MAGNETO-ELECTRICALLY INDUCED VIBRATION CONTROL OF A PLATE CONTACTED WITH FLUID

by

Kenan YILDIRIM [*] ***and Mehmet SIMSEK***

Mus Alparslan Universty, Mus, Turkey

Original scientific paper
https://doi.org/10.2298/TSCI2204973Y

In this paper, magneto-electrically induced vibration control of a magneto-electro-elastic plate in contact with fluid is studied by means of maximum principle. The performance index functional to be minimized at the predetermined control duration is considered as a modified kinetic energy of the magneto-electro-elastic plate and it is defined as a weighted quadratic functional of displacement and velocity and also includes as a penalty function of the control force spent in control duration. Two numerical examples are presented and results indicate that introduced control algorithm for damping the vibrations due to magneto-electric load on magneto-electro-elastic plate is very robust and effective.

Key words: *magneto-electro-elastic, vibration, optimal control, maximum principle*

Introduction and problem formulation

The definition, magneto-electro-elastic solid, is widely used to address to a kind of smart materials have capacity to transform reversibly their properties to respond external excitation such as temperature, moisture, stress, electric or magnetic fields [1]. Since last two decades, magneto-electro-elastic (MEE) composites have gained great importance due to their ability of transforming one form of energy to another, having simple geometry and economic design and being useful in smart or intelligent structure applications [1]. Much studies are done for examining on several properties of MEE structures and they can be summarized as follows, but not limited to [2-12]. The original contribution of the present paper is that magneto-electrically induced vibration control of a plate contacted with fluid is firstly studied by means of maximum principle in this paper. Specifically, the vibration suppression problem for damping the vibrations due to magneto-electric load on MEE plate is taken into account. In order to achieve the optimal control function, adjoint equation system and suitable terminal conditions are determined. Optimal control function is obtained by means of maximum principle, which converts to optimal control problem to solving an equation system subjected to initial-boundary-terminal conditions. The solution of the differential equation system is gained by means of computer aid and results are evaluated for two numerical examples and presented in the graphical forms. At the first, we consider a rectangular plate contacted with fluid, subject to the external magneto-electric load. Initially, the plate is assumed to be undeformed. The control problem is aimed to suppress the vibrations induced by magneto-electric load. The equation of the motion of the plate shown can be expressed [13]:

[*] Corresponding author, e-mail: k.yildirim@alparslan.edu.tr

$$(\rho_P h + M_f)W_{TT} + (D + E + M)\nabla^4 W = \overline{P}(X,Y,T) + \overline{C}(T)\overline{D}(X,Y) \tag{1}$$

where ρ_P is the mass density of the plate, h – the thickness of the plate, M_f – the fluid added mass, D, E, and M are the constants, representing the plate rigidity, effective rigidities due to the presence of electricity and magnetism, respectively, $W(X, Y, T)$ – the transversal displacement at $(X,Y,T) \in \breve{Q} = \{(X,Y,T) : (X,Y) \in \breve{S}, T \in (0,T_f)\}$, $\breve{S} = (0,\ell)\times(0,\ell)$ is an open bounded set with sufficiently smooth boundary $\partial\breve{S}$, T_f – the terminal time, $\overline{P}(X, Y, T)$ – the magneto-electric load function with indicating the distribution of the force over the plate, $\overline{C}(T) \in \overline{\mathfrak{C}}_{\mathfrak{ad}}$ is control function, $\overline{\mathfrak{C}}_{\mathfrak{ad}}$ – the set of admissible control functions, which are continuous and bounded functions, and $\overline{D}(X, Y)$ – the function showing the distribution of the control force over the plate:

$$\nabla^2 = \frac{\partial^2}{\partial X^2} + \frac{\partial^2}{\partial Y^2}$$

Equation (1) is subject to the following boundary conditions:

$$W(0,Y,T) = W(\ell,Y,T) = W(X,0,T) = W(X,\ell,T) = 0 \tag{2}$$

$$W_{XX}(0,Y,T) = 0,\ W_{XX}(\ell,Y,T) = 0 \tag{3}$$

$$W_{YY}(X,0,T) = 0,\ W_{YY}(X,\ell,T) = 0 \tag{4}$$

and the initial conditions:

