A NEW SCALING LAW HEAT CONDUCTION PROBLEM ASSOCIATED WITH THE KORCAK SCALING LAW

by

Xiao-Jun YANG [a,b,c*], ***Mahmoud ABDEL-ATY*** [c,d], **and *Moaiad A. A. KHDER*** [e]

[a] State Key Laboratory for Geomechanics and Deep Underground Engineering,
China University of Mining and Technology, Xuzhou,
Jiangsu, China
[b] School of Mathematics, China University of Mining and Technology,
Xuzhou, Jiangsu, China
[c] Mathematics Department, Faculty of Sciences, Sohag University,
Sohag, Egypt
[d] Center for Photonics and Smart Materials (CPSM), Zewail City of Science and Technology,
Zewail, Egypt
[e] College of Arts and Science, Applied Science University (ASU), East Al Ekir,
Manama, Kingdom of Bahrain

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In this article, we address a new model for the scaling law heat conduction problem by using the scaling law vector calculus associated with the Korcak scaling law. The scaling law heat conduction equations are discussed in detail. The scaling law vector calculus formulas are proposed as an efficiently mathematical tool to describe the Korcak scaling -law phenomena in heat transport system.

Key words: *scaling law heat conduction equation, Korcak scaling law, scaling law vector calculus*

Introduction

The theory of the classcial vector calculus [1] has played the important role in find ODE and PDE in the mathematical quantities of mathematical physics by the fundamental theorems, *i.e.*, Green [2] theorem, Gauss-Ostrogradski theorem [3-5], and Stokes theorem [6, 7]. There are many topics in mathematical physics, such as the electricity and magnetism [8], relativity [9], elasticity [10], heat conduction [11], fluid mechanics [12-15], *etc.*

The scaling laws in nature [16-18] are used to describe the complex behaviors in mathematical physics. For example, there are some scaling law models in mathematical physics, such as Darcy-like [19], telegraph [20], elasticity [21], and others [22].

Since there exist the scaling law behaviors of the heat conductions in the sheared granular [23] and carbon nanotube [24] materials, we have to find the best way to describe the scaling law heat conduction process. The main target of the paper is to propose the theory of the scaling law vector calculus associated with the Korcak scaling law, and to present the scaling law heat conduction problems.

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* Corresponding author, e-mail: dyangxiaojun@163.com

The scaling law vector calculus associated with the Korcak scaling law

The well-known Korcak scaling law, suggested by Korcak [18], reads:

$$\varpi(x) = \alpha x^{-D} \tag{1}$$

where $\alpha \in (0,+\infty), t \in (x,+\infty)$, and $D \in [0,+\infty)$ is the scaling exponent.

Let

$$\Re = \left\{ \Sigma_{\varpi}(x) \middle| \Sigma_{\varpi}(x) = (\Sigma \circ \varpi)(x) = \Sigma(\alpha x^{-D}) \right\} \tag{2}$$

Suppose that $\Lambda_{\varpi} \in \Re$, where $\varpi(x) = \alpha x^{-D}$.

The scaling law derivative of the function $\Lambda_{\varpi}(x)$ associated with the Korcak scaling law (2) is [19]:

$$^{\mathrm{KSL}}D_x^{(n)}\Lambda_{\varpi}(x) = \left(-\frac{x^{1+D}}{\alpha D}\frac{\mathrm{d}}{\mathrm{d}x} \right)^n \Lambda_{\varpi}(x) \tag{3}$$

The scaling law partial derivatives of $\Xi_{\varpi} = \Xi_{\varpi}(x,y,z) = \Xi(\alpha x^{-D}, \alpha y^{-D}, \alpha z^{-D})$ associated with the Korcak scaling law (2) reads [19]:

$$^{\mathrm{KSL}}\partial_x^{(1)}\Xi_{\varpi} = -\frac{t^{1+D}}{\alpha D}\frac{\partial \Xi_{\varpi}}{\partial x}, \quad ^{\mathrm{KSL}}\partial_y^{(1)}\Xi_{\varpi} = -\frac{t^{1+D}}{\alpha D}\frac{\partial \Xi_{\varpi}}{\partial y}, \quad ^{\mathrm{KSL}}\partial_z^{(1)}\Xi_{\varpi} = -\frac{t^{1+D}}{\alpha D}\frac{\partial \Xi_{\varpi}}{\partial z}$$

