A COMPRESSIBLE TURBULENCE MODEL FOR THE DISSIPATION RATE

by

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In this work, the ability of a Reynolds stress model to compute turbulent homogeneous shear flow with significant compressibility effects is discussed. Several studies of compressible turbulent flows carried out in the past years have shown that the pressure strain correlation is mainly responsible for the strong changes in the magnitude of the Reynolds stress anisotropies. Two recent compressible models of this term are considered in conjunction with the standard model of the dissipation rate of the turbulent kinetic energy to predict compressible homogeneous flow highly sheared are tested. It is found that deficiencies appear in the calculations even if the pressure strain model is improved by compressibility corrections. Consistent with earlier studies, this deficiency is attributed to the use of the incompressible model for turbulent dissipation. However, a compressibility correction of this equation model uncovers the main focus of the present study. This correction makes the standard coefficients of this equation depend on the turbulent gradient Mach and Mach numbers. The proposed model is tested for low and strong compressibility cases from the DNS results of Sarkar. A comparison of the proposed model predictions with the DNS results shows good qualitative agreement. Therefore, compressibility correction for the incomp- ressible model of the turbulent dissipation rate is found to be an important issue for the compressible homogeneous turbulent shear flow.

Key words: *turbulence, homogeneous, model, dissipation, pressure strain, compressibility, shear flow*

Introduction

Problems arise in several industrial applications, essentially from the hypersonic flight, supersonic combustion, and often environment involve compressible turbulence flows. Since 90's the compressibility phenomenon becomes of particular attention for several researchers. In this context, many theoretical and experimental studies have been developed in order to understand the significant compressibility effects on the behaviour of turbulence flows. Despite those the structural compresssibility effects are a complex issue for compressible turbulent flows, the past years are marked by an increase of modelling works that were focused, particularly on the Reynolds stress model. Based on the extension of the main contributions was started by Launder *et al*. [1], Speziale *et al*. [2], and Fu *et al*. [3], this practice has enjoyed

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an acceptable success in the calculations of the lower compressibility effects for different compressible turbulence configurations. Unfortunately, the Reynolds stress model is often unable to predict accurately the dramatic change in the turbulence flows at high compressibility. The deficiency of this model is widely analysed in several studies carried out in the few decades, most of which are centered on the role of the pressure velocity correlation. They identified the correlation pressure-strain correlation as a main term in the Reynolds stress transport equation which controls the compressibility effects on the turbulence anisotropy. Thus, it is suggested that this term must be modelled in order to account compressibility effects for more accuracy of the turbulent flow behaviours. Initially, the evaluation of this approach modelling is made in compressible homogeneous shear flow characterized by the mean velocity (Sx_2, 0, 0) where S is the constant mean shear. The work of Speziale *et al*. [4, 5] is one of the important contributions in this field. Based on the DNS results of Blaisdell *et al.* [6], they show that compressibility effects are not attributed to the non-zero divergent fluctuating velocity field, it can not be reflected by the dilatation correlations as the pressure-dilatation $p'd'$ and the compressible turbulent dissipation rate ε_c, terms appearing in the turbulent kinetic energy transport equation. The same results are confirmed by the DNS of Sarkar [7] and Simone *et al.* [8] which indicate that such terms have a negligible contribution the change of compressible flows. These terms are not able to reflect structural compressibility effects which are to be addressed to the change of the structure of the velocity field as it is reported in Fujihiro [9] and in [6, 7]. This is the main reason for which some compressibility corrections for the standard models [1-3], are derived by Fujihiro [9], Adumitroiae *et al*. [10], Hung *et al*. [11], Park *et al*. [12], Vreman *et al*. [13], and others [14-19]. In general, these models have been used in conjunction with the incompressible model of the turbulent dissipation which computes the solenoidal dissipation rate. Based on the Morkovin hypothesis, this approach of modelling considers the solenoidal dissipation rate which follows the same dynamic of the incompressible dissipation [4]. Probably, this is true when the ε_S is rather insensitive to the compressibility effects in low turbulent Mach number and this term can be described by the incompressible model equation. But, for the turbulent flows at a high Mach number, a great need for compressible turbulence modelling that is addressed to account for compressibility parameters for the incompressible turbulent dissipation rate model.

The gradient Mach number, M_g, introduced in [7, 8] has become essential as the parameter that determines structural compressibility effects, this number is defined by $M_g = S\ell/\overline{a}$, where $\overline{a}$ is the mean speed of sound and ℓ is the length scale of energetic turbulence motions. According to the DNS [7, 8], M_g shows a trend to become asymptotically constant after an initial slight increase with $S\ell$. In contrary, the turbulent Mach number, M_t:

$$M_t = \frac{(2K)^{1/2}}{\overline{a}} \text{where } K = \frac{0.5\overline{\rho u_i'' u_i''}}{\overline{\rho}}$$

is the turbulent kinetic energy, grows canstantly with St. From the DNS results [7, 8], it is clear that there is a similar trend between the structures of M_g and the turbulence. As a consequence, M_g seems to be an appropriate parameter to study compressibility effects on homogenous shear flow.

In this work, a compressibility modification for the classical equation model of the turbulent dissipation rate is proposed. This modification makes the standard coefficients of this equation a function of the compressibility parameters, M_g and M_t. The proposed model has been tested for low and strong compressibility cases from the DNS results of Sarkar [7] for compressible homogeneous shear flow.

Basic equations

The turbulence models used in this work are closely related to the standard Reynolds stress model from which The Favre averaged Reynolds stress:

$$R_{ij} = \frac{\overline{\rho u_i'' u_j''}}{\rho}$$

is described by:

$$\frac{\partial}{\partial t}(\bar{\rho} R_{ij}) + \frac{\partial}{\partial x_m}(\bar{\rho}\tilde{U}_m R_{ij}) = P_{ij} + D_{ij} + \phi_{ij} + \varepsilon_{ij} + V_{ij} \tag{1}$$

where the symbols P_{ij}, D_{ij}, ϕ_{ij}, ε_{ij}, and V_{ij} are the turbulent production, turbulent diffusion, pressure strain correlation, turbulent dissipation, and the mass flux variation, respectively:

$$P_{ij} = -\bar{\rho} R_{jm}\tilde{U}_{i,m} - \bar{\rho} R_{im}\tilde{U}_{j,m} D_{ijm} = -(\overline{\rho u_i'' u_j'' u_m''} + \overline{p' u_j''}\delta_{im} + \overline{p' u_i''}\delta_{jm} - \overline{\tau_{im}'' u_j''} - \overline{\tau_{jm}'' u_i''})$$

$$\phi_{ij} = \overline{p'(u_{i,j}'' + u_{j,i}'')} = \phi_{ij}^* + \frac{2}{3}\overline{p' u_{k,k}''}\delta_{ij}, \ \varepsilon_{ij} = \overline{\tau_{im}'' u_{j,m}''} - \overline{\tau_{jm}'' u_{i,m}''}$$

$$V_{ij} = -\bar{p}_{,j}\overline{u_i''} + -\bar{p}_{,i}\overline{u_j''} + \tilde{\tau}_{im,m}\overline{u_j''} + \tilde{\tau}_{jm,m}\overline{u_i''}$$