$$W(X,Y,0) = W_0(X,Y),\ W_T(X,Y,0) = W_1(X,Y) \tag{5}$$

Let $L^2(\breve{Q})$ denote the Hilbert space of real-valued square-integrable functions on the domain $\breve{Q}$ in the Lebesque sense with usual inner product and norm defined:

$$<\rho,\varrho>\breve{Q} = \int_{\breve{Q}} \rho(x,y,t)\varrho(x,y,t)\mathrm{d}\breve{Q},\ \|\rho\|^2 = <\rho,\rho>$$

respectively. Along the paper, it is assumed that W_0, W_1, $\overline{C}$, $\overline{D}$, and $\overline{P}$ are continuous and bounded functions on the domain $\overline{Q}$. For convenience, let us introduce the non-dimensional variables:

$$w = \frac{W}{\ell^2},\ t = \frac{T}{\ell^2}\sqrt{\frac{D+E+M}{(\rho_p h + M_f)}},\ x = \frac{X}{\ell},\ y = \frac{Y}{\ell},\ t_f = \frac{T_f}{\ell^2}\sqrt{\frac{D+E+M}{(\rho_p h + M_f)}}$$

$$P(x,y,t) = \frac{\ell^4}{D+E+M}\overline{P}(X,Y,T),\ C(t)D(X,Y) = \frac{\ell^4}{D+E+M}\overline{C}(T)\overline{D}(X,Y)$$

$$S = (0,1)\times(0,1) \text{ on } Q = \{(x,y,t) : (x,y) \in S, t \in (0,t_f)\}$$

Substituting new parameters into eqs. (1)-(5), we obtain non-dimensional equation of motion given:

$$w_{tt} + \nabla^4 w = P(x,y,t) + C(t)D(x,y) \tag{6}$$

subject to the following boundary conditions:

$$w(0,y,t) = w(1,y,t) = w(x,0,t) = w(x,1,t) = 0 \tag{7}$$

$$w_{xx}(0,y,t) = 0,\ w_{xx}(1,y,t) = 0 \tag{8}$$

$$w_{yy}(x,0,t)=0,\ \ w_{yy}(x,1,t)=0 \tag{9}$$

and the initial conditions:

$$w(x,y,0)=w_0(x,y),\ \ w_t(x,y,0)=w_1(x,y) \tag{10}$$

Optimal control problem

The objective of the control is the minimization of a given performance index at a terminal time t_f with a minimum expenditure of control force. The performance index is specified as the weighted dynamic response of the plate defined in terms of a quadratic functional of the deflection and its time derivative with the expenditure of the control energy added as a penalty term. The performance index is defined:

$$\mathcal{J}(C)=\iint_S \left\{\mu_1 w^2(x,y,t_f)+\mu_2 w_t^2(x,y,t_f)\right\}\mathrm{d}S+\int_0^{t_f}\mu_3 C^2(t)\mathrm{d}t \tag{11}$$

where $\mu_i \geq 0$ for i = 1, 2, 3 are weighted constants and $\mu_1 + \mu_2 \neq 0$. The first two terms on the right-hand side of eq. (11) are proportional to the kinetic energy of the plate and the last term measures the control effort that is consumed over [0, t_f]. The optimal control of the plate implies that the optimal control function $C^\circ(t)$ should be the solution of the minimization problem satisfies:

$$\mathcal{J}\left[C^\circ(t)\right]=\min_{C(t)\in\mathfrak{C}_{a\partial}}\mathcal{J}\left[C(t)\right] \tag{12}$$

where eq. (12) is subjects to the state equation eq. (6) as well as the boundary and initial conditions (7)-(10).

Adjoint operator and maximum principle

Let us define the adjoint variable, v, and this adjoint variable $v(x, y, t)$ satisfies:

$$v_{tt}+\nabla^4 v=0,\ \ 0\leq x,y\leq 1,\ \ 0\leq t\leq t_f \tag{13}$$

subject to the boundary conditions:

$$v(0,y,t)=v(1,y,t)=v_{xx}(0,y,t)=v_{xx}(1,x,t)=0 \tag{14a}$$

$$v(x,0,t)=v(x,1,t)=v_{yy}(x,0,t)=v_{yy}(x,1,t)=0 \tag{14b}$$

and the terminal conditions:

