ON THE TRAVELING WAVE SOLUTIONS OF PULSE PROPAGATION IN MONOMODE FIBER VIA THE EXTENDED KUDRYASHOV'S APPROACH

by

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In this research paper, we tackle with the solitary wave solutions to the pulse propagation in monomode optic fiber by defining non-linear Schrodinger equation with higher order. We applied the extended Kudryashov's method with Bernoulli-Riccati equation and successfully gained soliton solutions and their contour, 2-D and 3-D graphical representations, such as dark, singular, periodic and kink type solutions. We also discussed the obtained results in the related section.

Key words*: optical fiber, soliton pulse, periodic soliton, singular soliton, analytical soliton solution*

Introduction

In the past three decade, the importance of non-linear equations in describing and modelling many physical phenomena has gained great importance, and many researchers have focused on studies in this field. Genetics, biology, heat transfer, energy transfer, communication, physics, plasma physics, quantum physics, ocean engineering, wave propagation, optics, optical fibers, lighting, optical devices, *etc*. can be given as examples. Researchers have obtained many models and equations in these areas. Since these equations are non-linear in general, many models and solution methods have been developed, used, and still continue to be used for the solutions of these equations, such as ansatz method, modified simple equation integration, the improved Sardar sub-equation, the simplified Hirota's, the Backlund transformation, the extended sinh-Gordon equation expansion, the modified $\exp[-\phi(\zeta)]$-expansion function, $[G'/(G' + G + A]$-expansion, $R(z)$ function, extended Kudryashov's, generalized tanh, modified extended tanh expansion method enhanced with new Riccati solution, the generalized exponential rational function, Jacobi elliptical solution finder, variable separated ODE, the Riccati-Bernoulli sub-ODE, Sinc-Galerkin, modified extended mapping and so on [1-15]. Among the non-linear equations, the Schrodinger equation has its own importance. Because the Schrodinger equation is used in the modelling of many physical phenomena, especially in the examination of optics and optical wave behavior. Recently, many studies have been conducted examining the soliton behavior in optical fibers. For example, non-linear Schrodinger's equation (NLSE) with the Kudryashov's sextic power law, perturbed Fokas-Lenells, complex Ginzburg-Landau, Chen-Lee-Liu, Manakov, generalized Kudryashov's equation, Kundu-Mukherjee-Naskar, Sasa-Sat-

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suma, Biswas-Milovic, Hirota-Maccari system, Schrodinger-Hirota, Radhakrishnan-Kundu Lakshmanan, *etc*. [16-32].

The femto-second pulse propagation (PP) in monomode optical fiber (MOF) can be given by the higher order NLSE [33]:

$$i\left(\kappa_x+\lambda_1\kappa_t\right)-\frac{\lambda_2}{2}\kappa_{tt}+\beta\left|\kappa\right|^2\kappa+i\alpha_1\left(\left|\kappa\right|^2\kappa\right)_t+i\alpha_2\left(\left|\kappa\right|^2\right)_t\kappa-i\frac{\lambda_3}{6}\kappa_{ttt}=0 \tag{1}$$

$$\kappa\left(x,t\right)=\mathrm{e}^{i(ax-bt)}\kappa\left(\xi\right),\ \xi=-\eta x+t+\xi_0 \tag{2}$$

where $i=(-1)^{1/2}$, a, b, η, and ξ_0 are the non-zero real values, κ is the envelope of the electric field, λ_1, λ_2, λ_3, α_1, α_2, and β are the inverse of the group velocity (GV), the second-order dispersion (SOD), the third-order dispersion (TOD), the coefficient of the cubic term, the soliton self-frequency shift and the non-linear coefficient, respectively.

In this current study, we apply the extended Kudryashov method (EKM) with Riccati-Bernoulli equation seeking the wave solutions femto-second PP in MOF in eq. (1).

The EKM with Riccati-Bernoulli equation was applied to seek the wave solutions femto-second PP in MOF in eq. (1). The non-linear ordinary equation (NODE) form and the constraint equations are obtained depending on λ_1, λ_2, and λ_3. The EKM is implemented to investigated problem, and the graphical forms of the solution functions were presented in 3-D and 2-D graphics.

The NODE structure of the problem

Plugging the eq. (2) into eq. (1) and decomposing the resulted equation into imaginary and real parts we obtain:

$$-\frac{\mathrm{d}^3\kappa(\xi)}{\mathrm{d}\xi^3}\lambda_3+\left\{\left(18\alpha_1+12\alpha_2\right)\left[\kappa\left(\xi\right)\right]^2+3b^2\lambda_3+6b\lambda_2-6\eta+6\lambda_1\right\}\frac{\mathrm{d}\kappa\left(\xi\right)}{\mathrm{d}\xi}=0 \tag{3}$$

$$2\left(b\alpha_1+\beta\right)\left[\kappa\left(\xi\right)\right]^3+2\left(\frac{1}{6b^3\lambda_3}1+\frac{1}{2b^2\lambda_2}1+b\lambda_1-a\right)\kappa\left(\xi\right)-\left(b\lambda_3+\lambda_2\right)\frac{\mathrm{d}^2\kappa\left(\xi\right)}{\mathrm{d}\xi^2}=0 \tag{4}$$

