and Data Analyses*, Wiley, New York, USA, 1990
[15] Bagdonavicius, V., Nikulin, M., *Accelerated Life Models: Modelling and Sta- tistical Analysis*, Chapman Hall/CRC Press, Boca Raton, Fla., USA, 2002
[16] Abd-Elmougod, G. A., Mahmoud, E. E., Paramters Estimation of Compound Rayleigh Distribution under an Adaptive Type-II Progressively Hybrid Censored Data for Constant Partially Accelerated Life Tests, *Global Journal of Pure and Applied Mathematics, 13* (2016), Jan., pp. 8361-8372
[17] Algarni, A., *et al*., Partially Constant Stress Accelerates Life Test Model in Analyses Lifetime Competing Risks with a Bathtub Shape Lifetime Distribution in the Presence of Type-I, *Transylvanian Review*, (2019), ID202798080

[18] Ganguly, A., Kundu, D., Analysis of Simple Step-Stress Model in Presence of Competing Risks, *Journal of Statistical Computation and Simulation*, *8*6 (2016), 10, pp. 1989-2006
[19] Wang, R., Fei, H., Statistical Inference of Weibull Distribution for Tampered Failure Rate Model in Progressive Stress Accelerated Life Testing, *Journal of Systems Science and Complexity*, *17* (2004), 2, pp. 237-243
[20] Abdel-Hamid, A. H., Al-Hussaini, E. K., Progressive Stress Accelerated Life Tests under Finite Mixture Models, *Metrika*, *66* (2007), Jan., pp. 213-231
[21] Abu-Zinadah, H., *et al*., Competing Risks Model with Partially Step-Stress Accelerate Life Tests in Analyses Lifetime Chen Data under Type-II Censoring Scheme, *Open Phys*, *17* (2019), Apr., pp. 192-199
[22] Modhesh, A. A., *et al.*, Analysis of Progressive First-Failure-Censoring in the Burr XII Model for Competing Risks Data, *American Journal of Theoretical and Applied Statistics*, *4* (2015), 6, pp. 610-618
[23] Bakoban, R. A., Abd-Elmougod, G. A., The MCMC in Analysis of Progressively First Failure Censored Competing Risks Data for Gompertz Model, *Journal of Computational and Theoretical Nanoscience*, *13* (2016), 10, pp. 6662-6670
[24] Metropolis, N., *et a*l., Equations of State Calculations by Fast Computing Machines, *Journal Chemical Physics*, *21* (1953), 6, pp. 1087-1091
[25] Kundu, D., Ganguly, A., *Analysis of Step-Stress Models*, Elsavier, Amsterdam, The Netherlands, 2018

Paper submitted: September 8, 2022
Paper revised: October 27, 2022
Paper accepted: October 30, 2022