$$v_t(x,y,t_f)=-2\mu_1 w(x,y,t_f),\ \ v(x,y,t_f)=2\mu_2 w_t(x,y,t_f) \tag{15}$$

The maximum principle is stated as Theorem (Maximum principle): for the optimal control function $C^\circ(t)\in\mathfrak{C}_{a\partial}$, the corresponding optimal state function $w^\circ(x, y, t) = w(x, y, t, C^\circ)$ satisfy eqs. (6)-(10) and the adjoint variable $v^\circ(x, y, t) = v(x, y, t; C^\circ)$ satisfy eq. (13), boundary conditions eq. (14) and terminal conditions eq. (15). The maximum principle can be stated:

$$\mathcal{H}[t;C^\circ]=\max_{C\in\mathfrak{C}_{a\partial}}\mathcal{H}[t;C] \tag{16}$$

where Hamiltonian is given:

$$\mathcal{H}(t;C)=-C(t)G(t)-\mu_3 C^2(t) \tag{17}$$

and

$$G(t) = \int_0^1\int_0^1 D(x,y)v(x,y,t)\mathrm{d}x\mathrm{d}y$$

then

$$\mathcal{J}[C^\circ] \le \mathcal{J}[C], \ \forall C(t) \in \mathfrak{C}_{a\partial} \tag{18}$$

Proof. Before starting the proof, let us introduce the following operator and its adjoint operator, respectively:

$$\Psi(w) = w_{tt} + \nabla^4 w \tag{19}$$

$$\Psi^*(v) = v_{tt} + \nabla^4 v \tag{20}$$

Also, let us define the deviations in w and w_t:

$$\Delta w = w - w^\circ \ \text{ and } \ \Delta w_t = w_t - w_t^\circ$$

The operator $\Psi(\Delta w) = \Delta C(t)D(x, y)$ is subject to the boundary conditions:

$$\Delta w(0,y,t) = \Delta w(1,y,t) = \Delta w_{xx}(0,y,t) = \Delta w_{xx}(1,y,t) = 0 \tag{21a}$$

$$\Delta w(x,0,t) = \Delta w(x,1,t) = \Delta w_{yy}(x,0,t) = \Delta w_{yy}(x,1,t) = 0 \tag{21b}$$

and the initial conditions:

$$\Delta w(x,y,0) = 0, \quad \Delta w_t(x,y,0) = 0 \tag{22}$$

Consider the following functional:

$$\iiint_Q [v\Psi(\Delta w) - \Delta w \Psi^*(v)]\mathrm{d}Q = \iiint_Q v\Delta C(t)D(x,y)\mathrm{d}Q \tag{23}$$

The left side of eq. (23) can be written:

$$I_1 + I_2$$

where

$$I_1 = \iiint_Q \left(v\Delta w_{tt} - v_{tt}\Delta w\right)\mathrm{d}Q, \ I_2 = \iiint_Q \left[v\left(\nabla^4 \Delta w\right) - \Delta w\left(\nabla^4 v\right)\right]\mathrm{d}Q$$

Using the fact that:

$$I_1 = \iiint_Q \frac{\partial}{\partial t}\left(v\Delta w_t - v_t \Delta w\right)\mathrm{d}t\mathrm{d}x\mathrm{d}y$$

and making use of eq. (22) and terminal conditions eq. (15), I_1 becomes:

$$\begin{aligned} I_1 &= \iint_S \left[v(x,y,t_f)\Delta w_t(x,y,t_f) - v_t(x,y,t_f)\Delta w(x,y,t_f)\right]\mathrm{d}x\mathrm{d}y = \\ &= \iint_S \left[2\mu_2 w_t(x,y,t_f)\Delta w_t(x,y,t_f) 2\mu_1 w(x,y,t_f)\Delta w(x,y,t_f)\right]\mathrm{d}x\mathrm{d}y \end{aligned} \tag{24}$$

Using the boundary conditions eq. (21) and eq. (14), it is easy to verify by integration by parts:

$$I_2 = \iiint_Q [v(\nabla^4 \Delta w) - \Delta w(\nabla^4 v)]\mathrm{d}Q = 0 \tag{25}$$

Now, let us deal with the right side of eq. (23):

$$I_3 = \iiint_Q [v(x,y,t_f)\Delta C(t) D(x,y)] \mathrm{d}Q \tag{26}$$

In view of eqs. (24) and (25), eq. (23) becomes:

$$\iint_S \{2\mu_2 w(x,y,t_f)\Delta w_t(x,y,t_f) + 2\mu_1 w(x,y,t_f)\Delta w(x,y,t_f)\} \mathrm{d}x\mathrm{d}y = \int_0^{t_f} [\Delta C(t) G(t) \mathrm{d}t \tag{27}$$

