A FRACTAL APPROACH TO THE DIFFUSION PROCESS OF RED INK IN A SALINE WATER

by

Ji-Huan HE [a,b,c*] ***and Man-Yu QIAN*** [a]

[a] National Engineering Laboratory for Modern Silk, College of Textile and Clothing Engineering, Soochow University, Suzhou, China
[b] School of Mathematics and Information Science, Henan Polytechnic University, Jiaozuo, China
[c] School of Science, Xi'an University of Architecture and Technology, China

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The diffusion of the red ink in saline water is completely stochastic and highly unpredictable, and no differential model can precisely describe the process. This paper elucidates that the diffusion takes place in a molecule scale, therefore, the continuum assumption in fluid mechanics becomes totally invalid, and the two-scale fractal calculus has to be adopted to take into account the effects of the particles' size in the red ink and the properties of the saline solution including its concentration, water molecule's size and distribution on the diffusion process. On the molecule's scale, the diffusion becomes completely deterministic and predictable. An experiment is carefully designed and some phenomena, including optical observation and highly selective diffusion routine, are theoretically explained. This paper sheds light on modeling various contamination diffusion in air and water.

Key words: *fractal diffusion law, molecule diffusion, apical dominance, two-scale fractal dimension*

Introduction

At present, the increasingly serious problem of water/air pollution has attracted extensive attention from academic and industrial communities. It has become a major environmental problem, affecting all aspects of human productions and life.

The pollutant diffusion is completely stochastic and extremely unpredictable, and no differential model can exactly describe the process. Many experts and scholars have conducted in-depth studies on the diffusion mechanism of pollutants. Zhou *et al.* [1] conducted a sophisticated experiment using a drop of red ink as a point pollution source, and observed the diffusion and infiltration processes of the red ink in salt solutions of different concentrations. They found that the salt concentration greatly influences the osmotic process, and there is a threshold beyond which no osmosis will occur. Only the NaCl solution with a concentration of 5% has obvious selective permeability, while the NaCl solution with a concentration higher than 10% has no permeability, and only diffusion occurs. Finally, the results show that osmosis is the main pollutant dispersion, which is of great significance for theoretical analysis and pollution control. Farahbod [2] established a mathematical model for diffusion and decomposition of pollutants, he found that the wave depletion plays an important role at the initial

* Corresponding author, e-mail: hejihuan@suda.edu.cn

stage. Lin *et al.* [3-6] suggested a fractal diffusion oscillation for release of ions from a hollow fiber. Some researchers established a fractal differential models for diffusion-reaction processes. O'Shaughnessy and Procaccia [7] studied diffusion on fractals. Zheng *et al.* [8] proposed a fractal model for gas diffusion through a porous medium. Gouyet *et al.* [9] found the fractal pattern during the diffusion process. Stroock *et al.* [10] studied the molecular diffusion in microchannel. Dan *et al.* [11] suggested a fractional diffusion model. Tian and Liu [12] suggested a fractal propagation model in optical fibers.

In order to have a thorough understanding of pollutant diffusion so that the pollution problem can be easily dealt with, this paper uses a drop of red ink as a source of pollution in a NaCl solution.

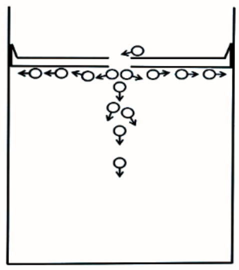

Figure 1. The experiment set-up for the diffusion process

The experiment

In our experiment, a bottle of red ink was bought in a supermarket in the campus of Soochow University as a pollution source, and 5%, 10%, 15%, and 20% NaCl solutions were prepared, and found that the 15% NaCl solution has almost the same density with that of the red ink. The experiment set-up is illustrated in fig. 1, where an aluminum paper was put on the surface the solution, and a small hole was cut in the center. A drop of red ink was gradually put into the hole, and the diffusion process was recorded.

When the red ink met the solution surface, there were two trends of the diffusion process at the initial stage, one was along the surface of the paper, and the other was diffused downwardly, fig. 2.