Integration of eq. (3) by taking the constant of integration as zero, gives:

$$2\left(3\alpha_1+2\alpha_2\right)\left[\kappa\left(\xi\right)\right]^3+3\left[b^2\lambda_3+2\left(b\lambda_2-\eta+\lambda_1\right)\right]\kappa\left(\xi\right)-\lambda_3\frac{\mathrm{d}^2\kappa\left(\xi\right)}{\mathrm{d}\xi^2}=0 \tag{5}$$

We can accept the eq. (4) or eq. (5) as NODE form of eq. (1):

$$\lambda_1=\frac{\left(8b^3\alpha_2{}^2+\left(-9\eta\alpha_1{}^2-24\beta\alpha_2\right)b^2+\left(18\beta^2-18\eta\beta\alpha_1+9\left(\alpha_1+2/3\alpha_2\right)\alpha_1 a\right)b+9\beta\left(-\eta\beta+a\left(\alpha_1+2/3\alpha_2\right)\right)\right)}{3\left(b\alpha_1+\beta\right)\left(2b\alpha_2-3\beta\right)} \tag{6}$$

$$\lambda_2=-2\frac{b\left(3\alpha_1+2\alpha_2\right)}{b\alpha_1+\beta}+2,\ \lambda_3=2\frac{3\alpha_1+2\alpha_2}{b\alpha_1+\beta} \tag{7}$$

Moreover, taking into account the terms $\kappa^3(\xi)$, $\kappa''(\xi)$ in eq. (4) or in eq. (5) by using the balancing rule gives $m+2=3m$, so $m=1$ which is the balancing constant.

Pen picture of the EKM

Step 1. Let us, take into account a non-linear partial differential equation (NLPDE) and a complex wave transformation:

$$H\left(\kappa_x,\kappa^2\kappa_{xx},\kappa_t,\kappa_{xt},...\right)=0 \tag{8}$$

$$\kappa\left(x,t\right)=e^{i(ax-bt)}\kappa\left(\xi\right),\ \ \xi=t-\eta x+\xi_0 \tag{9}$$

where η, ξ_0 are real values. Substitute eq. (9) into eq. (8), it is acquired a non-linear ordinary differential equation (NLODE):

$$N\left(\kappa\left(\xi\right),\kappa'\left(\xi\right),\kappa''\left(\xi\right),\kappa^2\left(\xi\right)\kappa'\left(\xi\right),...\right)=0 \tag{10}$$

In eq. (10) $\kappa'(\xi)$, $\kappa''(\xi)$ denotes:

$$\frac{\mathrm{d}\kappa\left(\xi\right)}{\mathrm{d}\xi},\ \frac{\mathrm{d}^2\kappa\left(\xi\right)}{\mathrm{d}\xi^2}$$

Step 2. The extended Kudryashov approach offers the solution of the eq. (10):

$$\kappa(\xi)=a_0+\sum_{k=1}^{m}\sum_{i+j=k}a_{ij}P^{i}\left(\xi\right)Q^{j}\left(\xi\right)+\sum_{k=1}^{m}\sum_{i+j=k}b_{ij}P^{-i}\left(\xi\right)Q^{-j}\left(\xi\right) \tag{11}$$

where m is the balancing term, A_{ij} and B_{ij} are coefficients to be determined. Moreover, $P(\xi)$ and $Q(\xi)$ are the solution of the Bernoulli and the Riccati equations:

$$\frac{\mathrm{d}P}{\mathrm{d}\xi}=P'(\xi)=K_2P^2(\xi)-K_1P(\xi),\ \ K_2\neq 0 \tag{12}$$

$$\frac{\mathrm{d}Q}{\mathrm{d}\xi}=Q'(\xi)=L_2Q^2(\xi)+L_1Q(\xi)+L_0,\ \ L_2\neq 0 \tag{13}$$

$$P(\xi)=\begin{cases}\dfrac{K_1}{K_2+K_1\mathrm{e}^{(K_1\xi+\xi_0)}}, & K_1\neq 0\\ -\dfrac{1}{K_2\xi+\xi_0}, & K_1=0\end{cases} \tag{14}$$

$$Q\left(\xi\right)=\begin{cases}-\dfrac{L_1}{2L_2}-\dfrac{\sqrt{\delta}}{2L_2}\tanh\left(\dfrac{\sqrt{\delta}}{2}\xi+\xi_0\right), & \delta>0\\ -\dfrac{L_1}{2L_2}-\dfrac{\sqrt{\delta}}{2L_2}\coth\left(\dfrac{\sqrt{\delta}}{2}\xi+\xi_0\right), & \delta>0\\ -\dfrac{L_1}{2L_2}+\dfrac{\sqrt{-\delta}}{2L_2}\tan\left(\dfrac{\sqrt{-\delta}}{2}\xi+\xi_0\right), & \delta<0\\ -\dfrac{L_1}{2L_2}-\dfrac{\sqrt{-\delta}}{2L_2}\cot\left(\dfrac{\sqrt{-\delta}}{2}\xi+\xi_0\right), & \delta<0\\ -\dfrac{L_1}{2L_2}-\dfrac{1}{L_2\xi+\xi_0}, & \delta=0\end{cases} \tag{15}$$

in which $\delta = L_1^2 - 4L_0L_2$. Equations (14) an (15) have six different cases. Case 1: $K_1 \neq 0$, $\delta > 0$; Case 2: $K_1 \neq 0$, $\delta < 0$; Case 3: $K_1 \neq 0$, $\delta = 0$; Case 4: $K_1 = 0$, $\delta > 0$; Case 5: $K_1 = 0$, $\delta < 0$; Case 6: $K_1 = 0$, $\delta = 0$.