Now, consider the difference of the performance index defined:

$$\mathcal{J}[C] = \mathcal{J}[C] - \mathcal{J}[C^\circ] = \iint_S \left\{ \mu_1 \left[w^2(x,y,t_f) - w^{\circ^2}(x,y,t_f) \right] + \mu_2 \left[w_t^2(x,y,t_f) - w_t^{\circ^2}(x,y,t_f) \right] \right\} \cdot \\ \cdot \mathrm{d}x\mathrm{d}y + \mu_3 \int_0^{t_f} \left[C^2 - C^{\circ^2} \right] \mathrm{d}t \tag{28}$$

Expanding $w^2(x, y, t_\mathrm{f})$ and $w_t^2(x, y, t_f)$ in Taylor series about $w^{\circ 2}(x, y, t_\mathrm{f})$ and$w_t^{\circ 2}(x, y, t_\mathrm{f})$, respectively, leads to:

$$w^2(x,y,t_f) - w^{\circ^2}(x,y,t_f) = 2w^\circ(x,y,t_f)\Delta w^\circ(x,y,t_f) + \gamma_1 \tag{29a}$$

$$w_t^2(x,y,t_f) - w_t^{\circ^2}(x,y,t_f) = 2w_t^\circ(x,y,t_f)\Delta w_t^\circ(x,y,t_f) + \gamma_2 \tag{29b}$$

where

$$\gamma_1 = 2(\Delta w)^2 + \text{higher order terms} > 0, \;\; \gamma_2 = 2(\Delta w_t)^2 + \text{higher order terms} > 0$$

Substituting eq. (29) into eq. (28) yields:

$$\Delta\mathcal{J}[C] = \iint_S \{\mu_1[2w^\circ(x,y,t_f)\Delta w(x,y,t_f) + \gamma_1] + \\ + \mu_2[2w_t^\circ(x,y,t_f)\Delta w_t(x,y,t_f) + \gamma_2]\} \mathrm{d}x\mathrm{d}y + \mu_3 \int_0^{t_f} [C^2 - C^{\circ^2}] \mathrm{d}t$$

From eq. (27) and because of $\mu_1\gamma_1 + \mu_2\gamma_2 > 0$, one observes:

$$\Delta\mathcal{J}[C] \geq \int_0^{t_f} \left[\Delta C(t) G(t) \right] \mathrm{d}t + \int_0^{t_f} \mu_3 \left[C^2(t) - C^{\circ^2}(t) \right] \mathrm{d}t \geq 0$$

which leads to:

$$C(t)G(t)\mu_3 C(t)^2 \geq C^\circ(t)G(t)\mu_3 C^{\circ^2}(t)$$

that is

$$\mathcal{H}[t;C^\circ] \geq \mathcal{H}[t;C]$$

Hence, we obtain:

$$\mathcal{J}[C] \geq \mathcal{J}[C^\circ], \;\; \forall \;\; C(t) \in \mathfrak{C}_{ad}$$

Taking the first variation of $\mathcal{H}$ in eq. (17), $C(t)$ gives the optimal control function:

$$C^\circ(t) = \frac{-G(t)}{2\mu_3} \tag{30}$$

To compute the control voltage C° in eq. (33), we need to evaluate $v(x, y, t)$ in eq. (13) that requires the solution of the optimal state function w of eqs. (6)-(10) subject to the mixed terminal conditions eq. (15). The details of this computation are given in the next section.

Numerical results and conclusion

In this section, in order to indicate the effectiveness and capability of the obtained theoretical results in the previous sections, following system of partial differential equations is solved by means of mathematical software. $0 \le x, y, \le 1$, $0 \le t \le t_f$:

$$w_{tt} + \nabla^4 w = P(x,y,t) + C(t)D(x,y),\ C^\circ(t) = \frac{-G(t)}{2\mu_3},\ G(t) = \int_0^1\int_0^1 D(x,y)v(x,y,t)\mathrm{d}x\mathrm{d}y \tag{31}$$