Classically, the second-order closure suggests the isotropic model for the dissipation term, namely:

$$\varepsilon_{ij} = \frac{2}{3}\varepsilon\delta_{ij} \tag{2}$$

Modelling the dissipation rate forcompressible turbulent flow

The turbulent viscous dissipation rate plays an important role in the exchange of energy, it assures the conversion of kinetic energy into thermal energy. In consequence, it appears to be an important physical property that needed to be correctly modelled for compressible turbulence flows. In this context, a more used compressible turbulence model for dissipation can be found in Zeman [20] and Sarkar *et al.* [21] in which the concept of dilatational dissipation was proposed:

$$\varepsilon = \varepsilon_s + \varepsilon_c \tag{3}$$

For homogeneous shear flow turbulence:

$$\bar{\rho}\varepsilon_s = \overline{\mu\omega_i'\omega_i'}$$

is the fluctuating vorticity, and

$$\varepsilon_c = \frac{4\overline{\mu u_{k,k}'^2}}{3}$$

The authors argued that the solenoidal part of the dissipation can be modelled by the traditional incompressible equation model, namely:

$$\frac{\partial}{\partial t}(\bar{\rho}\varepsilon_s) + \frac{\partial}{\partial x_k}(\bar{\rho}\varepsilon_s\tilde{U}_k) = \bar{\rho}\frac{\varepsilon_s}{K}\left(C_{\varepsilon 1} R_{km}\frac{\partial}{\partial x_m}\tilde{U}_k - C_{\varepsilon 2}\varepsilon_s\right) - \frac{\partial}{\partial x_k}\left(C_{\varepsilon 3}\bar{\rho}\frac{K}{\varepsilon_s} R_{km}\frac{\partial}{\partial x_m}\varepsilon_s\right) \tag{4}$$

The usual values of the numerical coefficients model as they are: $C_{\varepsilon 1}$= 1.44 and $C_{\varepsilon 1}$ = 1.92.

The compressible dissipation ε_c is determined by the commonalty used models [20, 21] as:

$$\varepsilon_c = h(M_t)\varepsilon_s \tag{5}$$

where h is the function of the turbulent Mach number. It is well known that the majority of existent compressible models as in [9-19] which are used in addition the incompressible ε-equation have shown an acceptable success in simulating a variety of compressible turbulent flows. On the other hand, this approach of modelling has not predicted correctly the structural compressibility effects at high speed shear flow. Thus, one can think that there is an incompatibility for high speed flows when the ε-equation model is employed for compressible turbulence closure without any modifications, the model constants are as for incompressible flows. Therefore, for flows of higher turbulent Mach number, a refined model taking into account compressibility effects for the dissipation is needed for accurate compressible turbulent flows predictions. On the other hand, the DNS results [7] show that compressibility significantly affects the turbulent production which plays a central role in the ε-equation model. Thus, a compressibility correction that affects this term is the major challenge in this study. Marking the beginning of this work is an equation for the fluctuation of dilatation, $d' = u'_{k,k}$ that can be obtained by subtracting the equation of the mean dilation $D = \tilde{U}_{k,k}$ from the instantaneous equation of the dilatation, $d = u_{k,k}$:

$$\frac{\mathrm{d}}{\mathrm{d}t}d' = -2\frac{\partial u_i'}{\partial x_j}\frac{\partial \tilde{U}_j}{\partial x_i} + \sum \tag{6}$$

where $\sum$ summarizes different terms containing the first and the second-derivative of the fluctuating and the mean of density and pressure. Multiplying both sides, and taking ensemble averaging, we have:

$$\frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}\overline{d'^2} = -2\overline{d'\frac{\partial u_i'}{\partial x_j}}\frac{\partial \tilde{U}_j}{\partial x_i} + \overline{d'\sum} \tag{7}$$

Thus, an equation for the compressible dissipation ε_c can be easily obtained from eq. (7):

$$\frac{\mathrm{d}}{\mathrm{d}t}\varepsilon_c = \left(\frac{1}{\overline{\nu}}\frac{\mathrm{d}}{\mathrm{d}t}\overline{\nu}\right)\varepsilon_c - \frac{16}{3}\overline{\nu}\left(\overline{d'\frac{\partial u_i'}{\partial x_j}}\right)\frac{\partial \tilde{U}_j}{\partial x_i} + \frac{8}{3}\overline{\nu}\,\overline{d'\sum} \tag{8}$$

On the other hand, the time derivative term (ε_c, t) can be split into two parts:

$$\frac{\mathrm{d}}{\mathrm{d}t}\varepsilon_c = \left(\frac{\mathrm{d}}{\mathrm{d}t}\varepsilon_c\right)_1 + \left(\frac{\mathrm{d}}{\mathrm{d}t}\varepsilon_c\right)_2 \tag{9}$$

where $(\varepsilon_c, t)_1$ is the linear of the mean strain and $(\varepsilon_c, t)_2$ is the remainder terms which seem to be not important in the shear flows and is not our motivation in the present study. Thus, seeing the right-hand side of eq. (8), one can know that $\overline{\nu} = \overline{\mu}/\overline{\rho}$ ideal gas, $\overline{\mu}$ is independent of the density and ($\overline{\mu}\,\overline{T}^n$, $n = 2/3$). Accordingly, the continuity equation: $(1/\overline{\rho})\overline{\rho}$, $t = -\tilde{U}_{m,n}$, the time derivative term $(1/\overline{\nu})(\overline{\nu}, t)$ involves the volume strain on the mean flows which would be important in the shock. The last term is expected to be small in homogeneous turbulent shear flow for which the effects of the mean shear come essentially from the mean velocity gradient. So, only the second term on the RSH of eq. (8) is retained and we can write:

$$\frac{\mathrm{d}}{\mathrm{d}t}\varepsilon_c = \left(\frac{\mathrm{d}}{\mathrm{d}t}\varepsilon_c\right)_1 = -\frac{16}{3}\overline{\nu}\left(\overline{d'\frac{\partial u_i'}{\partial x_j}}\right)\frac{\partial \tilde{U}_j}{\partial x_i} \tag{10}$$