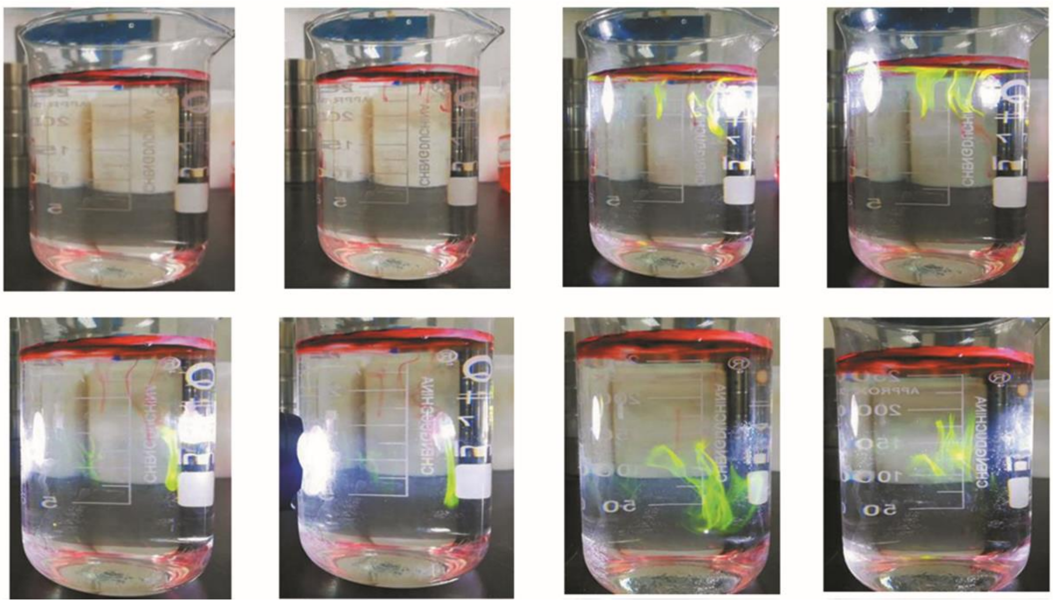

Figure 2. The sequential diffusion of the red ink in the salt solution within one minute

In order to observe the diffusion morphology, we used a strong light flashlight to observe the diffusion process. As the diffusion process went, a green line instead of red one was observed, and then a piece of green sheet was seen, and the 2-D sheet began to enlarge its area as time going. The mechanism of the 2-D diffusion process within one minute was given in fig. 3. As the time going, the diffusion process became much complex, and continued until the full bottle became red as illustrated in fig.4.

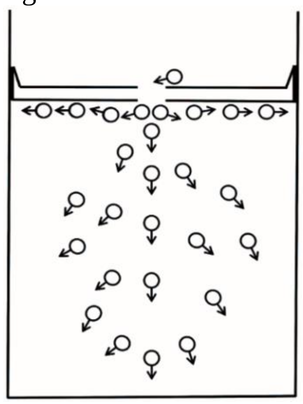

Figure 3. The 2-D diffusion process within one minute

Theoretical analysis

In our observation, an apical dominance-like phenomenon was seen, the diffusion process is similar to a sprouting seed, and the root-like distribution of the diffusion lines was unpredictable. In order to explain the phenomenon, the water molecules shown in fig. 5 can be considered as a Toda-like lattice [13-15], and a differential model with fractional or fractal derivatives [16-20] can describe the dynamics of the diffusion process.

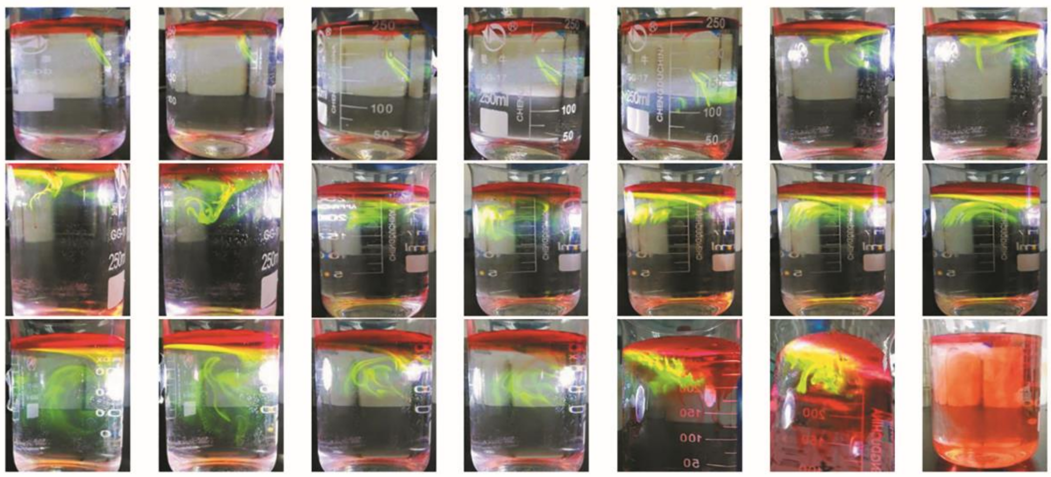

Figure 4. The sequential diffusion of the red ink in the salt solution from one minute to 36 minutes