$$v_{tt} + \nabla^4 v = 0 \tag{32}$$

$$w(0,y,t) = w(1,y,t) = w(x,0,t) = w(x,1,t) = 0 \tag{33}$$

$$w_{xx}(0,y,t) = w_{xx}(1,y,t) = w_{yy}(x,0,t) = w_{yy}(x,1,t) = 0 \tag{34}$$

$$w(x,y,0) = w_0(x,y),\ w_t(x,y,0) = w_1(x,y) \tag{35}$$

$$v(0,y,t) = v(1,y,t) = v_{xx}(0,y,t) = v_{xx}(1,x,t) = v(x,0,t) = v(x,1,t) = v_{yy}(x,0,t) = v_{yy}(x,1,t) = 0 \tag{36}$$

$$v_t(x,y,t_f) = -2\mu_1 w(x,y,t_f),\ v(x,y,t_f) = 2\mu_2 w_t(x,y,t_f) \tag{37}$$

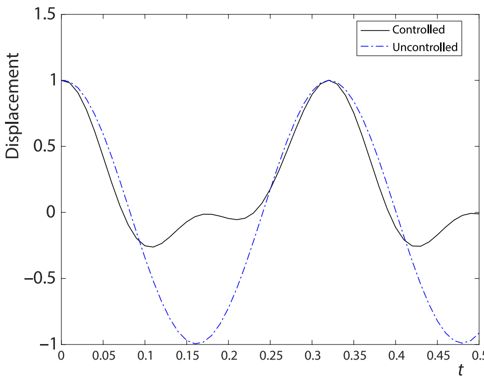

Figure 1. Un/controlled displacements at (0.5, 0.5) for *Example 1*

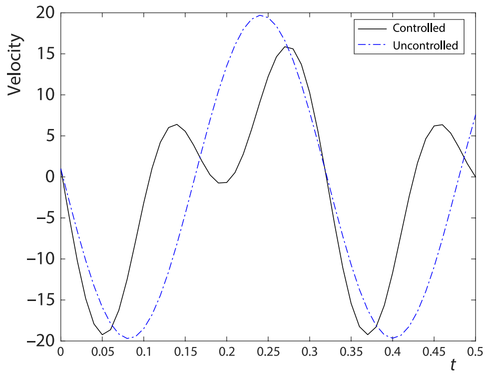

Figure 2. Un/controlled velocities at (0.5, 0.5) for *Example 1*

Before discussing the numerical results, take into account the optimal control function given by eq. (30) and focu the position of the μ_3. The value of the μ_3 is decreasing, the value of the $C(t)$ is increasing, vice versa. The displacement and velocity are computed at the mid-point (x = 0.5, y = 0.5) of the plate. Terminal time is taken into account as t_f = 0.5. The distribution function for the control function $D(x, y)$ = 1. The control function is obtained by computing $\mu_3 = 10^6$ and $\mu_3 = 10^{-6}$ for uncontrolled case and controlled case, respectively. Figures 1-4 show the curves of un/controlled displacements and velocities of the plate plotted against time $0 \le t \le t_f$ with the weighted coefficients taken as μ_1= 1, μ_2 = 1. In the *Example 1*, the magneto-electric load function is taken as $P(x, y, t) = \mathrm{e}^{2t}$ and the initial conditions are specified:

$$w_0 = \sqrt{2}\sin(\pi x)\sqrt{2}\sin(\pi y), w_1 = \sqrt{2}\sin(\pi x)\sqrt{2}\sin(\pi y)$$