Step 3. Considering the eqs. (11)-(13), together, gives a polynomial structure of $P^i(\xi)Q^j(\xi)$ and $P^{-i}(\xi)Q^{-j}(\xi)$. Arranging the coefficients of $P^i(\xi)Q^j(\xi)$ and $P^{-i}(\xi)Q^{-j}(\xi)$ and equating to zero, it is acquired an algebraic system for A_{ij}, B_{ij}, K_1, K_2, L_0, L_1, L_2, and η.

Step 4. Solving the system, produces some solution sets. Using the appropriate set with the eqs. (11), (14), (15), and eq. (9), we derive the soliton solution of eq. (8).

Constraint equations and implementation of the method

Considering the balancing constant $m = 1$, we use the following notations, $a_{10} = a_1$, $a_{01} = a_2$, $b_{10} = b_1$, and $b_{01} = b_2$. As a result, the solution form of eq. (11):

$$\kappa(\xi) = a_0 + a_1 R(\xi) + a_2 S(\xi) + \frac{b_1}{R(\xi)} + \frac{b_2}{S(\xi)} \tag{16}$$

Plugging the eq. (16) and its derivatives along with the eqs. (12) and (13) into eq. (4), it gives the system in terms of $R(\xi)^i S(\xi)^j$ and $R(\xi)^{-i} S(\xi)^{-j}$:

$$R(\xi)^0 S(\xi)^0 : \Big(\big(-2\alpha_2 b^3 + \big(3\beta + (12a_1b_1 + 12a_2b_2)\alpha_1\alpha_2 - 3\alpha_1\eta\big)b^2 +$$
$$+\Big(\big((12a_1b_1 + 12a_2b_2)\alpha_2 + (-18a_1b_1 - 18a_2b_2)\alpha_1 - 3\eta\big)\beta + 3\alpha_1 a\Big)b +$$
$$+3\Big(\big(-6a_1b_1 - 6a_2b_2\big)\beta + a\Big)\beta\Big)A_0\Big) +$$
$$+\Big(2(b\alpha_1 + \beta)(b\alpha_2 - 3/2\beta)a_0^{\,3} - 2(a_2L_0L_1 - b_1K_1K_2 + b_2L_1L_2)(b\alpha_2 - 3/2\beta)\Big) = 0$$

$$R(\xi)^{-1} S(\xi)^0 : \Big(-2/3\alpha_2 b^3 + \Big(\beta + \big(2a_0^{\,2} + 2a_1b_1 + 4a_2b_2\big)\alpha_1\alpha_2 - \alpha_1\eta\Big)b^2 +$$
$$+\Big(\big(-3a_0^{\,2} - 3a_1b_1 - 6a_2b_2\big)\beta + K_1^{\,2} + a\Big)\beta\Big) +$$
$$+\Big(\Big(\big(2A_0^{\,2} + 2a_1b_1 + 4a_2b_2\big)\alpha_2 + \big(-3a_0^{\,2} - 3a_1b_1 - 6a_2b_2\big)\alpha_1 - \eta\Big)\beta - 2/3K_1^{\,2}\alpha_2 + \alpha_1 a\Big)b = 0$$

$$R(\xi)^0 S(\xi)^1 : \Big(-2/3\alpha_2 b^3 + \Big(\beta + \big(2a_0^{\,2} + 4a_1b_1 + 2a_2b_2\big)\alpha_1\alpha_2 - \alpha_1\eta\Big)b^2 +$$
$$+\Big(\big(-3a_0^{\,2} - 6a_1b_1 - 3a_2b_2\big)\beta + L_1^{\,2} + 2L_0L_2 + a\Big)\beta\Big) + \Big(\Big(\big(2a_0^{\,2} + 4a_1b_1 + 2a_2b_2\big)\alpha_2 +$$
$$+\big(-3a_0^{\,2} - 6a_1b_1 - 3a_2b_2\big)\alpha_1 - \eta\Big)\beta + \big(-2/3L_1^{\,2} - 4/3L_0L_2\big)\alpha_2 + \alpha_1 a\Big)b = 0$$

