

SOLUTION METHOD OF MULTIPHASE SEEPAGE SEQUENCE IN HEAVY OIL RESERVOIR BY STEAM INJECTION CONSIDERING NUMERICAL OSCILLATION

by

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In order to improve the dynamic control ability of multiphase flow in heavy oil reservoir by steam injection, he put forward the sequential solution method of multiphase flow in heavy oil reservoir by steam injection based on numerical oscillation, established saturation equation and dynamic equation of multiphase flow in heavy oil reservoir by steam injection, solved the output pressure of multiphase flow in heavy oil reservoir by pressure equation based on volume conservation, and established the sequential solution model of all-component model of multiphase flow in heavy oil reservoir by steam injection. The output pressure of multiphase seepage in steam injection-production heavy oil reservoir is transformed into parabolic variable, and the saturation and composition are hyperbolic variable models. Through the analysis method of saturation and composition characteristics of all-component model, combined with the pressure signal analysis of numerical oscillation, empirical mode decomposition and spectrum analysis methods are adopted to realize the velocity parameter analysis and multiphase seepage sequence analysis of steam injection-production heavy oil reservoir, and the numerical oscillation analysis and parameter estimation of multiphase seepage in steam injection-production heavy oil reservoir are realized by establishing the sequential solution algorithm of steam injection-production heavy oil reservoir. The simulation results show that this method is used to solve the multiphase seepage sequence of heavy oil reservoir by steam injection, and the parameter estimation accuracy is high, which improves the calculation efficiency of numerical simulation of steam injection and production. The calculation efficiency and accuracy of this algorithm in 2-D and 3-D examples with gravity are verified by examples.

Keywords: *numerical oscillation, steam injection of heavy oil, reservoir, sequential solution of multiphase seepage*

Introduction

At present, with the gradual depletion of conventional oil and gas resources with high quality and easy exploitation, most of the remaining oil is of low quality, and quite a few of them are tight oil and heavy oil. Faced with huge energy demand, people began to pay more

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attention to unconventional oil and gas resources, such as the exploitation of tight oil, shale oil, heavy oil and super heavy oil [1, 2]. Heavy oil has high asphaltene and gum content, and poor fluidity. Because of the poor fluidity of heavy oil, its development difficulty is far greater than that of conventional oil and gas [3]. At present, the classification standard of viscosity as an index given by the second international symposium on heavy crude oil and asphalt is generally adopted internationally (heavy oil > 100 mPas, $10^\circ < \text{API} < 20^\circ$; $U > 1000$ mPa-s, $\text{API} < 10^\circ$) and the classification standard of API gravity ($\text{API} < 22.3^\circ$) proposed by American petroleum institute. Compared with foreign countries, the heavy oil in domestic oilfields is thicker, that is, the viscosity of domestic heavy oil at the same density is higher [4-6].

The methods to improve the fluidity of heavy oil mainly include reducing crude oil viscosity, increasing rock permeability, increasing production pressure difference, etc. Specific technical means include: hot water drive, steam drive, chemical drive, gas injection drive, burning oil layer, etc. Among them, chemical flooding and miscible flooding belong to cold recovery technology. The former improves the fluidity of heavy oil by injecting surfactant, alkali and polymer, while the latter uses the expansion, viscosity reduction and extraction of N₂, CO₂, flue gas and other gases under reservoir conditions to improve the output of heavy oil. Hot water flooding, steam flooding, and burning oil layer belong to thermal collection techniques, which mainly reduce the viscosity of crude oil by increasing the temperature, and improve the fluidity of heavy oil by thermal expansion, emulsification, and steam deoiling. In the total output of heavy oil in China, the total output produced by thermal recovery technology accounts for about two thirds. Therefore, this paper puts forward a numerical oscillation-based sequential solution method of multiphase seepage in heavy oil reservoir by steam injection [7, 8]. Firstly, the saturation equation and dynamic equation of multiphase seepage in heavy oil reservoir by steam injection are established, and the pressure equation based on volume conservation is used to solve the output pressure of multiphase seepage in heavy oil reservoir by steam injection, and then the sequential solution model of all-component model of multiphase seepage in heavy oil reservoir by steam injection is established. Empirical mode decomposition and spectrum analysis are used to analyze the velocity parameters and sequence of multiphase seepage in heavy oil reservoir by steam injection, and numerical oscillation analysis and parameter estimation of multiphase seepage in heavy oil reservoir by steam injection are realized by establishing the algorithm for solving sequence of heavy oil injection. Finally, the simulation test shows the superior performance of this method in improving the computational efficiency of steam injection and production numerical simulation.