The two-scale fractal theory observes the same phenomenon using two different scales. The large scale follows laws in the continuum mechanics, and the smaller scale considers the salt solution a discontinuous medium. On the molecule's scale, the solution becomes totally porous, as illustrated in fig. 5, some a particle in the red ink will certainly enter into the porosity of the solution, and an adjacent particle will follow the trajectory of the foregoing particle, and a diffusion line is formed. Multiple diffusion lines form a 2-D diffusion sheet, this results in a special optical phenomenon.

The diffusion process depends upon the molecules' distribution in the salt solution. The value of the fractal dimensions for pure water is D = 1.92, see page 12 in [21], this can explain the approximate 2-D diffusion sheet as illustrated in figs. 5(e) and 6.

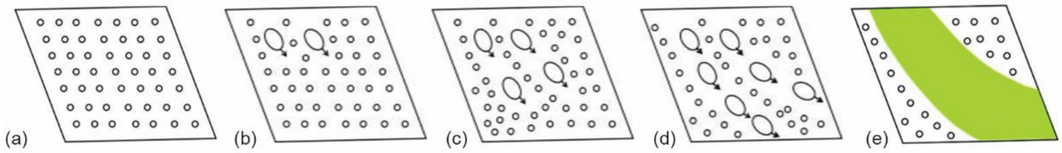

Figure 5. Diffusion trajectory of the red ink in a porous salt solution; the small circles represent molecules in the salt solution, the ellipses imply the particles in the red ink

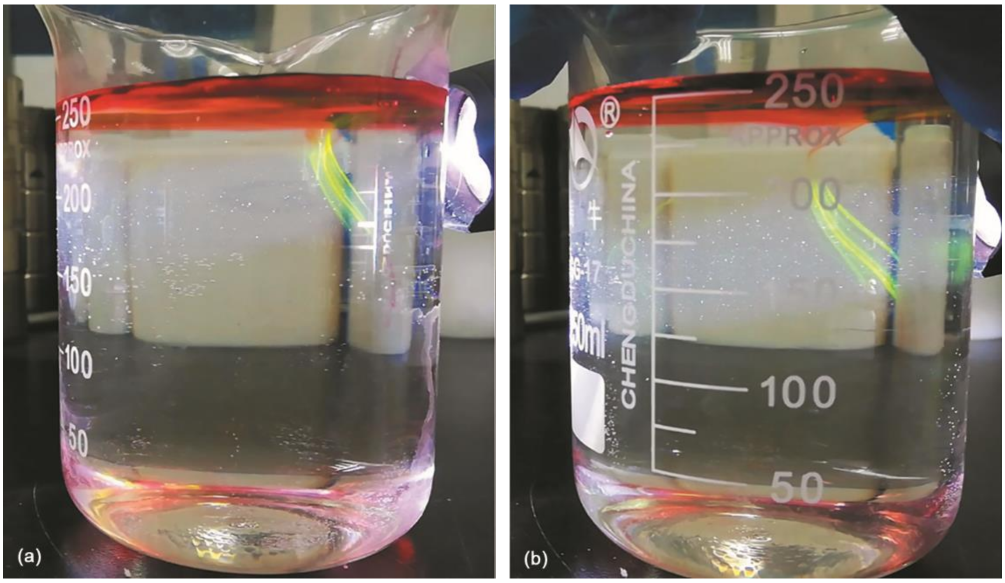

Figure 6. Almost 2-D diffusion sheet

Discussion and conclusions

In this paper, the diffusion of red ink in NaCl solution is studied to understand the pollution mechanism of pollutants in underwater/seabed areas and suggests a possible method for water pollution control. The results showed that the diffusion of red ink in NaCl solution was similar to that of germinated seeds, and there was a top advantage. The diffusion line is formed first, and then a 2-D diffusion surface is formed by several diffusion lines. In addition, we use the two-scale fractal theory to explain this phenomenon, which has a particular guiding significance for further understanding the pollution mechanism of pollutants in the underwater/seabed.

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