As can be seen in eq. (10), a dilatation-strain correlation $\overline{d'u'_{i,j}}$ modelled for the compressible dissipation rate ε_c. To estimate this correlation, the continuity equation $\rho,\, t = \rho u_{i,i}$ can be given for the fluctuating quantities in the approximation:

$$\frac{\rho'}{\tau_d} = -\overline{\rho}d' \tag{11}$$

where τ_d is the characteristic time scale of the density-dilatation fluctuation [22, 17]:

$$\tau_d \approx M_t^2 \frac{K}{\varepsilon} \tag{12}$$

and it is also assumed that the flow is isentropic:

$$\frac{p'}{\overline{p}} = \gamma \frac{\rho'}{\overline{\rho}} \tag{13}$$

From the aforementioned eqs. (11)-(13), the correlation $\overline{d'u'_{i,j}}$ can be directly related to $\overline{p'u'_{i,j}}$ correlation for which different models have been proposed. However, this is an exaggerated approximation because these correlations do not reflect the same phenomenon, so they cannot be governed by the same frequency range. In fact, the dilatation and the strain rate fluctuations are related to small dissipative eddies. While the pressure fluctuation is related to large eddies. In consequence, one can know that there is an interaction between dilatation and strain fluctuations. On the contrary, this trend is not observed for the pressure fluctuations. At a high turbulent Reynolds number $\mathrm{Re}_t = K^2/\nu\varepsilon$, the dilatation variance fluctuation is related to turbulent Mach number and Reynolds number as in [23]:

$$\overline{d'^2} M_t^2 \frac{\varepsilon}{K}\sqrt{\mathrm{Re}_t} \tag{14}$$

According to Fujiwara *et al.* [24], the strain rate fluctuation and the pressure variance are expressed, respectively:

$$\overline{u'^2_{i,j}} \frac{\varepsilon}{K}\sqrt{\mathrm{Re}_t} \tag{15}$$

$$\sqrt{\overline{p'^2}}\,\overline{\rho}K \tag{16}$$

From these equations, we can obtain:

$$\overline{d'\frac{\partial u'_j}{\partial x_i}} = \frac{H(\mathrm{Re}_t)}{\gamma\overline{p}\tau_d} M_t^2 \left(\left(\overline{p'\frac{\partial u'_j}{\partial x_i}} \right)^* + \frac{1}{3}\overline{p'd'}\delta_{ij} \right) \tag{17}$$

where

$$\left(\overline{p'u'_{i,j}}\right)^* = \overline{p'u'_{j,i}} - \frac{\overline{p'd'}\delta_{ij}}{3}$$

is the deviator part of the pressure strain correlation and $H(\mathrm{Re}_t)$ is a function of the turbulent Reynolds number. According to the analysis of Tennekes and Lumley [25], the coefficient correlation between the pressure and the strain rate fluctuations is approximated by $1/\mathrm{Re}_t^{1/2}$ for high turbulent Reynolds number. Following that, the function $H(\mathrm{Re}_t)$ should assure a strong interaction between dilatation and strain rate fluctuations. This gives a reason scale $H(\mathrm{Re}_t)$ with Re_t as:

$$H(\mathrm{Re}_t) \propto \mathrm{Re}_t \tag{18}$$

Taking the time scale in eq. (12), so, eq. (10) can be written:

$$\frac{\mathrm{d}}{\mathrm{d}t}\varepsilon_c = -M_t^2 \left(\overline{p' \frac{\partial u_j'}{\partial x_i}}\right)^* \frac{\partial \tilde{U}_j}{\partial x_i} \tag{19}$$

According to Adumitroiae *et al.* [10] and Hechmi *et al.* [16], the pressure strain can be split into two parts as shown:

$$\left(\overline{p' \frac{\partial u_j'}{\partial x_i}}\right)^* = \left(\overline{p' \frac{\partial u_j'}{\partial x_i}}\right)^{*\text{inc}} + \left(\overline{p' \frac{\partial u_j'}{\partial x_i}}\right)^{*\text{comp}} \tag{20}$$

The incompressible part:

$$\left(\overline{p' u_{j,i}'}\right)^{*\text{inc}}$$

is closed using standard models as has been already mentioned in the literature, there are some models for the compressible part of the pressure strain:

$$\left(\overline{p' u_{j,i}'}\right)^{*\text{comp}}$$

Pantano *et al.* [26] pointed out that for compressible homogeneous turbulence highly sheared, compressibility effects are closely linked with the turbulent Mach number and the gradient Mach number. They use two-time scales to propose a compressible model for the pressure strain by introducing a dumping function as shown:

$$\left(\overline{p' \frac{\partial u_j'}{\partial x_i}}\right)^{*\text{comp}} = f(M_t, M_g) \left(\overline{p' \frac{\partial u_j'}{\partial x_i}}\right)^{*\text{inc}} \tag{21}$$

where $f(M_t, M_g)$ is a function model of turbulent Mach number and gradient Mach number.

The isotropization of the pressure strain tensor is considered, only the slow part of the pressure strain which describes the return to isotropy process of turbulence is considered here. In general, we believe to use the model of Rotta [1] torepresent this process which is observed when the mean strain is removed by:

$$\left(\overline{p' \frac{\partial u_j'}{\partial x_i}}\right)^{*\text{inc}} = C_1 \bar{\rho} \varepsilon b ij \tag{22}$$

and eq. (21) can be written:

$$\left(\overline{p' \frac{\partial u_j'}{\partial x_i}}\right)^{*\text{comp}} = f(M_t, M_g) C_1 \bar{\rho} \varepsilon b_{ij} \tag{23}$$

From previously mentioned, eq. (19) can be written:

$$\frac{\mathrm{d}}{\mathrm{d}t}\varepsilon_c = -\bar{\rho} M_t^2 f(M_t, M_g) C_1 \varepsilon b_{ij} \frac{\partial \tilde{U}_j}{\partial x_i} \tag{24}$$

According to eq. (3) and adding the eqs. (4) and (24), the proposed compressible equation model for the total dissipation rate can be written:

$$\frac{\mathrm{d}}{\mathrm{d}t}\varepsilon = C_{\varepsilon 1}\left[1 - C_{d\varepsilon 1} M_t^2 f\left(Mt, Mg\right)\right]\frac{\varepsilon}{K} P - C_{\varepsilon 2}\frac{\varepsilon^2}{K} + D_\varepsilon \tag{25}$$

where $C_{d\varepsilon 1}$ is a constant model and $D\varepsilon$ – the diffusion term.