$$^{\mathrm{KSL}}\partial_x^{(1)}\left[^{\mathrm{KSL}}\partial_x^{(1)}\Xi_{\varpi} \right] = {}^{\mathrm{KSL}}\partial_x^{(2)}\Xi_{\varpi}, \quad ^{\mathrm{KSL}}\partial_x^{(1)}\left[^{\mathrm{KSL}}\partial_y^{(1)}\Xi_{\varpi} \right] = {}^{\mathrm{KSL}}\partial_{y,x}^{(2)}\Xi_{\varpi}$$

$$^{\mathrm{KSL}}\partial_x^{(1)}\left[^{\mathrm{KSL}}\partial_z^{(1)}\Xi_{\varpi} \right] = {}^{\mathrm{KSL}}\partial_{z,x}^{(2)}\Xi_{\varpi}$$

$$^{\mathrm{KSL}}\partial_y^{(1)}\left[^{\mathrm{KSL}}\partial_y^{(1)}\Xi_{\varpi} \right] = {}^{\mathrm{KSL}}\partial_y^{(2)}\Xi_{\varpi}, \quad ^{\mathrm{KSL}}\partial_y^{(1)}\left[^{\mathrm{KSL}}\partial_x^{(1)}\Xi_{\varpi} \right] = {}^{\mathrm{KSL}}\partial_{x,y}^{(2)}\Xi_{\varpi}$$

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$$^{\mathrm{KSL}}\partial_z^{(1)}\left[^{\mathrm{KSL}}\partial_y^{(1)}\Xi_{\varpi} \right] = {}^{\mathrm{KSL}}\partial_{y,z}^{(2)}\Xi_{\varpi}$$

The scaling law differential $\mathrm{d}\Lambda_{\varpi}(x)$ of the function $\Lambda_{\varpi}(x)$ associated with the Korcak scaling law (2) is [19]:

$$\mathrm{d}\Lambda_{\varpi}(x) = -\alpha D x^{-(D+1)}\, {}^{\mathrm{KSL}}D_x^{(1)}\Lambda_{\varpi}(x) \tag{4}$$

The scaling law integral of the function $\Theta_{\varpi}(x)$ associated with the Korcak scaling law (2) is [19]:

$$^{\mathrm{KSL}}_{\ a}I_b^{(1)}\Theta_{\varpi}(x) = (-\alpha D)\int_a^b \Theta_{\varpi}(x) x^{-(D+1)}\mathrm{d}x \tag{5}$$

The indefinite scaling law integral of the function $\Theta_{\varpi}(t)$ associated with the Korcak scaling law (2) is [19]:

$$ {}^{\mathrm{KSL}}I_x^{(1)}\Theta_{\varpi}(x) = (-\alpha D)\int \Theta_{\varpi}(x)x^{-(D+1)}\mathrm{d}x \tag{6} $$

The improper scaling law integrals of the function $\Theta_{\varpi}(t)$ associated with the Korcak scaling law (2) are:

$$ \lim_{a\to-\infty} {}^{\mathrm{KSL}}_{\ a}I_t^{(1)}\Theta_{\varpi}(x) = \lim_{a\to-\infty}\left[(-\alpha D)\int_a^x \Theta_{\varpi}(x)x^{-(D+1)}\mathrm{d}x\right] = (-\alpha D)\int_{-\infty}^x \Theta_{\varpi}(x)x^{-(D+1)}\mathrm{d}x \tag{7} $$

$$ \lim_{b\to\infty} {}^{\mathrm{KSL}}_{\ x}I_b^{(1)}\Theta_{\varpi}(x) = \lim_{b\to\infty}\left[(-\alpha D)\int_x^b \Theta_{\varpi}(x)x^{-(D+1)}\mathrm{d}x\right] = (-\alpha D)\int_x^\infty \Theta_{\varpi}(x)x^{-(D+1)}\mathrm{d}x \tag{8} $$

and

$$ \lim_{\substack{b\to\infty\\ a\to 0}} \left\{ {}^{\mathrm{KSL}}_{\ a}I_b^{(1)}\Theta_{\varpi}(x)\right\} = \lim_{\substack{b\to\infty\\ a\to 0}}\left[(-\alpha D)\int_a^b \Theta_{\varpi}(x)x^{-(D+1)}\mathrm{d}x\right] = (-\alpha D)\int_0^\infty \Theta_{\varpi}(x)x^{-(D+1)}\mathrm{d}x \tag{9} $$

For the detailed work for the scaling law calculus associated with the Korcak scaling law (2), see [19].