$$R(\xi)^1 S(\xi)^0 : \Big(-2/3\alpha_2 b^3 + \Big(\beta + \big(2a_0^{\,2} + 2a_1b_1 + 4a_2b_2\big)\alpha_1\alpha_2 - \alpha_1\eta\Big)b^2 +$$
$$+\Big(\big(-3a_0^{\,2} - 3a_1b_1 - 6a_2b_2\big)\beta + K_1^{\,2} + a\Big)\beta\Big) +$$
$$+\Big(\Big(\big(2a_0^{\,2} + 2a_1b_1 + 4a_2b_2\big)\alpha_2 + \big(-3a_0^{\,2} - 3a_1b_1 - 6a_2b_2\big)\alpha_1 - \eta\Big)\beta - 2/3K_1^{\,2}\alpha_2 + \alpha_1 a\Big)b = 0$$

$$R(\xi)^0 S(\xi)^{-1} : \Big(-2/3\alpha_2 b^3 + \Big(\beta + \big(2a_0^{\,2} + 4a_1b_1 + 2a_2b_2\big)\alpha_1\alpha_2 - \alpha_1\eta\Big)b^2 +$$
$$+\Big(\big(-3a_0^{\,2} - 6a_1b_1 - 3a_2b_2\big)\beta + L_1^{\,2} + 2L_0L_2 + a\Big)\beta\Big) + \Big(\Big(\big(2a_0^{\,2} + 4a_1b_1 + 2a_2b_2\big)\alpha_2 +$$
$$+\big(-3a_0^{\,2} - 6a_1b_1 - 3a_2b_2\big)\alpha_1 - \eta\Big)\beta + \big(-2/3L_1^{\,2} - 4/3L_0L_2\big)\alpha_2 + \alpha_1 a\Big)b = 0$$

$$R(\xi)^2 S(\xi)^0 :\left(a_0\left(b\alpha_1+\beta\right)a_1+K_1K_2\right)a_1=0,\ R(\xi)^0 S(\xi)^2 :\left(a_0\left(b\alpha_1+\beta\right)a_2-L_1L_2\right)a_2=0$$

$$R(\xi)^0 S(\xi)^{-2} : b_2\left(a_0\left(b\alpha_1+\beta\right)b_2-L_0L_1\right)=0,\ R(\xi)^3 S(\xi)^0 : a_1\left(\left(b\alpha_1+\beta\right)a_1^2-2K_2^2\right)=0$$

$$R(\xi)^0 S(\xi)^3 : a_2\left(\left(b\alpha_1+\beta\right)a_2^2-2L_2^2\right)=0,\ R(\xi)^1 S(\xi)^2 : a_1a_2^2\left(b\alpha_1+\beta\right)=0$$

$$R(\xi)^1 S(\xi)^{-2} : a_1b_2^2\left(b\alpha_1+\beta\right)=0,\ R(\xi)^2 S(\xi)^1 : a_1^2a_2\left(b\alpha_1+\beta\right)=0$$

$$R(\xi)^2 S(\xi)^{-1} : a_1^2b_2\left(b\alpha_1+\beta\right)=0,\ R(\xi)^{-2} S(\xi)^1 : a_2b_1^2\left(b\alpha_1+\beta\right)=0$$

$$R(\xi)^{-1} S(\xi)^2 : a_2^2b_1\left(b\alpha_1+\beta\right)=0,\ R(\xi)^{-1} S(\xi)^{-2} : b_1b_2^2\left(b\alpha_1+\beta\right)=0$$

$$R(\xi)^{-2} S(\xi)^{-1} : b_1^2b_2\left(b\alpha_1+\beta\right)=0,\ R(\xi)^1 S(\xi)^1 : a_0a_1a_2\left(b\alpha_1+\beta\right)=0$$

$$R(\xi)^1 S(\xi)^{-1} : a_0a_1b_2\left(b\alpha_1+\beta\right)=0,\ R(\xi)^{-1} S(\xi)^1 : a_0a_2b_1\left(b\alpha_1+\beta\right)=0$$

$$R(\xi)^{-1} S(\xi)^{-1} : a_0b_1b_2\left(b\alpha_1+\beta\right)=0,\ R(\xi)^0 S(\xi)^{-3} : \left(\left(b\alpha_1+\beta\right)b_2^2-2L_0^2\right)b_2=0$$

$$R(\xi)^{-2} S(\xi)^0 : a_0b_1^2\left(b\alpha_1+\beta\right)=0,\ R(\xi)^{-3} S(\xi)^0 : b_1^3\left(b\alpha_1+\beta\right)=0$$

The solution of the system produces many sets, we selected some of them which are given at the below. Set-i refers to Case i, $i = 1, 2,\ldots, 6$:

$$\text{Set1}=\text{Set2}=\begin{bmatrix} K_2=K_2,\ K_1=K_1,\ L_0=\dfrac{-b^3\lambda_3-3b^2\lambda_2-6b\lambda_1+6a}{\left(12b\lambda_3+12\lambda_2\right)L_2},\ L_1=0,\ L_2=L_2,\ a_0=0,\ a_1=0 \\ a_2=-\dfrac{L_2\sqrt{\left(b\lambda_3+\lambda_2\right)\left(b\alpha_1+\beta\right)}}{b\alpha_1+\beta},\ b_1=0,\ b_2=-\dfrac{-b^3\lambda_3-3b^2\lambda_2-6b\lambda_1+6a}{12L_2\sqrt{\left(b\lambda_3+\lambda_2\right)\left(b\alpha_1+\beta\right)}} \end{bmatrix} \tag{17}$$