Numerical simulation and parameter analysis of steam injection and production

Numerical simulation of multiphase seepage injection and production in heavy oil reservoir by steam injection and production

Sequential solution algorithm is a widely developed numerical solution format in recent years, which is used to solve non-linear governing equations in reservoir numerical simulation. Different from the total implicit algorithm, the sequential solution algorithm divides the whole non-linear system into several independent sub-stages by decoupling, and only one variable is solved in a single solution.

For the isothermal model, the sequential solution algorithm first solves the pressure equation to get the pressure, and then solves the saturation equation to get the saturation when the pressure is fixed. For the non-isothermal model, the energy equation needs to be solved after

the above steps. It is advantageous to the step-by-step solution method of sequential solution algorithm, which can greatly reduce the scale of equations solved in one time, reduce the memory occupied by the computer, and improve the efficiency of numerical simulation. At the same time, because the equations in different sub-stages are all composed of equations with the same mathematical characteristics (for example, the pressure equation is often elliptic or parabolic, and the saturation equation has the characteristics of hyperbolic equation), the sequential solution algorithm can choose a more appropriate numerical solution method in different sub-stages according to the mathematical characteristics of the equations [8]. Multi-phase seepage in steam injection-production heavy oil reservoir is a closed rigid frame structure. During the movement of multi-phase seepage in steam injection-production heavy oil reservoir, the radius of curvature has a random field separation effect on centripetal force. The hydrodynamic analysis of the flow field inside the impeller is carried out, and the numerical discretization of multi-phase seepage in steam injection-production heavy oil reservoir is carried out by using the rotating left side system, and the curve coordinate system of multi-phase seepage in steam injection-production heavy oil reservoir is constructed:

$$\begin{pmatrix} x_1(t) \\ \vdots \\ x_m(t) \end{pmatrix} = \begin{pmatrix} a_{11} \\ \vdots \\ a_{m1} \end{pmatrix} s_1(t) + \dots + \begin{pmatrix} a_{1n} \\ \vdots \\ a_{mn} \end{pmatrix} s_n(t) \quad (1)$$

where $s_1(t)$ is the implicit solution of physical quantities such as saturation and a_{mn} is the curvature velocity component of flow turbulence. When the upstream-downstream relationship formed by the full-field grid points according to their potential energy can form a directed graph, the topological sorting algorithm can be used to solve it, which can turn the problem of solving PDE of saturation into the problem of solving ODE at the point, thus greatly reducing the calculation time at this stage. The alternating direction method is introduced to solve the elliptic or parabolic pressure equation [9-12], the topological sorting algorithm is adopted in the calculation of saturation equation, and a single oil-water two-phase numerical solver with tens of millions of nodes is established by using the sequential solving algorithm [13-15]. The normalized 3-D transient image processing model is obtained:

$$d(t) = \begin{cases} \arctan\left(\frac{X_2'(t)}{X_1'(t)}\right), & X_1'(t) > 0 \\ \arctan\left(\frac{X_2'(t)}{X_1'(t)}\right) + \pi, & X_1'(t) < 0, \quad t = 1, 2, \dots, T \\ \pi / 2, & X_1'(t) = 0 \end{cases} \quad (2)$$

where $X_1'(t)$ is the contact finite element curvature characteristic coefficient of multiphase seepage in steam-injected heavy oil reservoir, $X_2'(t)$ – the sequential solution parameter of thermal recovery model of multiphase seepage in steam-injected heavy oil reservoir, T – the rotation cycle variable of multiphase seepage in steam-injected heavy oil reservoir, and it is a constant [16, 17] The saturation equation and dynamic equation of multiphase seepage in steam-injected heavy oil reservoir are established, and the pressure equation based on volume conservation is used to solve the characteristic value of output pressure of multiphase seepage in steam-injected heavy oil reservoir.