As can be seen, setting the compressibility function: $f(M_t, M_g)$ to zero will rejoint the incompressible model as it is described by eq. (4) for which $C_{\varepsilon 1} = 1.44$ and $C_{\varepsilon 3} = 1.9$.

It is clearly seen that the proposed correction for the standard dissipation model affects the production term that involves a compressibility function $f(M_t, M_g)$, this discrepancy is reached by several direct numerical simulation results as those conducted in homogeneous shear flow, see [7-9]. On the other hand, according to Sarkar [7], there is a similarity between the homogeneous shear flow and the mixing shear layers and the gradient Mach number, M_g, can be proportional to the convective Mach number, M_c, so we can deduce for the function $f(M_t, M_g)$ the expression, see [26]:

$$f(M_t, M_g) = M_t^2 \left(\frac{b - e^{\left[-(\alpha M_g - \beta)^2\right]}}{1 + bM_t^2} \right) \tag{26}$$

where b, α, and β are the numerical coefficients.

Simulation of compressible homogeneous shear flow

Compressible homogeneous shear flow is chosen in this tudy to evaluate the ability of the proposed model for the turbulent dissipation rate in the prediction of structural compressibilityeffects on the turbulence. For compressible homogeneous shear flow, the gradient of the mean velocity is given:

$$\tilde{U}_{i,j} = S\delta_{i1}\delta_{j2} \tag{27}$$

For homogeneous shear flow, $\bar{\rho} = cte$ and $\tilde{T} = \tilde{T}(t)$ is related to Reynolds-averaged of the pressure by the state equation for ideal gas:

$$\bar{P} = \bar{\rho}\mathrm{R}\tilde{T} \tag{28}$$

The Favre averaged basic second-order model equations are described:

$$\bar{\rho}\frac{\mathrm{d}}{\mathrm{d}t}R_{ij} = P_{ij} + \phi_{ij}^* - \frac{2}{3}\varepsilon\delta_{ij} + \frac{2}{3}\overline{p'd'}\delta_{ij} \tag{29}$$

$$\bar{\rho}\frac{\mathrm{d}}{\mathrm{d}t}\varepsilon_s = \bar{\rho}C_{\varepsilon 1}\frac{\varepsilon_s}{K}P - \bar{\rho}C_{\varepsilon 2}\frac{\varepsilon_s^2}{K} \tag{30}$$

Assuming that the mean specific heat is constant, the equivalent temperature equation for the Reynolds averaged energy may be written in a simplified form Speziale *et al.* [5], namely:

$$\overline{\rho c_v}\frac{\mathrm{d}}{\mathrm{d}t}\tilde{T} = \bar{\rho}\varepsilon - \overline{p'd'} \tag{31}$$

The contraction, $i = j$ in eq. (29), leads to write an equation for the Favre-averaged turbulent kinetic energy

$$K = \frac{0.5\overline{\rho u_i'' u_i''}}{\bar{\rho}}$$

as follows:

$$\bar{\rho}\frac{\mathrm{d}}{\mathrm{d}t}K = P - \bar{\rho}\varepsilon + \overline{p'd'} \tag{32}$$

where

$$P=-\overline{\rho}R_{ij}\tilde{U}_{i,j}$$

is the turbulent production.

The transport equation for turbulent Mach number:

$$M_t=\sqrt{\frac{2K}{\gamma \mathrm{R}\tilde{T}}}$$

can be obtained from combination between eqs. (31) and (32):

$$\frac{\mathrm{d}}{\mathrm{d}t}M_t=\frac{M_t}{2K}P+\frac{M_t}{2\overline{\rho}K}\left[1+0.5\gamma(\gamma-1)M_t^2\right]\left(\overline{p'd'}-\overline{\rho}\varepsilon\right) \tag{33}$$

– The final form of the proposal model for the turbulent dissipation

According to eqs. (25) and (26), the proposed model for the dissipation rate can be read:

$$\overline{\rho}\frac{d}{dt}\varepsilon=-C_{\varepsilon 1}\left[1-M_t^2\left(\frac{b-e^{\left[-(\alpha M_g-\beta)^2\right]}}{1+bM_t^2}\right)\right]\overline{\rho}\frac{\varepsilon}{K}P-C_{\varepsilon 2}\overline{\rho}\frac{\varepsilon^2}{K} \tag{34}$$

The numerical coefficients b, α, and β are: $b=2$, $\alpha=0.9$, $\beta=0$, $C_{\varepsilon 1}=1.4$, and $C_{\varepsilon 2}=1.9$.

As it is aforementioned, our attention is focused on the pressure strain correlation and the turbulent dissipation rate. Thus, the models to be applied for the pressure strain correlation-are:

– Model of Pantano and Sarkar [26]

In this study, we have modified the LRR model by using Pantano and Sarkar [26] to derive a new model for the pressure strain correlation in which M_t and M_g are used to express compressibility effects:

$$\phi_{ij}^*=-C_1\overline{\rho}\varepsilon_s b_{ij}+C_2\overline{\rho}K\left(\tilde{S}_{ij}-\frac{1}{3}\tilde{S}_{ll}\delta_{ij}\right)+C_3\overline{\rho}K\left[b_{ik}\tilde{S}_{jk}+b_{jk}\tilde{S}_{ik}-\frac{2}{3}b_{ml}\tilde{S}_{ml}\delta_{ij}\right]+ \\ +C_4\overline{\rho}K[b_{ik}\tilde{\Omega}_{jk}+b_{jk}\tilde{\Omega}_{ik}] \tag{35}$$

where

$$C_1=3.0,\ C_2=0.8,\ C_3=1.75\left(1-1.4M_t^2-0.012M_g^2\right),\ C_4=1.31\left(1-0.8M_t^2-0.00M_g^2\right)$$

$$b_{ij}=\frac{\dfrac{R_{ij}-2}{3\delta_{ij}}}{2K},\ \tilde{S}=\frac{\tilde{U}_{i,j}+\tilde{U}_{j,i}}{2},\ \text{and}\ \tilde{\Omega}_{ij}=\frac{\tilde{U}_{i,j}+\tilde{U}_{j,i}}{2}$$

– Model of Adumitroiae *et al.* [10]