$$\text{Set3}=$$

$$=\begin{bmatrix} K_2=\dfrac{a_1}{2}\sqrt{2b\alpha_1+2\beta},K_1=\dfrac{\sqrt{-2\left(2b\alpha_2-3\beta\right)\left(-2\alpha_2b^3-3b^2\eta\alpha_1+3b\alpha_1a+3b^2\beta-3b\beta\eta+3a\beta\right)}}{2b\alpha_2-3\beta}, \\ L_0=L_0,L_1=L_1,L_2=L_2 \\ a_0=-\dfrac{\sqrt{\left(-2\left(2b\alpha_2-3\beta\right)\left(-2\alpha_2b^3-3b^2\eta\alpha_1+3b\alpha_1a+3b^2\beta-3b\beta\eta+3a\beta\right)\right)\left(2b\alpha_1+2\beta\right)}}{\left(4b\alpha_2-6\beta\right)\left(b\alpha_1+\beta\right)}, \\ a_1=a_1,a_2=0,b_1=0,b_2=0 \end{bmatrix} \tag{18}$$

$$\text{Set4}=\text{Set5}=$$

$$=\begin{bmatrix} K_2=K_2,L_0=-\dfrac{b_2}{2}\sqrt{2b\alpha_1+2\beta},L_1=0,L_2=-\dfrac{-2\alpha_2b^3-3b^2\eta\alpha_1+3ab\alpha_1+3b^2\beta-3b\beta\eta+3a\beta}{b_2\left(2b\alpha_2-3\beta\right)\sqrt{2b\alpha_1+2\beta}} \\ a_0=0,a_1=0,a_2=0,b_1=0,b_2=b_2 \end{bmatrix} \tag{19}$$

$$\text{Set6}=$$

$$=\begin{bmatrix} a_0=-\dfrac{\sqrt{-\left(2b^2\alpha_2\alpha_1-3b\beta\alpha_1+2b\beta\alpha_2-3\beta^2\right)\left(-2\alpha_2b^3-3b^2\eta\alpha_1+3ab\alpha_1+3b^2\beta-3b\beta\eta+3a\beta\right)}}{2b^2\alpha_2\alpha_1-3b\beta\alpha_1+2b\beta\alpha_2-3\beta^2} \\ a_1=0,a_2=0,b_1=0,b_2=0,K_2=K_2,L_0=L_0,L_1=L_1,L_2=L_2 \end{bmatrix} \tag{20}$$

Case 1: ($K_1 \neq 0$, $\delta > 0$)

$$N_{11}(x,t) = \mathrm{e}^{i(ax-bt)}\left(a_0 + \frac{a_1 K_1}{K_2 + K_1 \mathrm{e}^{K_1(-\eta x+t)}} + a_2\left(-\frac{L_1+1}{2L_2}\sqrt{\delta}\tanh\left(\frac{-\eta x+t}{2}\sqrt{\delta}\right)\right) + \right.$$
$$\left. + \frac{b_1\left(K_2 + K_1 \mathrm{e}^{K_1(-\eta x+t)}\right)}{K_1} + b_2\left(-\frac{L_1+1}{2L_2}\sqrt{\delta}\tanh\left(\frac{-\eta x+t}{2}\sqrt{\delta}\right)\right)^{-1}\right) \tag{21}$$

$$N_{12}(x,t) = \mathrm{e}^{i(ax-bt)}\left(a_0 + \frac{a_1 K_1}{K_2 + K_1 \mathrm{e}^{K_1(-\eta x+t)}} + a_2\left(-\frac{L_1+1}{2L_2}\sqrt{\delta}\coth\left(\frac{-\eta x+t}{2}\sqrt{\delta}\right)\right) + \right.$$
$$\left. + \frac{b_1\left(K_2 + K_1 \mathrm{e}^{K_1(-\eta x+t)}\right)}{K_1} + b_2\left(-\frac{L_1+1}{2L_2}\sqrt{\delta}\coth\left(\frac{-\eta x+t}{2}\sqrt{\delta}\right)\right)^{-1}\right) \tag{22}$$

Case 2: ($K_1 \neq 0$, $\delta < 0$)

$$N_{21}(x,t) = \mathrm{e}^{i(ax-bt)}\left(a_0 + \frac{a_1 K_1}{K_2 + K_1 \mathrm{e}^{K_1(-\eta x+t)}} + a_2\left(-\frac{L_1+1}{2L_2}\sqrt{-\delta}\tan\left(\frac{-\eta x+t}{2}\sqrt{-\delta}\right)\right) + \right.$$
$$\left. + \frac{b_1\left(K_2 + K_1 \mathrm{e}^{K_1(-\eta x+t)}\right)}{K_1} + b_2\left(-\frac{L_1+1}{2L_2}\sqrt{-\delta}\tan\left(\frac{-\eta x+t}{2}\sqrt{-\delta}\right)\right)^{-1}\right) \tag{23}$$

$$N_{22}(x,t) = \mathrm{e}^{i(ax-bt)}\left(a_0 + \frac{a_1 K_1}{K_2 + K_1 \mathrm{e}^{K_1(-\eta x+t)}} + a_2\left(-\frac{L_1+1}{2L_2}\sqrt{-\delta}\cot\left(\frac{-\eta x+t}{2}\sqrt{-\delta}\right)\right) + \right.$$
$$\left. + \frac{b_1\left(K_2 + K_1 \mathrm{e}^{K_1(-\eta x+t)}\right)}{K_1} + b_2\left(-\frac{L_1+1}{2L_2}\sqrt{-\delta}\cot\left(\frac{-\eta x+t}{2}\sqrt{-\delta}\right)\right)^{-1}\right) \tag{24}$$