Theory of the scaling law vector calculus associated with the Korcak scaling law

The total scaling law differential of the Mandelbrot scaling law scalar field $\Xi_{\varpi} = \Xi_{\varpi}(x,y,z) = \Xi(\alpha x^{-D}, \alpha y^{-D}, \alpha z^{-D})$ associated with the Korcak scaling law (2) is defined by:

$$ \mathrm{d}\Xi_{\varpi} = (-\alpha D)\left\{\left[x^{-(D+1)}\,{}^{\mathrm{KSL}}\partial_x^{(1)}\Xi_{\varpi}\right]\mathrm{d}x + \left[y^{-(D+1)}\,{}^{\mathrm{KSL}}\partial_y^{(1)}\Xi_{\varpi}\right]\mathrm{d}y + \left[z^{-(D+1)}\,{}^{\mathrm{KSL}}\partial_z^{(1)}\Xi_{\varpi}\right]\mathrm{d}z\right\} \tag{10} $$

The scaling law gradient with respect to the Korcak scaling law

The scaling law gradient with respect to the Korcak scaling law (2) in a Cartesian co-ordinate system is defined by:

$$ {}^{\mathrm{KSL}}\nabla^{D} = (-\alpha D)\left[\mathbf{i}x^{-(D+1)}\,{}^{\mathrm{KSL}}\partial_x^{(1)} + \mathbf{j}y^{-(D+1)}\,{}^{\mathrm{KSL}}\partial_y^{(1)} + \mathbf{k}z^{-(D+1)}\,{}^{\mathrm{KSL}}\partial_z^{(1)}\right] \tag{11} $$

where $\mathbf{i}$, $\mathbf{j}$, and $\mathbf{k}$ are the unite vectors in a Cartesian co-ordinate system.

Thus, eq. (10) can be represented:

$$ \mathrm{d}\Xi_{\varpi} = {}^{\mathrm{KSL}}\nabla^{D}\Xi_{\varpi}\mathbf{n}\mathrm{d}r = {}^{\mathrm{KSL}}\nabla^{D}\Xi_{\varpi}\mathrm{d}\mathbf{r} \tag{12} $$

where $\mathbf{n}$ is the unit vector, dr is a distance measured along the normal direction, d$\mathbf{r}$ = $\mathbf{n}$dr = = $\mathbf{i}$dx + $\mathbf{j}$dy + $\mathbf{k}$dz with d$\mathbf{r}$ = $\mathbf{n}dr$.

The scaling law Laplace-like operator with respect to the Korcak scaling law

The scaling law Laplace-like operator of the scaling law scalar field Φ associated with the Korcak scaling law (2) is:

$$ {}^{\mathrm{KSL}}\nabla^{2D}\Xi_{\varpi} = (\alpha D)^2 \left[x^{-(D+1)\ \mathrm{KSL}} \right]^2 \partial_x^{(2)}\Xi_{\varpi} + \left[x^{-(D+1)\ \mathrm{KSL}} \right]^2 \partial_y^{(2)}\Xi_{\varpi} + \left[x^{-(D+1)\ \mathrm{KSL}} \right]^2 \partial_z^{(2)}\Xi_{\varpi} \quad (13) $$

The properties for the scaling law gradient and Laplace-like operator associated with the Korcak scaling law (2) reads:

$$ {}^{\mathrm{KSL}}\Delta^{D}\Xi_{\varpi} = {}^{\mathrm{KSL}}\nabla^{2D}\Xi_{\varpi} = \left[{}^{\mathrm{KSL}}\nabla^{D}\ {}^{\mathrm{KSL}}\nabla^{D} \right]\Xi_{\varpi} \quad (14) $$

$$ {}^{\mathrm{KSL}}\nabla^{D}(\Xi_{\varpi}\Theta_{\varpi}) = \Theta_{\varpi}\ {}^{\mathrm{KSL}}\nabla^{D}(\Xi_{\varpi}) + \Xi_{\varpi}\ {}^{\mathrm{KSL}}\nabla^{D}(\Theta_{\varpi}) \quad (15) $$

$$ {}^{\mathrm{KSL}}\nabla^{D}(\Theta_{\varpi}\ {}^{\mathrm{KSL}}\nabla^{D}\Xi_{\varpi}) = \Theta_{\varpi}\ {}^{\mathrm{KSL}}\Delta^{2D}\Xi_{\varpi} + {}^{\mathrm{KSL}}\nabla^{D}\Xi_{\varpi}\ {}^{\mathrm{KSL}}\nabla^{D}\Theta_{\varpi} \quad (16) $$