Adumitroiae *et al.* [10]. develop a compressible model for the pressure strain, their model is written:

$$\phi_{ij}^*=-C_1\overline{\rho}\varepsilon_s b_{ij}+\left(\frac{4}{5}+\frac{2}{5}d_1\right)\overline{\rho}K\left(\tilde{S}_{ij}-\frac{1}{3}\tilde{S}_{ll}\delta_{ij}\right)+2\overline{\rho}K\left(1-C_3+2d_2\right)\cdot \\ \cdot\left[b_{ik}\tilde{S}_{jk}+b_{jk}\tilde{S}_{ik}-\frac{2}{3}b_{ml}\tilde{S}_{ml}\delta_{ij}\right]-\overline{\rho}K\left(1-C_4-2d_2\right) \tag{36}$$

where

$$\tilde{S}_{ij} = 0.5(\tilde{U}_{i,j} + \tilde{U}_{j,i}),\ \tilde{\Omega}_{ij} = 0.5(\tilde{U}_{i,j} - \tilde{U}_{j,i}),\ b_{ij} = \frac{R_{ij}}{2K} - \frac{1}{3\delta_{ij}}1$$

The compressible coefficients d_1 and d_2 are determined from some compressible closures for the pressure-dilatation correlation, see [10]. In this study, the model [21] is used for the pressure-dilatation:

$$\overline{p'd'} = 0.15 M_t \overline{\rho}\left(R_{ij} - \frac{2}{3}K\delta_{ij}\right) + 0.2\overline{\rho} M_t^2 \varepsilon \tag{37}$$

Results and discussion

As described in this study, compressible homogeneous shear flow is mainly an important motivation for some authors. This is because this flow summarizes some of the critical compressibility effects features found in mixing and boundary-layers [7] are presented in a simplified setting without including the complexity of modelling walls (the complex effects of turbulent diffusion are not considered). The conputations for this flow, (the mean velocity gradient is given in matrix form by $\tilde{U}_{ij}$ = (0, S, 0), see fig. 1, will now be considered for large non-dimensional time, St = 0-20.

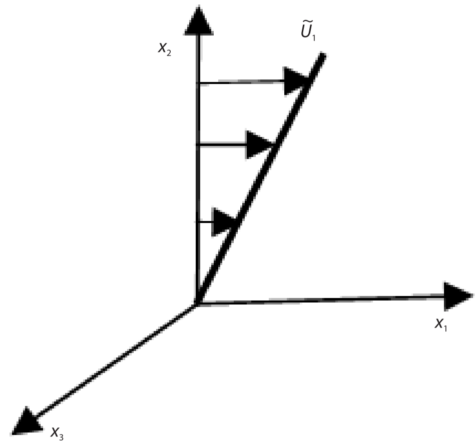

Figure 1. The mean velocity homogeneous shear flow

Thus, the transport eqs. (29)-(34) incorporating turbulence models, as discussed previously, were solved numerically by the fourth order accurate Runge-Kutta numerical integration scheme using a discretization mesh of N = 200 steeps in the time. More clearly, we consider in the canon-ic forms, the St-time transport equations for the components of the Reynolds stress anisotropies deduced from eq. (29): b_{11}, b_{22}, b_{12}, the turbulent M_t, eq. (33), the normalized turbulent kinetic energy K/K_0, eq. (32), and its turbulent dissipation $\varepsilon/\varepsilon_0$, eq. (34) by its initial values. The initial conditions correspond to a state of isotropic turbulence where b_{ij} = 0, M_t = 0.4, K/K_0 = 1, and $\varepsilon/\varepsilon_0$ = 1 at St = 0.

To see the performance level of the proposed model for the turbulent dissipationin predicting compressible homogeneous turbulent shear flow is now discussed. Two compressible models for the pressure strain correlation are considered: the model deduced from the Pantano ans Sarkar [26] and the model of Adumitroiae *et al.* [10] noticed AR and PS model, respectively.

Table 1. Initial conditions for homogeneous shear flow: DNS [7]

Case	$Mt0$	M_{g0}	$(SK/\varepsilon)_0$	b_{11}	b_{22}	b_{12}
A_1	0.4	0.22	1.8	0	0	0
A_2	0.4	0.32	3.6	0	0	0
A_3	0.4	0.66	5.4	0	0	0
A_4	0.4	1.32	10.8	0	0	0

As discussed earlier, these models are used with the ε-present model, (see eq. (34) and with the ε-standard model. Figures 2-9 show a comparison between the predictions obtained by

AR and PS models with the ε-present model and those from these models with the ε-standard model and the DNS results of Sarkar [7] for Cases: A_1, A_2, A_3, and A_4. These cases listed in tab. 1, show that compressibility effects in homogeneous shear flows were seen to evolve with the initial conditions and could be parameterized by M_t and M_g. Thus, they appear to be suitable for evaluating the proposed model which is explicitly expressed with the extra compressibility parameters: the turbulent Mach number and the gradient Mach number.

In fig. 2, the time evolution of the normalized dissipation ε/SK predicted by the AR and PS models with the ε-standard model is displayed for cases A_1 to A_4 from the DNS results of Sarkar [7]. It is clearly seen that the two models are nearly similar in the prediction of ε/SK for all cases. In cases A_1 and A_2, the models appear to be able to predict as it is shown in figs. 2(a) and 2(b). For cases A_3 and A_4, as the results are shown in figs. 2(c) and 2(d), the two models over-predict ε/SK. One can see unphysical equilibrium values predictions of ε/SK showing a systematic increase with the time in disagreement with the DNS results for the earlier time.