Oil-water two-phase flow model of multiphase seepage in steam injection-production heavy oil reservoir

Based on the oil-water two-phase flow model, this paper will revise the energy conservation calculation of the sequential solution algorithm of steam injection and production of heavy oil by numerical and theoretical analysis. In the process of establishing the sequential solution algorithm of steam injection and production of heavy oil, this paper found that using the original energy equation to solve in the oil-water two-phase region would lead to large calculation errors and even abnormal solutions [18-20]. In this paper, the error sources of using the original energy equation are analyzed, and at last, a modified calculation equation is proposed, and the model equation of oil-water two-phase flow is obtained:

$$\begin{pmatrix} x_1(t) \\ \vdots \\ x_m(t) \end{pmatrix} = \begin{pmatrix} a_{1i} \\ \vdots \\ a_{mi} \end{pmatrix} s_1(t) \Rightarrow \frac{x_1(t)}{a_{1i}} = \dots = \frac{x_m(t)}{a_{mi}} = s_i(t) \quad (3)$$

It can be seen from eq. (1) that when the mixture reaches phase equilibrium, it should meet the following requirements:

- material conservation of each component of multiphase seepage in heavy oil reservoir by steam injection,
- the temperature and pressure of each phase are the same, and
- the chemical potential of each component in each phase is equal.

Besides, in the equilibrium state, the energy of multiphase percolation system in heavy oil reservoir by steam injection should be at the lowest value of all possible conditions, which can be expressed:

$$(\partial G)_{p,T} = 0 \quad (4)$$

$$(\partial^2 G)_{p,T} > 0 \quad (5)$$

where G is Gibbs energy of the system. For a hypothetical multiphase seepage single-phase fluid in heavy oil reservoir produced by steam injection, the process of judging whether the Gibbs energy of the mixture is minimum under given conditions is called stability analysis. The governing equation of steam injection-production three-phase fluid contains two mass conservation equations and one energy conservation equation:

$$g(Y_1, Y_2 \dots Y_n) = 1 + \sum_{i=1}^n Y_i [\ln Y_i + \ln(\phi)_2 - \ln z_i - \ln(\phi)_0 - 1] \quad (6)$$

where Y_i is a measure of multiphase seepage in heavy oil reservoir produced by steam injection to measure the amount of each component in new phase, ϕ – the fugacity coefficient of each component, 0 and 2 represent the original single phase and the possible new phase, respectively, and z_i – the mole fraction of each component in the original phase of multiphase seepage in steam injection-production heavy oil reservoir.

When the multiphase seepage mixture of heavy oil reservoir produced by steam injection is stable under given conditions, the aforementioned formula is always greater than or equal to 0 at all solutions, so it is necessary to search for its global minimum value. If it converges to $\min(g) < 0$ or $\sum_{i=1}^n Y_i > 1$ the mixture is unstable, and the equilibrium flash calculation is required [21-23].

For the vapor-liquid two-phase analysis of multiphase seepage in heavy oil reservoir produced by steam injection, the composition of new phase can be estimated.

For liquid-like mixtures:

$$Y_i = K_i z_i \quad (7)$$

For gas-like mixtures:

$$Y_i = \frac{z_i}{K_i} \quad (8)$$

where K_i is the phase equilibrium constant of multiphase seepage in heavy oil reservoir produced by steam injection.

The sequential solution model of all-component model of multiphase flow in heavy oil reservoir by steam injection is established. The output pressure of multiphase flow in heavy oil reservoir by steam injection is transformed into parabolic variable, and the saturation and component are hyperbolic variable models [24].

Optimization of multiphase seepage sequence in steam injection-production heavy oil reservoir

Analysis of multiphase seepage parameters of steam injection-production heavy oil reservoir

Through the analysis method of saturation and component characteristics of the full component model, combined with the pressure signal analysis of numerical oscillation, empirical mode decomposition and spectrum analysis methods, the viscous flux control equation is constructed, and the multiphase seepage analysis of steam injection-production heavy oil reservoir is realized. The expression of single variable viscous flux with relative permeability as saturation is obtained:

$$avg_{BWP}(k) = \frac{1}{n} \sum_{j=1}^k \sum_{i=1}^{n_j} \omega BWP(j,i) \quad (9)$$

where j, i are the curvature vector controlled by steam injection-production three-phase flow, BWP – the numerical model expression of saturation-related flow turbulence of each phase, ω – the rotational angular velocity, and n – the univariate component of saturation. Based on the viscous flux design, the expression of multiphase seepage control equation of steam injection-production heavy oil reservoir is obtained:

$$V(a_1, \dots, a_m) = \left(a_i^{j-1} \right)_{i,j=1}^m = \begin{pmatrix} 1 & a_1 & \dots & a_1^{m-1} \\ 1 & a_2 & \dots & a_2^{m-1} \\ \vdots & \vdots & \ddots & \vdots \\ 1 & a_m & \dots & a_m^{m-1} \end{pmatrix} \quad (10)$$

where V is the decomposition matrix of multiphase seepage state of steam injection-production heavy oil reservoir and a_i^{j-1} – the conservation coefficient of velocity fraction, and then the grid interface turbulence equation for controlling steam injection-production three-phase flow is obtained:

$$\begin{aligned}
 \Lambda_j(k) &= P[z(k)/m_j(k), z^{k-1}] \\
 &= P[z(k)/m_j(k), \hat{x}^{0j}(k-1/k-1), P^{0j}(k-1/k-1)] \\
 &= N\{[z^j(k) - z^j(k/k-1)] | 0, S^j(k)\}
 \end{aligned}
 \tag{11}$$

where $z_j(k)$ is the displacement tensor of multiphase seepage model of steam injection-production heavy oil reservoir considering numerical oscillation, \hat{x}^{0j} – the rotation tensor of multiphase seepage in steam injection-production heavy oil reservoir, and $S_j(k)$ – the flux of multiphase seepage in steam injection-production heavy oil reservoir. Through the previous research, the finite volume method is used for discrete treatment of multiphase seepage in steam injection-production heavy oil reservoir. The specific process is shown in fig. 1.

Viscous flux control equation is constructed, which provides the original model foundation for the next step of constructing multiphase seepage analysis model of steam injection-production heavy oil reservoir based on finite element method and flow turbulence curvature correction [25].

Solution of multiphase seepage sequence in steam injection-production heavy oil reservoir

On the basis of the aforementioned control equation, in order to realize the flow field analysis of multiphase flow in steam injection-production heavy oil reservoir, multi-block structured grid analysis of flow turbulence according is designed to low Reynolds number turbulence model [26]. Firstly, based on Houston method, the multi-body system rigid structure topology of multiphase flow in steam injection-production heavy oil reservoir was carried out, and the pressure signal analysis of multiphase flow in steam injection-production heavy oil reservoir combined with numerical oscillation was obtained. Empirical mode decomposition and spectral analysis methods were adopted. Based on the multi-body system theory, the unit body of multi-phase seepage impeller in steam injection-production heavy oil reservoir is designed to realize the analysis of velocity parameters and sequence of multi-phase seepage in steam injection-production heavy oil reservoir.

By adopting the equivalent topological method of the characteristic distribution of rows and rows in the grid structure and the calibration of element vector growth sequence, the randomness characteristics of rows and columns of multi-phase seepage output grid in steam injection-production heavy oil reservoir are obtained:

$$\begin{aligned}
 V(a_1, \dots, a_m) [E | V(a_1, \dots, a_m)^{-1} V(b_1, \dots, b_m)] &= \\
 &= [V(a_1, \dots, a_m) | V(b_1, \dots, b_m)]
 \end{aligned}
 \tag{12}$$

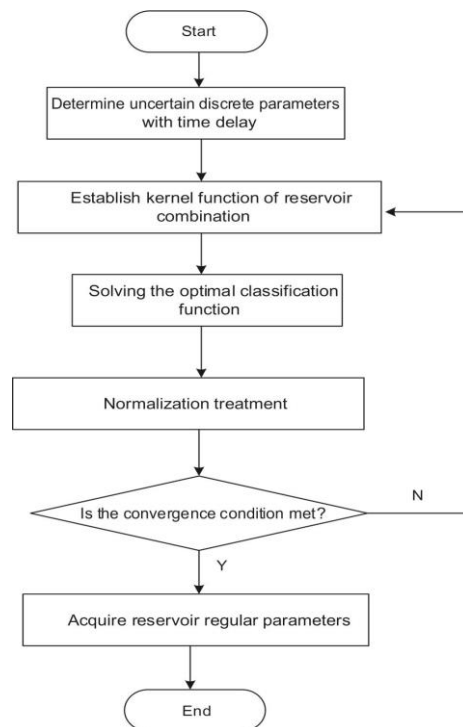


Figure 1. Discrete treatment process of multiphase seepage in heavy oil reservoir by steam injection