where

$$ \Xi_{\varpi} = \Xi_{\varpi}(x,y,z) = \Xi(\alpha x^{-D}, \alpha y^{-D}, \alpha z^{-D}) \quad (17) $$

and

$$ \Theta_{\varpi} = \Theta_{\varpi}(x,y,z) = \Theta(\alpha x^{-D}, \alpha y^{-D}, \alpha z^{-D}) \quad (18) $$

The element of the scaling law vector line

By using eq. (11), the element of the scaling law vector line:

$$ \mathbf{l} = \mathbf{l}(\alpha x^{-D}, \alpha y^{-D}, \alpha z^{-D}) = \mathbf{i}\ell_x + \mathbf{j}\ell_y + \mathbf{k}\ell_z \quad (19) $$

is defined by

$$ \begin{aligned} \mathrm{d}\mathbf{l} &= \mathbf{m}\mathrm{d}\ell \\ &= \mathbf{i}(-\alpha D)\left[x^{-(D+1)\ \mathrm{KSL}} \partial_x^{(1)}\ell_x \right]\mathrm{d}x + \mathbf{j}(-\alpha D)\left[y^{-(D+1)\ \mathrm{KSL}} \partial_y^{(1)}\ell_y \right]\mathrm{d}y \\ &+ \mathbf{k}(-\alpha D)\left[z^{-(D+1)\ \mathrm{KSL}} \partial_z^{(1)}\ell_z \right]\mathrm{d}z \end{aligned} \quad (20) $$

in which

$$ \mathrm{d}\ell = (\alpha D)\sqrt{\left[x^{-(D+1)\ \mathrm{KSL}} \partial_x^{(1)}\ell_x \mathrm{d}x \right]^2 + \left[y^{-(D+1)\ \mathrm{KSL}} \partial_y^{(1)}\ell_y \mathrm{d}y \right]^2 + \left[z^{-(D+1)\ \mathrm{KSL}} \partial_z^{(1)}\ell_z \mathrm{d}z \right]^2} \quad (21) $$

and

$$ \mathbf{m} = \frac{\mathrm{d}\mathbf{l}}{\mathrm{d}\ell} = \frac{\mathbf{i}\left[x^{-(D+1)\ \mathrm{KSL}} \partial_x^{(1)}\ell_x \right]\mathrm{d}x + \mathbf{j}\left[y^{-(D+1)\ \mathrm{KSL}} \partial_y^{(1)}\ell_y \right]\mathrm{d}y + \mathbf{k}\left[z^{-(D+1)\ \mathrm{KSL}} \partial_z^{(1)}\ell_z \right]\mathrm{d}z}{\sqrt{\left[x^{-(D+1)\ \mathrm{KSL}} \partial_x^{(1)}\ell_x \mathrm{d}x \right]^2 + \left[y^{-(D+1)\ \mathrm{KSL}} \partial_y^{(1)}\ell_y \mathrm{d}y \right]^2 + \left[z^{-(D+1)\ \mathrm{KSL}} \partial_z^{(1)}\ell_z \mathrm{d}z \right]^2}} \quad (22) $$

where

$$\mathrm{d}\ell_x = \frac{\mathbf{i}\left[x^{-(D+1)\ \mathrm{KSL}}\partial_x^{(1)}\ell_x\right]\mathrm{d}x}{\sqrt{\left[x^{-(D+1)\ \mathrm{KSL}}\partial_x^{(1)}\ell_x\mathrm{d}x\right]^2+\left[y^{-(D+1)\ \mathrm{KSL}}\partial_y^{(1)}\ell_y\mathrm{d}y\right]^2+\left[z^{-(D+1)\ \mathrm{KSL}}\partial_z^{(1)}\ell_z\mathrm{d}z\right]^2}} \tag{23}$$

$$\mathrm{d}\ell_y = \frac{\mathbf{j}\left[y^{-(D+1)\ \mathrm{KSL}}\partial_y^{(1)}\ell_y\right]\mathrm{d}y}{\sqrt{\left[x^{-(D+1)\ \mathrm{KSL}}\partial_x^{(1)}\ell_x\mathrm{d}x\right]^2+\left[y^{-(D+1)\ \mathrm{KSL}}\partial_y^{(1)}\ell_y\mathrm{d}y\right]^2+\left[z^{-(D+1)\ \mathrm{KSL}}\partial_z^{(1)}\ell_z\mathrm{d}z\right]^2}} \tag{24}$$