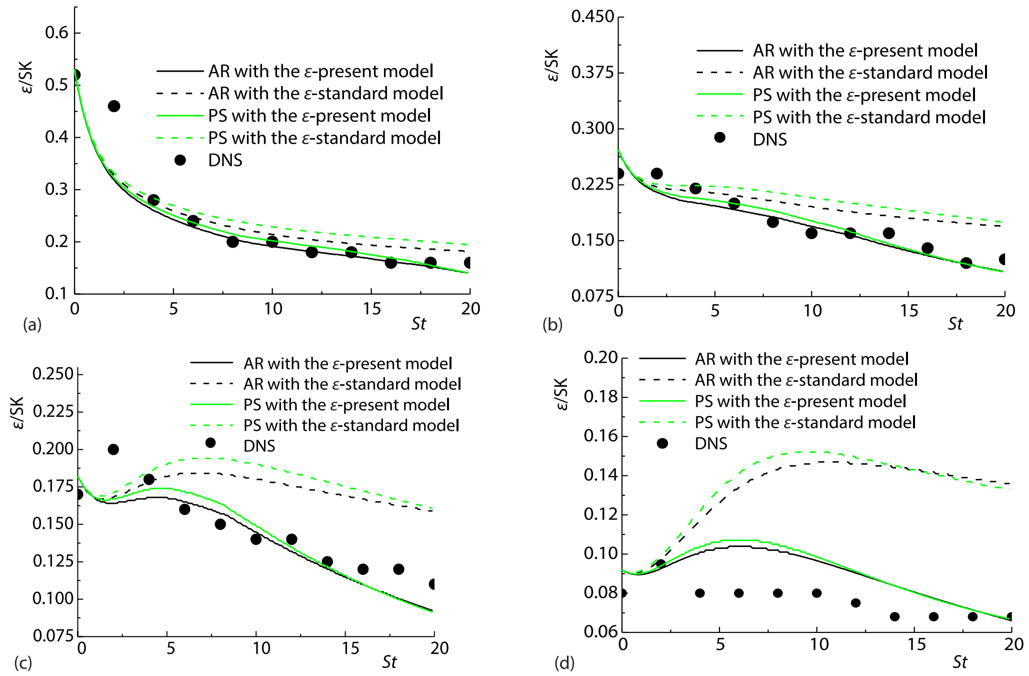

Figure 2. Time evolution of the normalized dissipation ε/*SK* in the cases; (a) A_1, (b) A_2, (c) A_3, and (d) A_4

However, the previous DNS results show tha ε/SK tstrongly decreases when M_{g0} increases, since compressibility effects cause significant reduction in the turbulent production from numerical simulation cases A_1 to A_4. Regarding $\varepsilon/SK = -2b_{12}(\varepsilon/P)$, the DNS results [7] show that ε/P is insensitive to compressibility effects and shows little differences between cases A_1 to A_4. This implies that the decrease of ε/SK is attributed to the strong reduction in the shear stress anisotropy, b_{12} when the gradient Mach number increases.

Figures 3-5 show the non-dimensional time St variation of the Reynolds stress anisotropies b_{11}, b_{22}, and b_{12} for cases A_1 and A_4. Figures 3(a) and 3(b) present the computed results from AR and PS models with the ε-standard of the stream wise b_{11}. These results show that the model (PS) better predicts the asymptotic trend of b_{11} than the AR model which is unable to properly reproduces this discrepancy. Figures 4(a) and 4(b) show that the two models under-predict the transverse b_{22} of Reynolds stress anisotropy for cases A_1 and A_4.

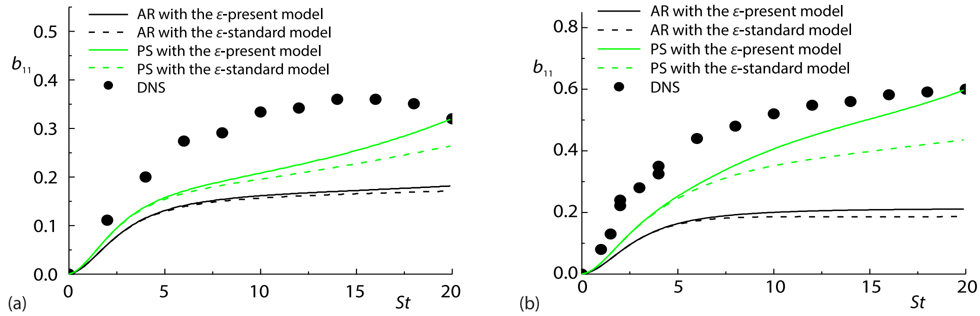

Figure 3. Time evolution of the streamwise Reynolds stress anisotropy b_{11} in the cases; (a) A_1 and (b) A_4

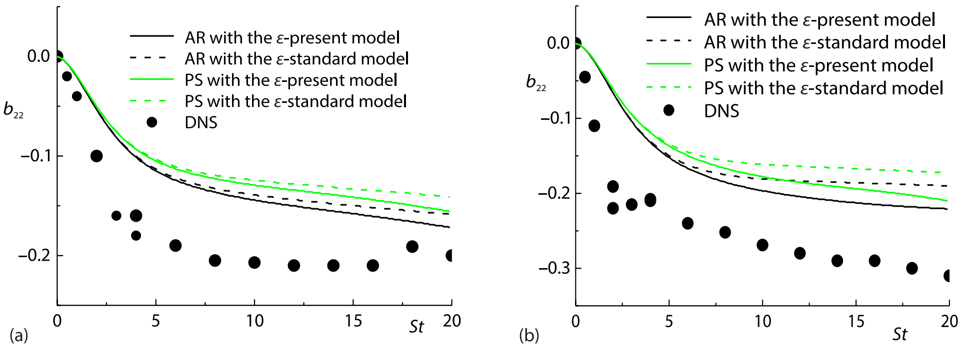

Figure 4. Time evolution of the transverse Reynolds stress anisotropy b_{22} in the cases; (a) A_1, and (b) A_4

From figs. 5(a) and 5(b), it is clear that the models over-predict the magnitude of the Reynolds shear stress anisotropy b_{12} for the case A_4, in the contrary, for the case A_1, all the two modelsprovide an acceptable performance in reproducing the DNS results for this case.

In figs. 6-8 the predictions of the pressure strain components $\phi_{ij}^{*}(i, j = 1,2)$ from the models of AR and PS with the ε-standard are displayed for cases A_1 and A_4. Clearly seen, the models have nearly similar behaviours in the predictions of the compressible homogeneous

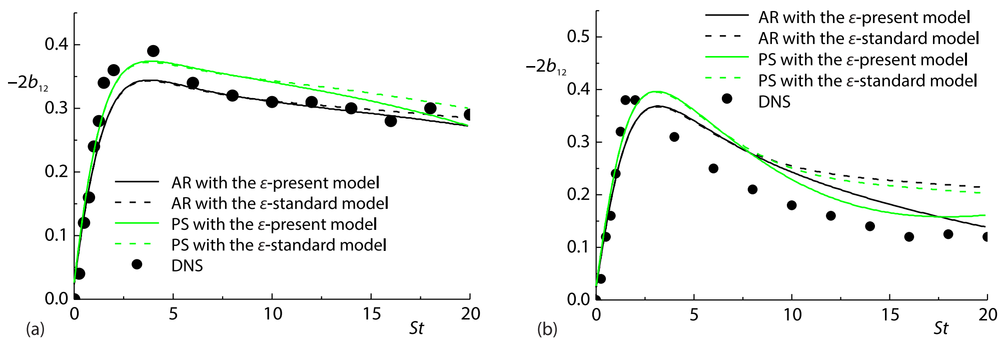

Figure 5. Time evolution of the shear Reynolds stress anisotropy b_{12} in the cases; (a) A_1 and (b) A_4

shear flow. Except in fig. 6(b), there are differences between the model's predictions, thePS model follows the DNS data better than the AR model. From figs. 6(a), 7(a). and 8(a), the models' AR and PS are in qualitative acceptable agreement with the DNS results.

