CHEMICAL REACTION AND RADIATION ON BOUNDARY-LAYER FLOW OF ELECTRICALLY CONDUCTION MICROPOLAR FLUID THROUGH A POROUS SHRINKING SHEET

by

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The flow of an electrically conducting micropolar fluid with a radiative heat source and mixed chemically reactive species is considered. The stretching/shrinking surface under the influence of the applied magnetic field in the normal direction is used. Appropriate similarity functions are used for the numerical solution of highly non-linear governing equations of the flow problem, and the behaviors of the flow, temperature and concentration function under the influence of various physical parameters are revealed graphically.

Key words: *micropolar fluids, shrinking sheet, radiation effects, magnetohydrodynamic boundary-layer, chemical reaction, similarity function*

Introduction

Eringen [1, 2] first formulated the micropolar fluid theory and derived the constitutive laws for the fluids with micro-structure. This theory provided a mathematical model for the non-Newtonian behavior which could be observed in certain liquids such as polymers, colloidal suspensions, animal blood, liquids crystals, *etc.* A thorough review of the subject and applications of micropolar fluid mechanics was provided by Ariman *et al.* [3] and Eringen [4]. Ramachandram *et al.* [5] studied the heat transfer in the flow of micropolar fluids past a curved surface with suction and injection using Van Dyke's singular perturbation technique. Jena [6] deals with a steady 2-D laminar flow of a viscous incompressible electrically conducting fluid over a shrinking sheet in the presence of a uniform transverse magnetic field with viscous dissipation. Thiagarajan *et al.* [7] studied MHD boundary-layer flow of Casson fluid due to permeable shrinking sheet. Haramgari and Sulochana [8] studied the influence of thermal radiation and chemical reaction on the 2-D steady MHD flow of a nanofluid past a permeable stretching/shrinking sheet in the presence of suction/injection. Haile and Shankar [9] investigated the boundary-layer flow of nanofluid through a porous medium subject to a magnetic field, thermal radiation, viscous dissipation, and chemical reaction effects. Tripathy *et al.* [10] considered unsteady MHD free convection in boundary-layer flow of an electrically

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conducting fluid through porous medium subject to uniform transverse magnetic field over a moving vertical plate in the presence of heat source and chemical reaction. Medikare *et al.* [11] studied MHD stagnation point flow of Casson fluid over a non-linearly stretching sheet with viscous dissipation. Bakr *et al.* [12] studied the effects of a chemical reaction and thermal radiation on unsteady free convection flow of a micropolar fluid past a semi-infinite vertical plate embedded in a porous medium in the presence of heat absorption with Newtonian heating. Heat and mass transfer in MHD non-Newtonian bio-convection flow over a rotating cone/plate with cross-diffusion has been analyzed by Raju and Sandeep [13]. Mixed convective heat transfer for MHD viscoelastic fluid flow over a porous wedge with thermal radiation is considered by Rashidi *et al.* [14]. Raju *et al.* [15] considered dual solutions of MHD boundary-layer flow past an exponentially stretching sheet with non-uniform heat source/sink. Sharma *et al.* [16] investigated heat transfer due to an exponentially shrinking sheet in the existence of the thermal radiation among mass suction of the boundary-layer flow of a viscous fluid. Khan *et al.* [17] studied the thermodiffusion results on stagnation position flow of the nanofluid in the direction of an elongating surface with applied magnetics subject is introduced. Dash *et al.* [18] studied the MHD flow heat and mass diffusion of electrically conducting stagnation point flow past a stretching/shrinking sheet and chemical reaction. This article attempts a numerical solution of the MHD boundary-layer stagnation point flow of a micropolar fluid.

Mathematical model

We consider the flow of electrically conducting micropolar fluids due to a linearly moving boundary surface that stretches/shrinks. The flow is steady, 2-D and incompressible. A magnetic field of uniform strength $\vec{B}_0 = B_0(0,\, B_0,\, 0)$ is applied in the normal direction of xy-plane. The sheet is along x-axis under the action of two equal forces in opposite directions. A chemical reactive species of concentration with first order chemical reaction mixed in the fluid. The fluid temperature is T and temperature of surface is T_w. The vectors $\vec{\mathrm{v}} = v(u, v)$ and $\vec{\omega} = \omega(0, 0, \omega_3)$ represent, respectively, the velocity and micromotion. Governing equations of the problems are:

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 0 \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = U\frac{\partial U}{\partial y} + (\mu + k)\frac{\partial^2 u}{\partial y^2} + k\frac{\partial \omega_3}{\partial x} - \frac{\sigma B_0^2 (U - u)}{\rho} \tag{2}$$

$$\gamma\left(u\frac{\partial^2 \omega_3}{\partial y^2}\right) - k\left(\frac{\partial u}{\partial y} + 2\omega_3\right) = \rho j\left(u\frac{\partial \omega_3}{\partial x} + v\frac{\partial \omega_3}{\partial y}\right) \tag{3}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{K}{\rho C_p}\frac{\partial^2 T}{\partial y^2} + \frac{Q(T - T_\infty)}{\rho C_p} + \frac{16\alpha}{2\beta\rho C_p}\frac{\partial^2 T}{\partial y^2} \tag{4}$$

$$u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D\frac{\partial^2 u}{\partial y^2} - R(C - C_\infty) \tag{5}$$

with boundary conditions:

$$\begin{gathered} \omega_3(x, 0) = 0, \quad u(x, 0) = bx, \quad v(x, 0) = 0, \quad T(x, 0) = T_w, \quad C(x, 0) = C_w \\ \omega_3(x, \infty) = 0, \quad u(x, \infty) = ax, \quad v(x, \infty) = -ay, \quad T(x, \infty) = T_\infty, \quad C(x, \infty) = C_\infty \end{gathered} \tag{6}$$

where u and v are the velocity components and ω_3 is the spin friction. We also add the radiation in energy equation such as radiation effect results from radiation heat exchange between human bodies and surrounding surface, such as wall and ceiling.

Using the similarity transformation, the velocity components are described in terms of stream function $\psi(x, y)$, where:

$$u = \frac{\partial \psi}{\partial y}, \quad v = -\frac{\partial \psi}{\partial x}, \quad \psi(x,y) = x\sqrt{a\nu}\, f(\eta), \quad \eta = y\sqrt{\frac{a}{\nu}}, \quad u = xaf',$$

$$v = -\sqrt{a\nu}\, f, \quad \omega_3 = \frac{a^{3/2}}{\nu^{1/2}} xL(\eta), \quad \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}$$

$$(1+d_1) f''' + d_1 L' - M(f'-1) + 1 = f'^2 - ff'' \tag{7}$$

$$d_3 L'' + 2d_1 d_2 L - d_1 d_2 f'' = f'L - fL' \tag{8}$$

$$(4+3R_n)\theta'' + 3R_n \Pr(f\theta' + \Pr S\theta) \tag{9}$$

$$\phi'' + \mathrm{Sc}(f\theta' - \beta\phi) = 0 \tag{10}$$

The associated boundary conditions (6) are:

$$f'(0) = \frac{b}{a}, \quad f(0) = f_w, \quad L(0) = 0, \quad \theta(0) = 1, \quad \phi(0) = 1$$

$$f'(\infty) = 1, \quad L(\infty) = 0, \quad \theta(\infty) = 0, \quad \phi(\infty) = 1 \tag{11}$$

The system given in eqs. (7)-(11) can be analytically solved by some analytical methods [19, 20], *e.g.* the variational iteration method [21-23], the direct algebraic method [24], the exp-function method [25], the Fourier spectral method [26], the reproducing kernel method [27], and the variational principle [28-32]. In this paper, we study the system numerically by mathematical software.

Results and discussion

The flow problem is governed by the set of non-linear ODE (7)-(11), is difficult for any closed-form solutions. An effective, straight forward and efficient coding has been made in software to obtain the numerical solution of the problem. The higher-order derivative involved in the governing model has been reduced to the first-order form. Then the numerical results for the velocity, micro motion, temperature distribution, and concentration function have been obtained for suitable ranges of the physical parameter namely thermal slip parameter, β, radiation parameter, R, Prandtl number, Pr, heat source parameter, S, magnetic parameter, M, suction parameter, f_w, velocity ratio parameter b/a, and the particularly micropolar parameter, d_1. When the micro spin components w_3 and d_1 become zero, the flow problem resembles Newtonian fluids flow. The graph for the results has been presented.

Using mathematical software, we represent the graphical solution of the governing eqs. (7)-(11). It is noticed in figs. 1 and 2 that increase in shrinking parameter and micropolar parameter causes increase in magnitude of tangential velocity f', but increase magnetic field intensity reduce the fluid flow as depicted in fig. 3. It is due to opposing effect of Lorentz force. Figure 4 presents the microrotation, L, under the effect of the parameter d_1. It is noticed that d_1 has an increasing effect on L. Figures 5 and 6, respectively, show that the temperature distribution, $\theta(\eta)$ increases with an increase in the value of radiation parameter, R_n, and heat source parameter, S. But the temperature $\theta(\eta)$ decreases with an increase in Prandtl number as shown in fig. 7.

The concentration function, $\phi(\eta)$, increases, the reaction rate of the thermal slip parameter, β, as indicated in fig. 8. But it decreases with an increase in the Schmidt number as presented in fig. 9.

