A NEW FRACTIONAL THERMAL MODEL FOR THE Cu/LOW-K INTERCONNECTS IN NANOMETER INTEGRATED CIRCUIT

by

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In this paper, the Cu/Low-k interconnects in a nanoscale integrated circuit are considered. A new fractal conventional heat transfer equation is suggested using He's fractal derivative. The two-scale transform method is applied for solving the equation approximately. The new findings, which the traditional differential models can never reveal, shed a bright light on the optimal design of a nanoscale integrated circuit.

Key words: *He's fractal derivative, fractional thermal model, interconnects, two-scale transform method*

Introduction

After 1990's, the development of large-scale integrated circuit technology still followed Moore's law, and the device size was reduced by 2/3 every three years, the chip area was increased by about 1.5 times, and the number of transistors in the chip was increased by 4 times [1, 2]. The integrated circuit technology has now developed to a very large-scale stage, namely the ULSI stage. The number of components contained in each chip has reached 100 million. Its micro-processing technology has reached as small as 10 nm. It has continued to develop in the direction of 7 nm, and finally to 5 nm. In the deep submicron ULSI, a chip requires 7-8 layers of wiring. The total length of its internal wiring can reach several kilometers. Any a point interconnect defect is fatal to the chip [3, 4]. With the further decrease of the interconnection system's size and the interconnection system's size, the current density and the number of layers of metal wire are further increased. The thermal problems on the metal interconnects are becoming more and more serious, especially the continuous rise of the temperature on the metal interconnects has become an important limiting factor in the design of high performance integrated circuit chips. The high temperature distribution on the metal wire will directly deteriorate the circuit performance and result in a circuit electromigration failure, which will affect the reliability of the device [5-8].

As in fig.1, under steady-state conditions, the heat conduction equations on the metal wire and the through hole can be obtained, respectively [9]:

$$\frac{\mathrm{d}^2U_\mathrm{m}(x)}{\mathrm{d}x^2}-\frac{\mathrm{d}U_\mathrm{m}(x)-U_\mathrm{sub}}{L_{H,\mathrm{m}}^2}=\frac{q_\mathrm{m}}{k_\mathrm{m}},\quad \frac{\mathrm{d}^2U_v(x)}{\mathrm{d}y^2}-\frac{\mathrm{d}U_v(x)-U_\mathrm{sub}}{L_{H,v}^2}=-\frac{q_v}{k_v} \tag{1}$$

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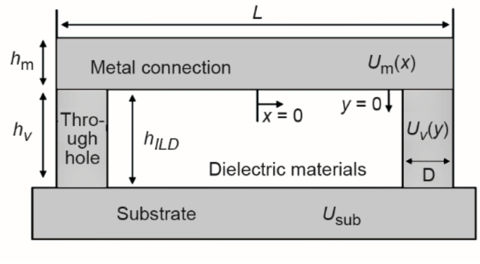

Figure 1. Metal interconnection in nanometer integrated circuit

where $q_\mathrm{m} = j^2_{\mathrm{rms,m}}\rho_\mathrm{m}$ and $q_v = j^2_{\mathrm{rms},v}\rho_v$ represent the Joule heat produced by the wire and the through-hole in the unit volume, ρ_m – the resistivity of the metal wire, ρ_v – the resistivity of the through hole, k_m – the heat conduction coefficient of the metal wire, k_v – the heat conduction coefficient of the through hole, and $L_{H,\mathrm{m}}$ and $L_{H,v}$ – the characteristic length of thermal diffusion of the metal wire and the through-hole, respectively, which can be expressed, respectively:

$$L_{H,\mathrm{m}} = \left(\frac{k_\mathrm{m}A_\mathrm{m}}{k_\mathrm{ILD}s_\mathrm{m}}\right)^{\frac{1}{2}} = \left(\frac{k_\mathrm{m}h_\mathrm{m}w}{k_\mathrm{ILD}s_\mathrm{m}}\right)^{\frac{1}{2}} \tag{2}$$

$$L_{H,v} = \left(\frac{k_\mathrm{m}A_v}{k_\mathrm{ILD}s_v}\right)^{\frac{1}{2}} = \left(\frac{k_\mathrm{m}\dfrac{\pi D^2}{4}}{k_\mathrm{ILD}s_v}\right)^{\frac{1}{2}} \tag{3}$$

where w is the width of the metal wire, D – the diameter of the through-hole, and s_m and s_v – the shape parameters of the unit length wire and the through-hole respectively, which are used to modify the case that only 1-D heat conduction is considered in the metal wire and through-hole, respectively.