where a_m is the heat dissipation coefficient of rock and b_m – the low-order array. Through the previous topological structure construction, a low Reynolds number turbulent model of multiphase flow in heavy oil reservoir by steam injection is obtained, and the expression is:

$$f(x) = \begin{cases} x/P_1, & x \in I_1 \\ (x-P_1)/P_2, & x \in I_2 \\ \dots\dots & \dots\dots \\ (x-\sum_{i=1}^{n-1} P_i)/P_n, & x \in I_n \end{cases} \quad (13)$$

where I_n is the expression error of seepage sequence in the process of coordinate transformation and P_i – the model parameters that the pressure and temperature meet the saturated vapor pressure relationship, thus obtaining the multi-block structured grid model of multiphase seepage in heavy oil reservoir by steam injection production as:

$$w(j,i) = \frac{1}{n_j - 1} \sum_{q=1, q \neq i}^{n_j} \|x_q^{(j)} - x_i^{(j)}\|^2 \quad (14)$$

where n_j is the internal energy enthalpy, $x_q^{(j)}$ – the internal energy entropy of water and steam, $x_i^{(j)}$ – the enthalpy of saturated steam, and the established enthalpy of steam is a bivariate function of temperature and pressure. For temperature and pressure, the discrete format is obtained in a finite volume way in space, and the mixture of combined control state parameters is obtained. The equilibrium flash calculation should include:

- the number of phases,
- vapor fraction, and
- composition of each phase (y_1, y_2, \dots, y_n).

Vapor-liquid two-phase flash evaporation (x_1, x_2, \dots, x_n), which generally used to represent vapor phase composition and liquid phase composition. The x_i and y_i can be represented by:

$$x_i = \frac{z_i}{1 + (K_i - 1)\beta} \quad (15)$$

$$y_i = \frac{K_i z_i}{1 + (K_i - 1)\beta} \quad (16)$$

In order to solve it conveniently by iterative method, Rachford-Rice equation is generally chosen, and its value decreases monotonically with the increase of β :

$$F(\beta) = \sum_{i=1}^n \frac{z_i (K_i - 1)}{1 + (K_i - 1)\beta} = 0 \quad (17)$$

If $K_i z_i > 1$ and $(z_i/K_i) > 1$, then the previous formula will get a physically meaningful root between 0 and 1. Combined with the pressure signal analysis of numerical oscillation, empirical mode decomposition and spectrum analysis are adopted to realize the velocity parameter analysis and multiphase seepage sequence analysis of steam injection-production heavy oil reservoir, and the numerical oscillation analysis and parameter estimation of multiphase seepage of steam injection-production heavy oil reservoir are realized by establishing the solution algorithm of steam injection-production heavy oil sequence.

Test results and analysis

In order to verify the application of this method in multiphase flow analysis and sequential solution of heavy oil reservoir produced by steam injection, a 2-D conceptual model of one injection and one production with a length of 192 m and a width of 192 m is considered. The model x is evenly divided into 64 grids, and y is evenly divided into 64 grids, each of which has a length and width of 3 m. The constant pressure injection well is located in the center of (3 m, 3 m) grid point, and the saturated wet steam composed of 98% steam and 2% water is continuously injected at 4 MPa. The constant pressure production well is located at (192 m, 192 m), and the continuous production is at 1 MPa. The full-field initial conditions are set at 2 MPa pressure and 400.15 K temperature, and the initial saturation field is set at 0.7 oil phase saturation and 0.3 water phase saturation. See tab. 1 for the settings of other physical quantities.

Table 1. Parameter setting

Parameter	Value
X-direction permeability	0.574
Y-direction permeability	0.005
Porosity	0.829
Irreducible water saturation residual oil saturation	0.452

The eccentric water distribution pipe string of multiphase seepage in steam injection-production heavy oil reservoir is mainly composed of eccentric water distributor, compression packer, ball seat and tubing, which can realize multi-stage fine water distribution, generally divided into 4-6 layers, up to 11 layers at most. Under the condition of not moving the pipe string, the downhole water distribution nozzle can be arbitrarily changed and layered testing can be carried out, thus reducing the workload of adjustment and combination testing of water injection wells, and when testing the water injection rate of any layer, the water injection of other layers will not be affected. According to the previous parameter settings, the finite element method proposed in this paper is used to model the multiphase seepage in steam injection-production heavy oil reservoir, and the numerical oscillation distribution of multiphase seepage flow field in steam injection-production heavy oil reservoir is obtained as shown in fig. 2.