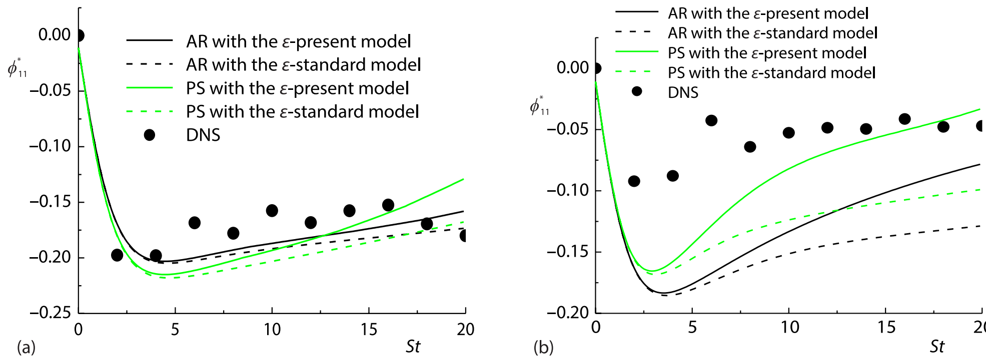

Figure 6. Time evolution of the streamwise pressure strain $\phi^*_{11}/2SK$ in the cases; (a) A_1 and (b) A_4

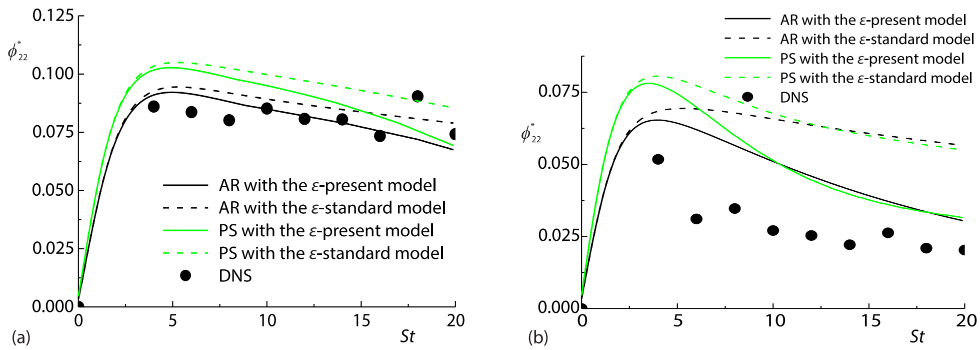

Figure 7. Time evolution of the transverse pressure strain $\phi^*_{22}/2SK$ in the cases; (a) A_1 and (b) A_4

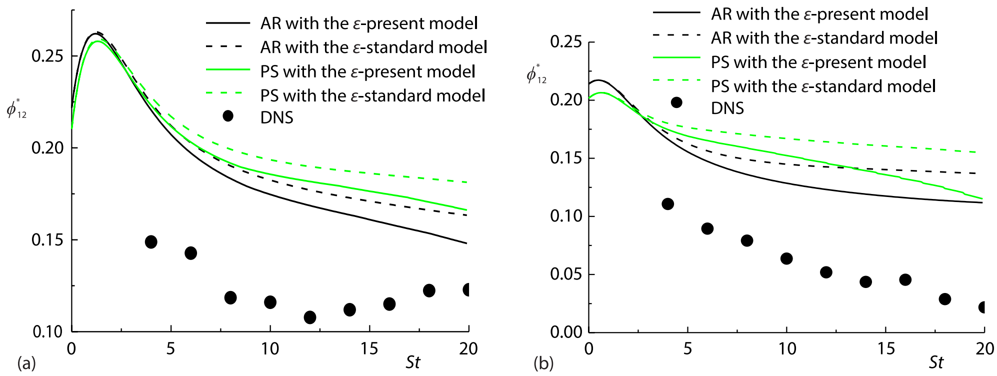

Figure 8. Time evolution of the shear pressure strain $\phi^*_{12}/2SK$ in the cases; (a) A_1 and (b) A_4

Figures 7(b), 8(b), and 9(b) show the predictions of the non-dimensionless components $\phi^*_{11}/2SK$, $\phi^*_{22}/2SK$, and $\phi^*_{12}/2SK$, respectively from the ε-standard with the PS and AR models for case A_4. Inregards to the latter, a notably unphysical equilibrium turbulence components of the pressure strain are predicted. From these figures, one can see that the ε-standard with the PS and AR models overpredict all the components of the pressure strain.

Evaluation of the proposed compressible model for the turbulent dissipation rate will now be considered with the use of the DNS data of Sarkar [7] for compressible homogeneous shear flow. The proposed model called ε-present model is tested in conjunction with the AR model and PS models for the pressure strain. Computed results for the PS and AR models with the addition of the ε-present model are compared with the DNS data and with those of these models with the ε-standard model. From the all figures, it is clear that the PS and PR models with the ε-present model lead to a remarkably good improvement, this is true for the major characteristic parameters for the compressible homogeneous shear flow for all the DNS cases [7].

In figs. 2(a)-2(d), the normalized dissipation is well predicted by the proposed model, one can see that the PS and AR models with the ε-present model yield good agreement with the DNS results for the all cases.

Clearly seen, figs. 3(a), 3(b), 4(a), 4(b), and 5(a), 5(b) show that the present model leads to good predictions for the stream wise. The b_{11} and the shear stress b_{12} the model remains still to correctly predict the transverse b_{22}.

Figures 6-8 indicate that there is a substantial improvement in the present model predictions for the normalized pressure strain components. For Case A_1, the models AR and PS with the ε-present model yield reasonable agreement with the DNS data for ϕ^*_{11}, ϕ^*_{22}, the shear component ϕ^*_{12} is over-predicted by these models. On the other hand, noticeably, for caseA4,the PS model with the ε-present model leads to predictions that are in good agreement with the DNS results for ϕ^*_{11}.

Figures 9(a)-9(c) the predictions of the equilibrium values of the normalized pressure strain by the AR and PS models with ε-present model *vs.* with the gradient Mach number are displayed. From these figures, it is clear that the ε-present model with the AR and PS models yields reasonable agreement with the DNS results [7] and clearly predict the reduced of the pressure strain components better than these models with the ε-standard model. This is reached by the DNS results [7-9] in which it is argued that the pressure fluctuations and all components of the pressure strain correlation show monotone decrease with increasing gradient Mach number.