Case 3: ($K_1 \neq 0$, $\delta = 0$),

$$N_3(x,t) = \mathrm{e}^{i(ax-bt)}\left(a_0 + \frac{a_1 K_1}{K_2 + K_1 \mathrm{e}^{K_1(-\eta x+t)}} + a_2\left(-\frac{L_1}{2L_2} - \frac{1}{L_2(-\eta x+t)}\right) + \right.$$
$$\left. + \frac{b_1\left(K_2 + K_1 \mathrm{e}^{K_1(-\eta x+t)}\right)}{K_1} + b_2\left(-\frac{L_1}{2L_2} - \frac{1}{L_2(-\eta x+t)}\right)^{-1}\right) \tag{25}$$

Case 4: ($K_1 = 0$, $\delta > 0$)

$$N_{41}(x,t) = \mathrm{e}^{i(ax-bt)}\left(a_0 - \frac{a_1}{K_2(-\eta x+t)} + a_2\left(-\frac{L_1+1}{2L_2}\sqrt{\delta}\tanh\left(\frac{-\eta x+t}{2}\sqrt{\delta}\right)\right) - \right.$$
$$\left. - b_1 K_2(-\eta x+t) + b_2\left(-\frac{L_1+1}{2L_2}\sqrt{\delta}\tanh\left(\frac{-\eta x+t}{2}\sqrt{\delta}\right)\right)^{-1}\right) \tag{26}$$

$$N_{42}(x,t)=\mathrm{e}^{i(ax-bt)}\left(a_0-\frac{a_1}{K_2(-\eta x+t)}+a_2\left(-\frac{L_1+1}{2L_2}\sqrt{\delta}\coth\left(\frac{-\eta x+t}{2}\sqrt{\delta}\right)\right)-\right.$$
$$\left.-b_1K_2(-\eta x+t)+b_2\left(-\frac{L_1+1}{2L_2}\sqrt{\delta}\coth\left(\frac{-\eta x+t}{2}\sqrt{\delta}\right)\right)^{-1}\right) \tag{27}$$

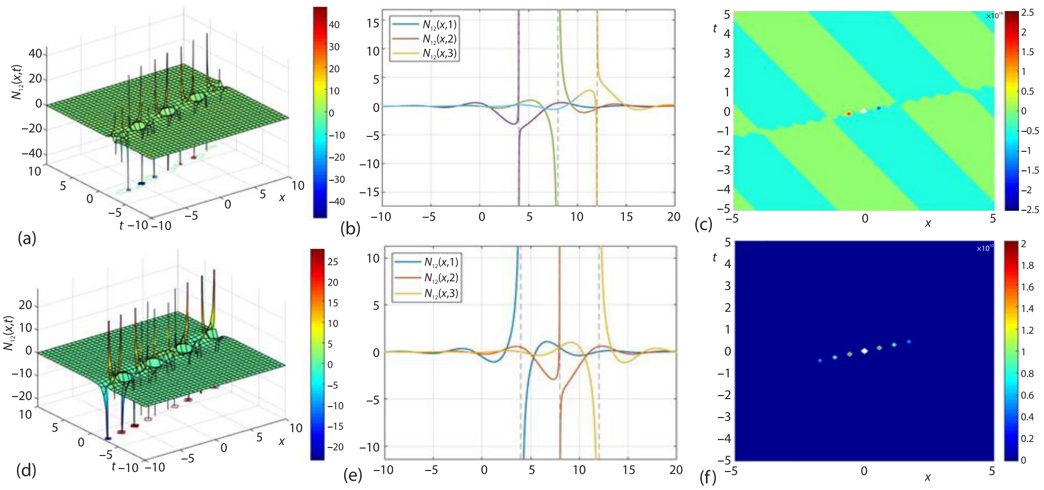

Figure 1. The 3-D (a) and (d), 2-D (b) and (e), and contour (c) and (f) plot of $N_{12}(x, t)$ in eq. (22) for $a = -1$, $b = 0.75$, $\alpha_1 = 0.8$, $\alpha_2 = 0.5$, $\eta = 0.25$, and $\beta = 2$; (a)-(c) Re[$N_{12}(x, t)$], and (d)-(f) Im [$N_{12}(x, t)$]