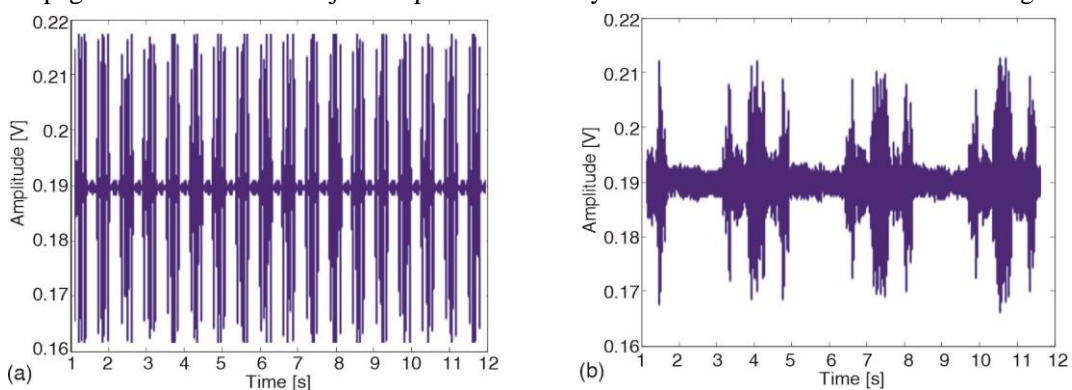


Figure 2. Numerical oscillation distribution of multiphase seepage flow field in heavy oil reservoir by steam injection; (a) inlet seepage and (b) outlet seepage

From the analysis of the previous numerical oscillation results, it can be seen that although the distribution of physical quantities calculated by the sequential solution algorithm after the modified energy equation is different from that of the fully hidden algorithm to some

extent, its displacement trend is basically the same, because only a small number of external iterations are used in this example. At this time, if the number of external cycles is appropriately increased, the calculation results can be consistent with that of the fully hidden algorithm. However, under the same conditions, the numerical results that do not conform to the physical facts appear in the results calculated by using the energy equation before correction. At this time, if the calculation is continued, the numerical results will fail after more days of simulation, and the displacement trend can be calculated stably by using the energy equation after correction. The example shows that the modified energy equation can further optimize the decoupling between mass and energy system and reduce the error caused by decoupling. On this basis, the distribution of multiphase seepage flow field in steam injection-production heavy oil reservoir is tested, and the solution result of multiphase seepage sequence in steam injection-production heavy oil reservoir is shown in fig. 3.