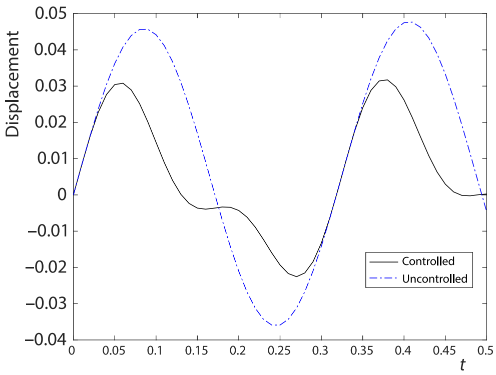

Figure 3. Un/controlled displacements at (0.5, 0.5) for *Example 2*

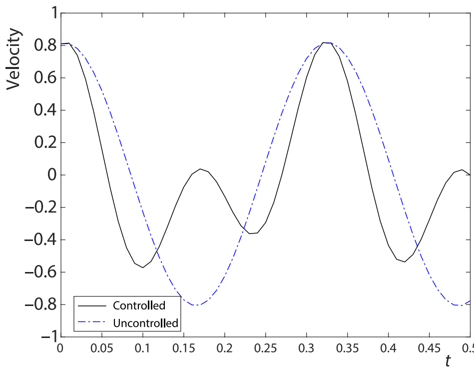

Figure 4. Un/controlled velocities at (0.5, 0.5) for *Example 2*

The un/controlled displacement and velocity are plotted in figs. 1 and 2, respectively. In the *Example 2*, the magneto-electric load function is taken as $P(x, y, t) = e^{2t}$ and the initial conditions are specified as $w_0 = 0$, $w_1 = x + y$. The un/controlled displacement and velocity are plotted in figs. 3 and 4, respectively.

Conclusion

In this paper, magneto-electrically induced vibration control of a MEE plate in contact with fluid is considered and optimal control function is obtained by means of maximum principle, which transforms the control problem to solving a system of PDE systems linked by initial-boundary-terminal conditions. The system is solved by mathematical software and results are presented for two numerical examples. After observing the numerical resultsi, it is concluded that the deflection and the velocity of the vibrating plate, contacted with fluid and subjected to magneto-electric load, can be effectively damped out.

References

[1] Yildirim, K., Vibration Suppression of a Micro Beam Subjected to Magneto-Electric Load, Sigma J. Eng. Nat. Sci., *39* (2021), 4, pp. 404-413
[2] Vaezi, M., *et al.*, Free Vibration Analysis of Magneto-Electro-Elastic Microbeams Subjected to Magneto-Electric Loads, *Physics E*, *75* (2016), Jan., pp. 208-286
[3] Chen, J., *et al.*, Modal Analysis of Magneto-Electro-Elastic Plates Using the State-Vector Approach, *Journal of Sound and Vibration*, *304* (2007), 3-5, pp. 722-734
[4] Zhou, L., *et al.*, The Multi-Physic Cell-Based Smoothedfinite Element Method for Dynamiccharacterization of Magneto-Electro-Elastic Structures under Thermal Conditions, *Composite Structures*, *240* (2020), 112045
[5] Zhang, X. L., *et al.*, Non-Linear Analyses of Magneto-Electro-Elastic Laminated Beams in Thermal Environments, *Composite Structures*, *234* (2020), 111524
[6] Liu, H. T., *et al.*, Investigation of Non-Local Theory Solution a Three-Dimensional Rectangular Permeable Crack in Magneto-Electro-Elastic-Materials, *Int J. Mech. Sci.*, *134* (2017), Oct., pp. 460-478
[7] Vinyas, M., Finite Element Evaluation of Free Vibration Characteristics of Magneto-Electro-Elastic Rectangular Plates in Hygrothermal Environment Using Higher-Order Shear Deformation Theory, *Compos. Struct.*, *202* (2018), June, pp. 1339-1352
[8] Shojaeefard, M. H., *et al.*, Free Vibration of an Ultra-Fast-Rotating-Induced Cylindrical Nano-Shell Resting on a Winkler Foundation under Thermo-Electro-Magneto-Elastic Condition, *Appl. Math. Model*, *61* (2018), Apr., pp. 255-279
[9] [Pan, E., Exact Solution for Simply Supported and Multilayered Magneto-Electro-Elastic Plates, *Journal of Applied Mechanics*, *Trans. ASME*, *68* (2001), 4, pp. 608-618

[10] Pan, E., *et al.*, Free Vibrations of Simply Supported and Multilayered Magneto-Electro-Elastic Plates, *Journal Sound Vib*., *252* (2002), 3, pp. 429-442
[11] Annigeri, A. R., *et al*., Free Vibration Behaviour of Multi-Phase and Layered Magneto-Electro-Elastic Beam, *Journal Sound Vib*., *299* (2007), 1-2, pp. 44-63
[12] Lage, R. G., *et al.*, Layerwise Partial Mixed Finite Element Analysis of Magneto-Electro-Elastic Plates, *Compos. Struct*., *82* (2004), 17-19, pp. 1293-1301
[13] Chang, T. P., On the Natural Frequency of Transversely Isotropic Magneto-Electro-Elastic Plates in Contact with Fluid, *App. Math. Mod*., *37* (2013), 4, pp. 2503-2515

Paper submitted: June 11, 2021
Paper revised: October 30, 2021
Paper accepted: May 6, 2022