In the case of the single metal connection, we have:

$$s_\mathrm{m} = 1.86\left[\log\left(1+\frac{h_\mathrm{ILD}}{w}\right)\right]^{-0.66}\left(\frac{w}{h_\mathrm{m}}\right)^{-0.1} \quad \text{and} \quad s_v = \frac{2\pi}{\left[\ln\left(\dfrac{4h_\mathrm{ILD}}{D}\right)\right]}$$

The fractal modification

It is known that the integer order derivatives are not suitable for porous problems accurately. On the contrary, the fractional derivative is more suitable than integer orders to describe many complex phenomenon, such as fractional soliton [10], cold plasma [11], fractal circuit [12], fractal filter [13-15], MEMS systems [16, 17], fractal disease model [18], fractal variational theory [19-32], fractal heat transfer through a porous cocoon [33], fractal approach to biology [34], fractal Hall-Petch law [35], fractal boundary [36], fractal vibration [37, 38], fractional KdV equation [39], fractional advection-reaction-diffusion [40], fractional Gardner equation [41], and fractional Sasa-Satsuma equation [42].

As pointed out in the introduction section that eq. (1) can well describe the steady heat conduction model of the Cu/Low-k interconnects in nanometer integrated circuit. However, when the interconnects and through-hole are porous medium, eq. (1) becomes invalid, so a new fractal model is needed, which takes the following form:

$$\frac{\mathrm{d}^2U_\mathrm{m}(x)}{\mathrm{d}x^{2\chi}}-\frac{\mathrm{d}U_\mathrm{m}(x)-U_\mathrm{sub}}{L_{H,\mathrm{m}}^2}=\frac{q_\mathrm{m}}{k_\mathrm{m}},\qquad \frac{\mathrm{d}^2U_v(x)}{\mathrm{d}y^{2\gamma}}-\frac{\mathrm{d}U_v(x)-U_\mathrm{sub}}{L_{H,v}^2}=-\frac{q_v}{k_v} \tag{4}$$

where $0<\chi\le 1$, $0<\gamma\le 1$, $\mathrm{d}/\mathrm{d}x^\chi$ and $\mathrm{d}/\mathrm{d}y^\gamma$ are He's fractal derivatives [43]:

$$\frac{\partial U}{\partial x^\chi}(x_0)=\Gamma(1+\chi)\lim_{\substack{x-x_0=\Delta x\to\psi\\ \Delta x\neq 0}}\frac{U(x)-U(x_0)}{(x-x_0)^\chi},\qquad \frac{\partial U}{\partial y^\gamma}(y_0)=\Gamma(1+\gamma)\lim_{\substack{y-y_0=\Delta x\to\psi\\ \Delta y\neq 0}}\frac{U(y)-U(y_0)}{(y-y_0)^\gamma}$$

where ψ is the smallest porous size and χ and γ – the fractal dimensions of the porous structure, respectively.

The two-scale transform method

The two-scale transform method extends the fractional complex transform [44-47] and is used widely in fractal calculus as a powerful computational analysis tool.

Consider the fractal equation:

$$\frac{\mathrm{d}^2U}{\mathrm{d}t^{2\alpha}}+F(U)=0 \tag{5}$$

In order to use the two-scale transform method, we assume:

$$T=t^\alpha \tag{6}$$

where t is for the small scale and T for the large scale. Then, we can convert eq. (5) into:

$$\frac{\mathrm{d}^2U}{\mathrm{d}T^2}+F(U)=0 \tag{7}$$

Then eq. (7) can be solved by many classical methods.