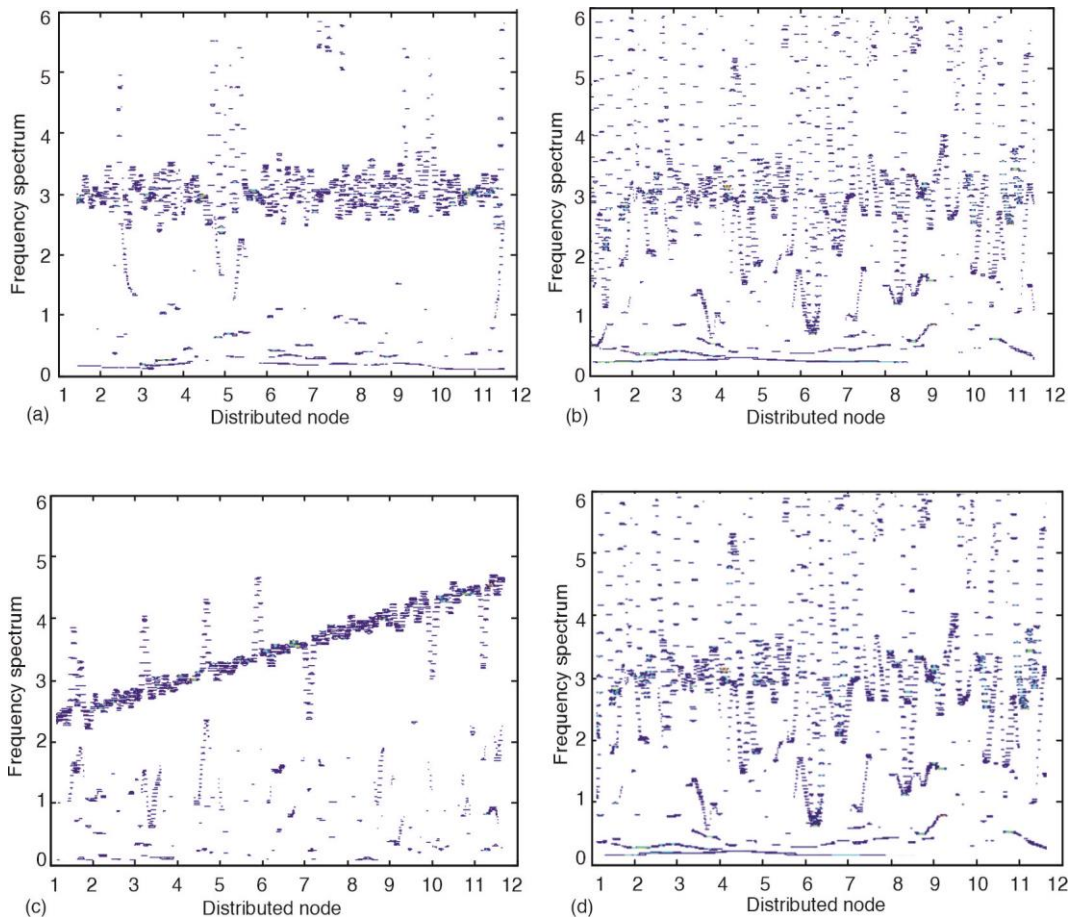


Figure 3. Solution results of multiphase seepage sequence in heavy oil reservoir by steam injection; (a) first order, (b) second order, (c) third order, and (d) fourth order

According to the analysis of fig. 3, the method in this paper establishes a complete sequential solution algorithm of steam injection and production of heavy oil, and realizes efficient solution of steam injection and production of heavy oil. To realize the effective de-

coupling between equations, different solutions are selected according to the equations with different mathematical characteristics, and a complete sequential solution algorithm for steam injection and production of heavy oil is established under the modified energy solution system. 2-D and 3-D examples show that the comparison results of efficiency are shown in tab. 2. The analysis shows that the calculation efficiency of using the sequential solution algorithm established in this paper to solve the problem of steam injection and production of heavy oil can be greatly improved by about 2-4 times compared with the fully hidden results.

Table 2. Comparison results of calculation efficiency (%)

Iterative steps	This method	[4]	[5]
10	99.18	79.14	59.98
20	93.44	76.78	59.82
30	90.77	76.77	59.75
40	95.18	77.58	59.87

Conclusions

In this paper, the sequential solution algorithm of steam injection and recovery of heavy oil is studied, and an efficient solution system of steam injection and recovery of heavy oil is established. The pressure equation of steam injection and recovery of heavy oil is constructed by the Gaussian elimination method, and the influence of other physical quantities except pressure in the time discrete term in the pressure equation is eliminated. The saturation equation based on volume flux conservation constructed by the total seepage velocity is used to solve the saturation, and the upwind style is used to solve it. For the use of sequential solution of energy equation in steam injection and recovery of heavy oil, in the energy conservation equation of oil-water two-phase region, the influence of mass conservation error caused by temperature change on energy conservation calculation is eliminated. In this paper, the topological sorting algorithm is applied to the solution of the energy equation, which further improves the computational efficiency.

In the process of establishing the sequential solution algorithm for heavy oil production by steam injection, the following conclusions are obtained.

- The sequential solution algorithm requires effective decoupling between the variables of the equations. The saturation equation was established based on the conservation of volume flux by using the total seepage velocity. It can effectively realize the decoupling between pressure and saturation, and ensure the stability of the sequential solution algorithm.
- In the problem of steam injection and production, the solution of the energy conservation system in the sequential solution algorithm is modified in this paper. In this paper, the original energy conservation equation is modified by combining the mass and energy conservation equations. Numerical examples show that the modified sequential solution algorithm can control the error of energy solution to a certain extent and realize further decoupling of mass and energy systems.
- A complete sequential solution algorithm for steam injection and recovery of heavy oil is established to realize the efficient solution of steam injection and recovery of heavy oil. The 2-D and 3-D numerical examples show that the problem of steam injection and recovery of heavy oil can be solved by the sequential solving algorithm established in this chapter.

Compared with the fully implicit result, the computational efficiency can be greatly improved, and the efficiency is improved by about 2-4 times.

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