and

$$\mathrm{d}\ell_z = \frac{\mathbf{k}\left[z^{-(D+1)\ \mathrm{KSL}}\partial_z^{(1)}\ell_z\right]\mathrm{d}z}{\sqrt{\left[x^{-(D+1)\ \mathrm{KSL}}\partial_x^{(1)}\ell_x\mathrm{d}x\right]^2+\left[y^{-(D+1)\ \mathrm{KSL}}\partial_y^{(1)}\ell_y\mathrm{d}y\right]^2+\left[z^{-(D+1)\ \mathrm{KSL}}\partial_z^{(1)}\ell_z\mathrm{d}z\right]^2}} \tag{25}$$

The scaling law arc length

The scaling law arc length:

$$\ell = \int_0^{\ell}\mathrm{d}\ell$$

from $t = a$ to $t = b$ is:

$$\ell = (\alpha D)\int_a^b \sqrt{\left[x^{-(D+1)\ \mathrm{KSL}}\partial_x^{(1)}\ell_x\frac{\partial x}{\partial t}\right]^2+\left[y^{-(D+1)\ \mathrm{KSL}}\partial_y^{(1)}\ell_y\frac{\partial y}{\partial t}\right]^2+\left[z^{-(D+1)\ \mathrm{KSL}}\partial_z^{(1)}\ell_z\frac{\partial z}{\partial t}\right]^2}\,\mathrm{d}t \tag{26}$$

The scaling law vector line integral

The scaling law vector line integral of:

$$\mathbf{\Pi}_{\varpi} = \mathbf{\Pi}_{\varpi}(x,y,z) = \mathbf{\Pi}(\alpha x^{-D},\alpha y^{-D},\alpha z^{-D}) = \mathbf{i}\Pi_{\varpi,x}+\mathbf{j}\Pi_{\varpi,y}+\mathbf{k}\Pi_{\varpi,z} \tag{27}$$

associated with the Korcak scaling law (2) is defined by:

$$\int_L \mathbf{\Pi}_{\varpi}\mathrm{d}\mathbf{l} = \int_{L(t)}\mathbf{\Pi}_{\varpi}\frac{\mathrm{d}\mathbf{l}}{\mathrm{d}t}\mathrm{d}t \tag{28}$$

where

$$\frac{\mathrm{d}\mathbf{l}}{\mathrm{d}t} = \mathbf{i}x^{-(D+1)\ \mathrm{KSL}}\partial_x^{(1)}\ell_x\frac{\partial x}{\partial t}+\mathbf{j}y^{-(D+1)\ \mathrm{KSL}}\partial_y^{(1)}\ell_y\frac{\partial y}{\partial t}+\mathbf{k}z^{-(D+1)\ \mathrm{KSL}}\partial_z^{(1)}\ell_z\frac{\partial z}{\partial t} \tag{29}$$

Thus,

$$\int_L \mathbf{\Pi}_\varpi \mathrm{d}\mathbf{l} = (-\alpha D)\times\left[\int_L \Pi_{\varpi,x}\left[x^{-(D+1)\ \mathrm{KSL}}\partial_x^{(1)}\ell_x\right]\mathrm{d}x + \right.$$

$$\left. + \int_L \Pi_{\varpi,y}\left[y^{-(D+1)\ \mathrm{KSL}}\partial_y^{(1)}\ell_y\right]\mathrm{d}y + \int_L \Pi_{\varpi,z}\mathrm{d}\left[z^{-(D+1)\ \mathrm{KSL}}\partial_z^{(1)}\ell_z\right]\mathrm{d}z\right] \tag{30}$$

The scaling law volume integral

The scaling law volume integral of the Korcak-scaling law scalar field $\Xi_\varpi = \Xi_\varpi(x,y,z) = \Xi(\alpha x^{-D}, \alpha y^{-D}, \alpha z^{-D})$ is defined by:

$$\mathrm{M}(\Xi_\varpi) = \iiint_\Omega \Xi_\varpi \mathrm{d}V \tag{31}$$

in which

$$\mathrm{d}V = (-\alpha D)x^{-(D+1)}(-\alpha D)y^{-(D+1)}(-\alpha D)z^{-(D+1)}\mathrm{d}x\mathrm{d}y\mathrm{d}z =$$

$$= -(\alpha D)^3(xyz)^{-(D+1)}\mathrm{d}x\mathrm{d}y\mathrm{d}z = \mathrm{d}o(x)\mathrm{d}p(y)\mathrm{d}q(z) \tag{32}$$

where $o(x) = \alpha x^{-D}$, $p(y) = \alpha y^{-D}$, and $q(z) = \alpha z^{-0}$.