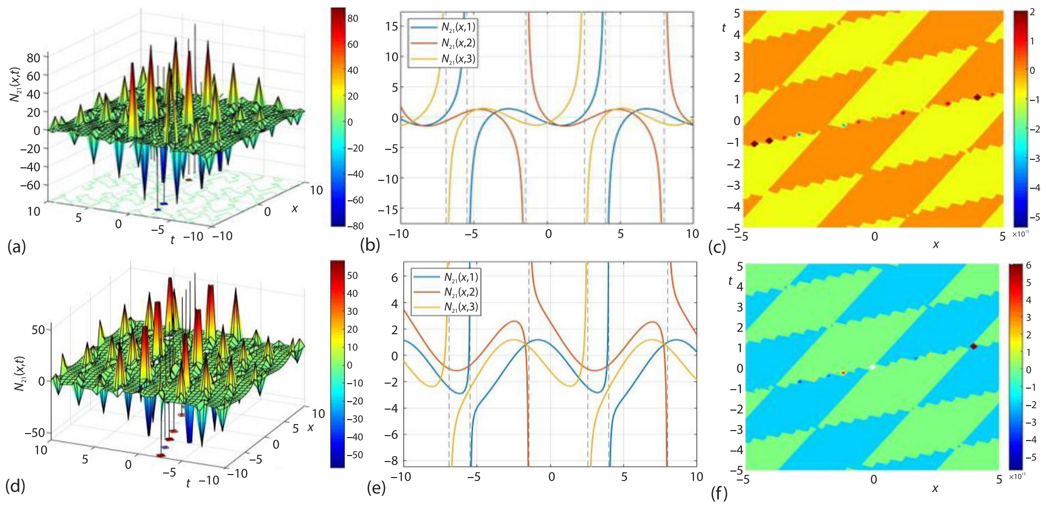

Figure. 2 The 3-D (a) and (d), 2-D (b) and (e), and contour (c) and (f) plot of $N_{21}(x, t)$ in eq. (23) for $a = 1$, $b = 0.75$, $\alpha_1 = 0.8$, $\alpha_2 = 0.5$, $\eta = 0.25$, and $\beta = 2$; (a)-(c) Re[$N_{21}(x, t)$], (d)-(f) Im [$N_{21}(x, t)$]

Case 5: ($K_1 = 0$, $\delta < 0$)

$$N_{51}(x,t) = \mathrm{e}^{i(ax-bt)}\left(a_0 - \frac{a_1}{K_2(-\eta x+t)} + a_2\left(-\frac{L_1+1}{2L_2}\sqrt{-\delta}\tan\left(\frac{-\eta x+t}{2}\sqrt{-\delta}\right)\right) - \right.$$
$$\left. -b_1K_2(-\eta x+t) + b_2\left(-\frac{L_1+1}{2L_2}\sqrt{-\delta}\tan\left(\frac{-\eta x+t}{2}\sqrt{-\delta}\right)\right)^{-1}\right) \tag{28}$$

$$N_{52}(x,t) = \mathrm{e}^{i(ax-bt)}\left(a_0 - \frac{a_1}{K_2(-\eta x+t)} + a_2\left(-\frac{L_1+1}{2L_2}\sqrt{-\delta}\cot\left(\frac{-\eta x+t}{2}\sqrt{-\delta}\right)\right) - \right.$$
$$\left. -b_1K_2(-\eta x+t) + b_2\left(-\frac{L_1+1}{2L_2}\sqrt{-\delta}\cot\left(\frac{-\eta x+t}{2}\sqrt{-\delta}\right)\right)^{-1}\right) \tag{29}$$

Case 6: ($K_1 = 0$, $\delta = 0$)

$$N_6(x,t) = \mathrm{e}^{i(ax-bt)}\left(a_0 - \frac{a_1}{K_2(-\eta x+t)} + a_2\left(-\frac{L_1}{2L_2} - \frac{1}{L_2(-\eta x+t)}\right) - \right.$$
$$\left. -b_1K_2(-\eta x+t) + b_2\left(-\frac{L_1}{2L_2} - \frac{1}{L_2(-\eta x+t)}\right)^{-1}\right) \tag{30}$$

Considering the eq. (16) and substituting the Set *i* into the solution functions in Case *i* (i = 1, 2,..6) given by eqs. (17)-(30) and using the eq. (2), the solutions of eq. (1) are acquired. Some depictions of the obtained solutions are presented in eqs. (21)-(30). The obtained results are given figs. 1-5.

Conclusion

In this research paper, we studied the PP in MOF by considering the λ_1, λ_2, and λ_3 are as the constraint equations. We reveals a plethora of soliton solutions and graphical representations to the PP in MOF by applying the extended Kudryashov's scheme with Bernoulli-Riccati equation. The EKM has been applied for the first time to investigated problem and the obtained results show that the extended Kudryashov's scheme method is effective, reliable and applicable to such problems. Besides, we gained the dark, singular, periodic and kink type solitons and depicted in 3-D, 2-D and contour graphs. We hope that the obtained results in the article might be helpful not only for the PP in MOF, but also for the researches on the optical PP in fibers.