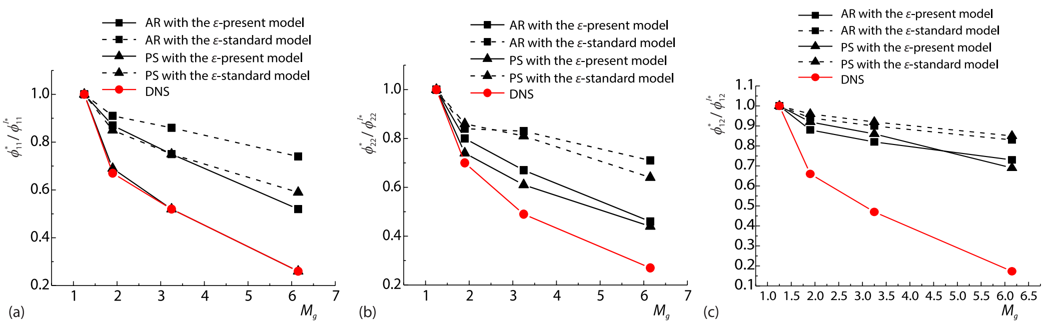

Figure 9. The long-time values of the normalized pressure strain $\phi^*_{ij}/\phi^{1*}_{ij}$ vs. M_g

According to DNS results of Sarkar [7], one can see that the compressible homogeneous shear flow seems to evolve towards equilibrium states. Obviously, it is well known that this trend of the flow may be useful tests in evaluation and calibration of the turbulence models. So, the ability of the proposed compressibility correction model for the turbulent dissipation in capturing the compressibility effects on the turbulent equilibrium values of ε/SK is now discussed by considering different published results of several references. Thus, clearly seen in

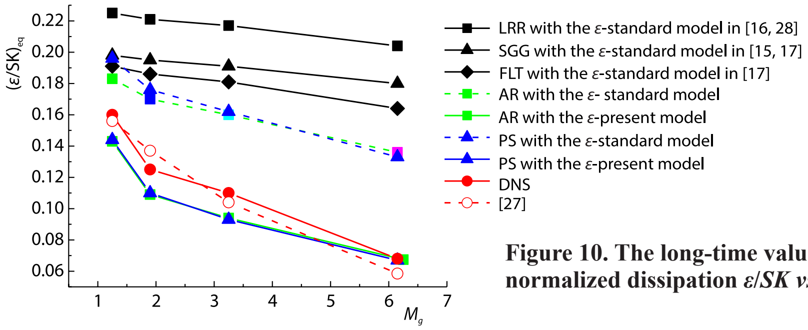

Figure 10. The long-time values of the normalized dissipation ε/SK *vs.* M_g

fig. 10 that shows a systematic comparison between the proposed model predictions for the long-time (St = 20) of ε/SK *vs.* the M_g, the DNS [7], the results given by the formula of Stefan Heinz [27], namely:

$$\frac{\varepsilon}{SK} = 0.2\mathrm{e}^{-0.2M_g} \tag{38}$$

and the published results [15-17, 28] obtained by the standard model of the turbulent dissipation in addition with the LRR [1], SSG [2], and FLT [3] models for the pressure strain correlation, respectively. According to fig. 10, it seems that the proposed model appears to be able to reproduce accurately compressibility effects on the equilibrium values of ε/SK for compressible homogeneous shear flow.

Conclusion

In this study, the Favre Reynolds stress model has been used for the prediction compressibility effects on the homogeneous shear flow. Evaluation of the compressible models developed by Adumitroaie *et al.* [10] and Pantano and Sharkar [26] for the pressure strain correlation were examined in conjunction with the incompressible ε-equation model. Consistent with earlier results, it was shown from the previous results that for case A_1 which corresponds to low compressibility, both models are nearly similar. They predict correct the majority turbulence characteristic parameters of compressible homogeneous shear flow. At high compressibility, non-e of the two models are able to correctly predict the strong changes arising from compressibility effects as it can be seen in case of A_4 [7]. A revision of the incompressible turbulent dissipation model makes the coefficients of this model in function of the gradient Mach number in addition turbulent Mach number is considered. Application of the proposed model for the dissipation rate with the models [10, 26] for the pressure strain leads to substanstially improved predictions that are in satisfactory agreement with available DNS data for the structural compressibility effects on homogeneous shear flow as the significant decrease of the normalized dissipation and the magnitude of the Reynolds shear stress, the increase of the streamwise and the transverse of the Reynolds stress and the reduction of the pressure strain components with increasing initial values of the gradient Mach number. It is clear that the proposed model for the turbulent dissipation leads to predictions that are better than those obtained by the standard model. Therefore, as a priority, the compressibility correction for the incompressible ε –model with an eventual revision of the existent models for the pressure strain correlation by including other compressibility parameters as the gradient Mach number in addition turbulent Mach number is found out to be an important issue in the modelling of the compressible homogeneous turbulent shear flow.

Nomenclature

a – speed of sound $[=(\gamma R\tilde{T})^{1/2}]$, [ms^{-1}]
b_{11} – Reynolds stress anisotropy, [–]
c_p – specific heat at constant pressure, [JK^{-1}kg^{-1}]
c_v – specific heat at constant volume, [JK^{-1}kg^{-1}]
d' – fluctuation of the dilatation, [s^{-1}]
P – pressure, [Pa]
R – specific gas constant, [JK^{-1}mol^{-1}]
S – shear rate $=(u_{i,j}u_{j,i})^{1/2}$, [s^{-1}]
T – temperature, [K]
t – time, [s]
u_i – velocity in the x_i-direction, [ms^{-1}]

Greek symbols

γ – specific heat ratio (= c_p/c_v), [–]
δ_{ij} – Kronecker delta
ε – dissipation rate of turbulent kinetic energy, [m^{2}s^{-3}]
ε_s – solenoidal part of the dissipation, [m^{2}s^{-3}]
ε_c – compressible part of the issipation, [m^{2}s^{-3}]
ν – kinematic viscosity, [m^{2}kgm^{-3}]
ρ – density, [kgm^{-3}]

Subscripts

$''$ – favre fluctuation
$'$ – Reynolds fluctuation
i – spatial gradient
t – time derivative
0 – initial value

Other Symbols

$\bar{}$ – Reynolds mean
$\tilde{}$ – favre average

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