Solution of the fractal modification

Taking the two-scale transform as:

$$X=x^\chi \tag{8}$$

$$Y=y^\gamma \tag{9}$$

By using the previous transform, eq. (4) can be converted into the following form:

$$\frac{\mathrm{d}^2U_\mathrm{m}(x)}{\mathrm{d}X^2}-\frac{\mathrm{d}U_\mathrm{m}(x)-U_\mathrm{sub}}{L_{H,\mathrm{m}}^2}=\frac{q_\mathrm{m}}{k_\mathrm{m}}$$

$$\frac{\mathrm{d}^2U_v(x)}{\mathrm{d}Y^2}-\frac{\mathrm{d}U_v(x)-U_\mathrm{sub}}{L_{H,v}^2}=-\frac{q_v}{k_v} \tag{10}$$

The solution of the previous equation can be obtained as [9]:

$$U_{\mathrm{m}}(X)=U_{\mathrm{sub}}+\theta_{\mathrm{j}}\frac{\cosh\dfrac{X}{L_{H,\mathrm{m}}}}{\cosh\dfrac{L}{2L_{H,\mathrm{m}}}}+\frac{q_{\mathrm{m}}L_{H,v}^{2}}{k_{\mathrm{m}}}\left(1-\frac{\cosh\dfrac{X}{L_{H,\mathrm{m}}}}{\cosh\dfrac{L}{2L_{H,\mathrm{m}}}}\right) \tag{11}$$

$$U_{\mathrm{m}}(Y)=U_{\mathrm{sub}}+\theta_{\mathrm{j}}\frac{\sinh\dfrac{h_v-Y}{L_{H,v}}}{\sinh\dfrac{h_v}{L_{H,v}}}+\frac{q_{\mathrm{m}}L_{H,v}^{2}}{k_v}\left(1-\frac{\sinh\dfrac{Y}{L_{H,v}}+\sinh\dfrac{h_v-Y}{L_{H,v}}}{\sinh\dfrac{h_v}{2L_{H,v}}}\right) \tag{12}$$

where θ_{j} represents the temperature difference between the junction of the upper metal wire and the through-hole relative to the substrate temperature U_{sub}:

$$\theta_{\mathrm{j}}=\frac{A_{\mathrm{m}}L_{H,\mathrm{m}}\tanh\dfrac{L}{2L_{H,\mathrm{m}}}}{\dfrac{k_{\mathrm{m}}A_{\mathrm{m}}}{L_{H,\mathrm{m}}}\tanh\dfrac{L}{2L_{H,\mathrm{m}}}+\dfrac{k_vA_v}{L_{H,v}}\coth\dfrac{h_v}{L_{H,v}}}+\frac{A_vL_{H,v}q_v\left(\coth\dfrac{h_v}{L_{H,v}}-\operatorname{csc}h\dfrac{h_v}{L_{H,v}}\right)}{\dfrac{k_mA_m}{L_{H,m}}\tanh\dfrac{L}{2L_{H,\mathrm{m}}}+\dfrac{k_vA_v}{L_{H,v}}\coth\dfrac{h_v}{L_{H,v}}} \tag{13}$$

So, we can get the solution of the fractal modification in eq. (4) *via* the transforms in eqs. (8) and (9):

$$u_{\mathrm{m}}(x)=U_{\mathrm{sub}}+\theta_{\mathrm{j}}\frac{\cosh\dfrac{x^{\chi}}{L_{H,\mathrm{m}}}}{\cosh\dfrac{L}{2L_{H,\mathrm{m}}}}+\frac{q_{\mathrm{m}}L_{H,v}^{2}}{k_{\mathrm{m}}}\left(1-\frac{\cosh\dfrac{x^{\chi}}{L_{H,\mathrm{m}}}}{\cosh\dfrac{L}{2L_{H,\mathrm{m}}}}\right) \tag{14}$$

$$u_{\mathrm{m}}(y)=U_{\mathrm{sub}}+\theta_{\mathrm{j}}\frac{\sinh\dfrac{h_v-y^{\gamma}}{L_{H,v}}}{\sinh\dfrac{h_v}{L_{H,v}}}+\frac{q_{\mathrm{m}}L_{H,v}^{2}}{k_v}\left(1-\frac{\sinh\dfrac{y^{\gamma}}{L_{H,v}}+\sinh\dfrac{h_v-y^{\gamma}}{L_{H,v}}}{\sinh\dfrac{h_v}{2L_{H,v}}}\right) \tag{15}$$

Conclusion

This paper presented a fractional thermal model for the Cu/Low-k interconnects in a nanometer integrated circuit for the first time. The two-scale transform method is applied to convert the fractional equation into the ordinary equation, and an approximate analytical solution is obtained. The obtained results in this paper are expected to shed a bright light on practical applications of fractal calculus.

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