Thus, (31) can be rewritten:

$$\begin{aligned}\iiint_\Omega \Xi_\varpi \mathrm{d}V &= \int_\alpha^\beta (-\alpha D)z^{-(D+1)}\mathrm{d}z\int_c^d (-\alpha D)y^{-(D+1)}\mathrm{d}y\int_a^b \Xi_\varpi(-\alpha D)x^{-(D+1)}\mathrm{d}x \\ &= \int_c^d (-\alpha D)x^{-(D+1)}\mathrm{d}x\int_a^b (-\alpha D)z^{-(D+1)}\mathrm{d}z\int_\alpha^\beta \Xi_\varpi(-\alpha D)y^{-(D+1)}\mathrm{d}y \\ &= \int_c^d (-\alpha D)y^{-(D+1)}\mathrm{d}y\int_a^b (-\alpha D)x^{-(D+1)}\mathrm{d}x\int_\alpha^\beta \Xi_\varpi(-\alpha D)z^{-(D+1)}\mathrm{d}z \\ &= \int_a^b \mathrm{d}o(x)\int_c^d \mathrm{d}p(y)\int_\alpha^\beta \Xi_\varpi \mathrm{d}q(z),\end{aligned} \tag{33}$$

where $x \in [a,b], y \in [c,d],\ z \in [\alpha,\beta]$.

The scaling law surface integral

The scaling law surface integral of the Korcak scaling law vector field:

$$\mathbf{H}_\varpi = \mathbf{H}_\varpi(x,y,z) = \mathbf{H}(\alpha x^{-D}, \alpha y^{-D}, \alpha z^{-D})$$

is defined by:

$$\Sigma(\mathbf{H}_\varpi) = \iint_\mathbf{S} \mathbf{H}_\varpi \mathrm{d}\mathbf{S} = \iint_\mathbf{S} \mathbf{H}_\varpi \mathbf{n}\mathrm{d}S \tag{34}$$

where $\mathbf{n} = \mathrm{d}\mathbf{S}/\mathrm{d}S$ with $\mathbf{S} = \mathbf{S}(\alpha x^{-D}, \alpha y^{-D}, \alpha z^{-D})$.

We have:

$$\mathbf{n} = \frac{\mathrm{d}\mathbf{S}}{|\mathrm{d}\mathbf{S}|} = \frac{\mathrm{d}\mathbf{S}}{\mathrm{d}S}, \quad \mathrm{d}S = |\mathrm{d}\mathbf{S}| \tag{35}$$

in which

$$\mathrm{d}\mathbf{S} = \mathrm{d}p(y)\mathrm{d}q(z)\mathbf{i} + \mathrm{d}o(x)\mathrm{d}q(z)\mathbf{j} + \mathrm{d}o(x)\mathrm{d}p(y)\mathbf{k} =$$

$$= (\alpha D)^2 \left[\mathbf{i}y^{-(D+1)} z^{-(D+1)} \mathrm{d}y\mathrm{d}z + \mathbf{j}x^{-(D+1)} z^{-(D+1)} \mathrm{d}x\mathrm{d}z + \mathbf{k}x^{-(D+1)} y^{-(D+1)} \mathrm{d}x\mathrm{d}y \right] \tag{36}$$

in which

$$\mathrm{d}p(y)\mathrm{d}q(z) = \left[(-\alpha D)y^{-(D+1)} (-\alpha D)z^{-(D+1)} \right] \mathrm{d}y\mathrm{d}z = (\alpha D)^2 y^{-(D+1)} z^{-(D+1)} \mathrm{d}y\mathrm{d}z \tag{37}$$

$$\mathrm{d}o(x)\mathrm{d}q(z) = \left[(-\alpha D)x^{-(D+1)} (-\alpha D)z^{-(D+1)} \right] \mathrm{d}x\mathrm{d}z = (\alpha D)^2 x^{-(D+1)} z^{-(D+1)} \mathrm{d}x\mathrm{d}z \tag{38}$$

and

$$\mathrm{d}o(x)\mathrm{d}q(y) = \left[(-\alpha D)x^{-(D+1)} (-\alpha D)y^{-(D+1)} \right] \mathrm{d}x\mathrm{d}y = (\alpha D)^2 x^{-(D+1)} y^{-(D+1)} \mathrm{d}x\mathrm{d}y \tag{39}$$