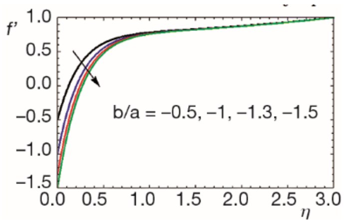

Figure 1. The plot for curves of *f'* for different values of *b*/*a*

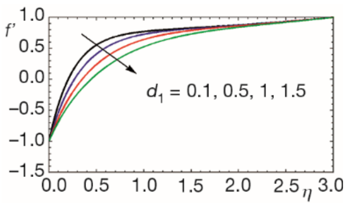

Figure 2. The plot for curves of *f'* under the effect of micropolar parameter, d_1

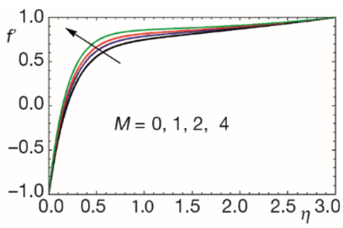

Figure 3. The plot for curves of *f'* under the effect of magnetic parameter, *M*

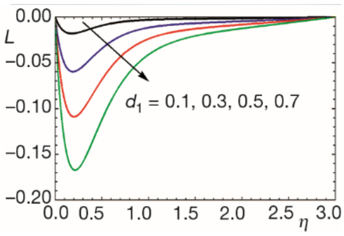

Figure 4. The plot for curves of *L* under the effect of micropolar parameter, d_1

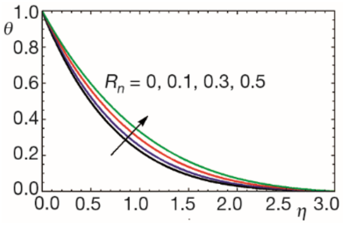

Figure 5. The plot for curves of *θ* under the effect of radiation parameter, R_n

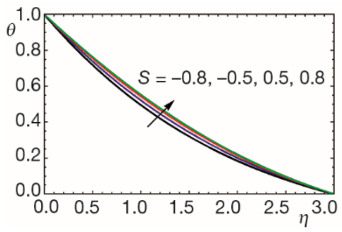

Figure 6. The plot for curves of *θ* under the effect of *S*

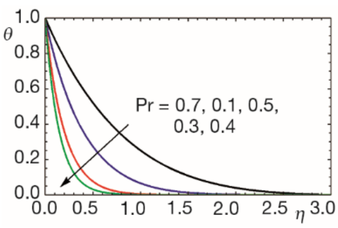

Figure 7. The plot for curves of *θ* under the effect of Prandtl number

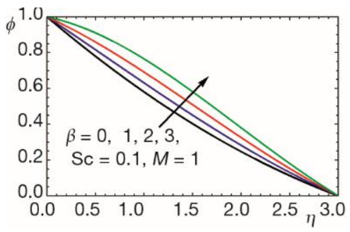

Figure 8. The plot for curves of *ϕ* under the effect of *β*

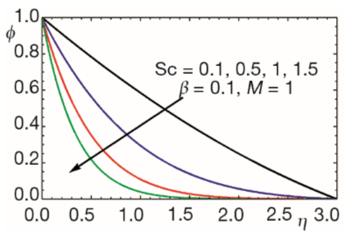

Figure 9. The plot for curves of *ϕ* under the effect of Sc

Conclusion

The paper investigated theoretically the chemical reaction and radiation effects on boundary-layer flow of electrically conducting micro polar fluids owing to the porous shrinking sheet.

The salient findings of the problem are as follows.

- The increase in the shrinking parameter and the micro polar parameter causes an increase in the magnitude of tangential velocity, f'.
- The increase in magnetic field intensity reduces the fluid-flow.
- The micropolar parameter, d_1, has an increasing effect on the micro rotation.
- The temperature distribution, $\theta(\eta)$, increases with an increase in the value of the radiation parameter, R, and heat source parameter, S.
- The temperature distribution, $\theta(\eta)$, decreases with an increase in Prandtl number.
- The concentration function, $\theta(\eta)$, increases with the reaction rate of thermal slip parameter, β, but it decreases with an increase in the Schmidt number.

Nomenclature

b – constant
B_0 – constant magnetic field
C_p – specific heat at constant pressure
f – dimensionless fluid velocity in r-direction
f' – dimensionless fluid velocity in s-direction
Pr – Prandtl number
R – radius of curvature
S – dimensionless mass transfer parameter
Sc – Schmidt number
T – temperature
T_∞ – ambient fluid temperature
T_w – surface temperature
u – velocity component in the s-direction
v – velocity component in the r-direction

Greek symbols

β – thermal slip parameter
γ – Biot number
θ – dimensionless fluid temperature
μ – dynamic viscosity of the fluid
ν – kinematics viscosity of the fluid
ρ – density of fluid

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