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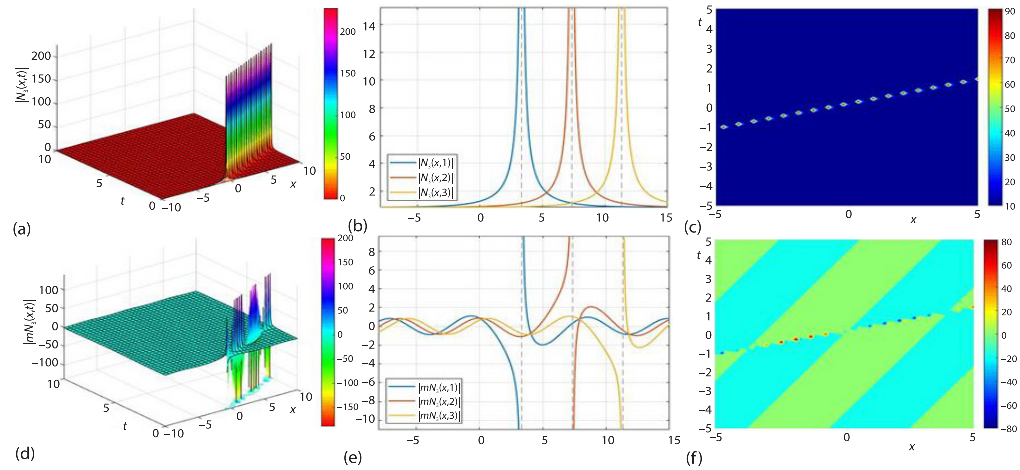

Figure 3. The 3-D (a) and (d), 2-D (b) and (e), and contour (c) and (f) plot of modulus and imaginary component of $N_3(x, t)$ in eq. (25) for $a = 1$, $b = 0.75$, $\alpha_1 = 0.8$, $\alpha_2 = 0.5$, $\eta = 0.25$, and $\beta = 2$, and $a_1 = 1.2$

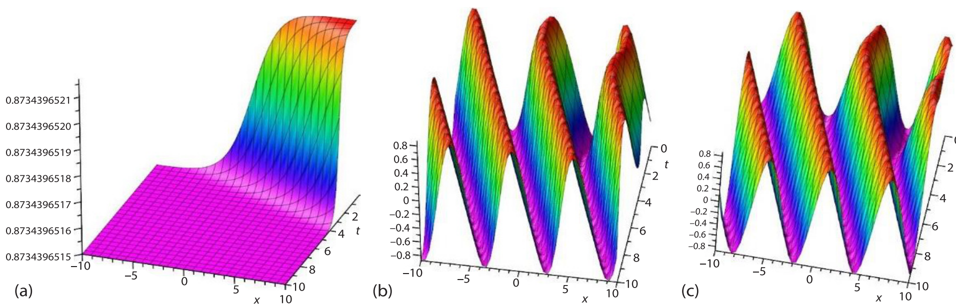

Figure 4. The 3-D plot of modulus (a), real (b), and imaginary (c) component of $N_3(x, t)$ in eq. (25) for $a = 1$, $b = 0.75$, $\alpha_1 = 0.8$, $\alpha_2 = 0.5$, $\eta = 0.25$, and $\beta = -2$, and $a_1 = 2$

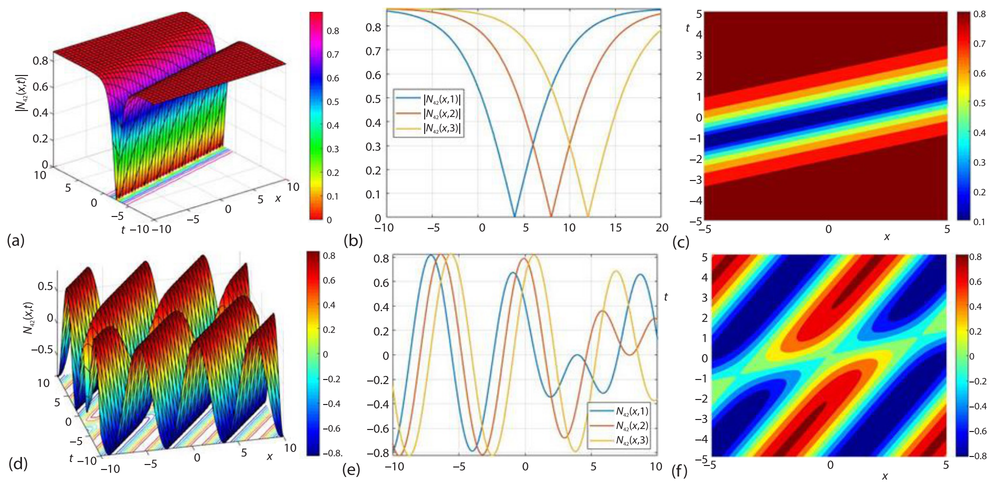

Figure 5. The 3-D (a) and (d), 2-D (b) and (e), and contour (c) and (f) plot of modulus and imaginary component of $N_{42}(x, t)$ in eq. (27) for $a = 1$, $b = 0.75$, $\alpha_1 = 0.8$, $\alpha_2 = 0.5$, $\eta = 0.25$, and $\beta = 2$

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