It is easy to obtain

$$\Sigma(\mathbf{H}_\varpi) = \iint_{\mathbf{S}} \mathbf{H}_\varpi \mathrm{d}\mathbf{S} = \iint_S H_{\varpi,x}\mathrm{d}p(y)\mathrm{d}q(z) + H_{\varpi,y}\mathrm{d}o(x)\mathrm{d}q(z) + H_{\varpi,z}\mathrm{d}o(x)\mathrm{d}p(x) \tag{40}$$

where

$$\mathbf{H}_\varpi = \mathbf{H}_\varpi(x,y,z) = \mathbf{H}(\alpha x^{-D}, \alpha y^{-D}, \alpha z^{-D}) = \mathbf{i}H_{\varpi,x} + \mathbf{j}H_{\varpi,y} + \mathbf{k}H_{\varpi,z} \tag{41}$$

It is also easy to find that:

$$\iint_S H_{\varpi,z}\mathrm{d}S = \iint_S H_{\varpi,z}\mathrm{d}o(x)\mathrm{d}p(y) = (\alpha D)^2 \int_a^b \left[\int_c^d y^{-(D+1)} H_{\varpi,z}\mathrm{d}y \right] x^{-(D+1)}\mathrm{d}x \tag{42}$$

The scaling law divergence

Let:

$$\Delta S = \Delta o(x)\Delta p(y) = (\alpha D)^2 x^{-(D+1)} y^{-(D+1)} \Delta x \Delta y \tag{43}$$

and

$$\Delta V = (-\alpha D)x^{-(D+1)}(-\alpha D)y^{-(D+1)}(-\alpha D)z^{-(D+1)}\Delta x\Delta y\Delta z = \Delta o(x)\Delta p(y)\Delta q(z) \tag{44}$$

The scaling law divergence of the Korcak scaling law vector field $\mathbf{H}$ is defined by:

$$^{\mathrm{KSL}}\nabla^{D}\mathbf{H}_{\varpi}=\lim_{\Delta V_m\to 0}\frac{1}{\Delta V_m}\oiint_{\Delta\mathbf{S}_m}\mathbf{H}_{\varpi}\mathrm{d}\mathbf{S} \tag{45}$$

where the Korcak scaling law volume V is divided into a large number of small sub-volumes ΔV_m with the Korcak scaling law surfaces $\Delta\mathbf{S}_m$ and d$\mathbf{S}$ is the element of the Korcak scaling law surface $\mathbf{S}$.

The scaling law divergence (27) associated with the Korcak scaling law (2) in a Cartesian co-ordinate system is defined:

$$^{KSL}\nabla^{D}\mathbf{H}_{\varpi}=(-\alpha D)\left[\mathbf{i}x^{-(D+1)}\,{}^{KSL}\partial_x^{(1)}H_{\varpi,x}+\mathbf{j}y^{-(D+1)}\,{}^{KSL}\partial_y^{(1)}H_{\varpi,y}+\mathbf{k}z^{-(D+1)}\,{}^{KSL}\partial_z^{(1)}H_{\varpi,z}\right] \tag{46}$$

where

$$\mathbf{H}_{\varpi}=\mathbf{H}_{\varpi}(x,y,z)=\mathbf{H}(\alpha x^{-D},\alpha y^{-D},\alpha z^{-D})=\mathbf{i}H_{\varpi,x}+\mathbf{j}H_{\varpi,y}+\mathbf{k}H_{\varpi,z}$$

The scaling law curl

The scaling law curl of the Korcak-scaling law vector field $\mathbf{H}_{\varpi}$ is defined:

$$^{KSL}\nabla^{D}\times\mathbf{H}_{\varpi}=\lim_{\Delta V_m\to 0}\frac{1}{\Delta V_m}\oiint_{\Delta\mathbf{S}_m}\mathbf{H}_{\varpi}\times\mathrm{d}\mathbf{S} \tag{47}$$

where the Korcak-scaling law volume V is divided into a large number of small sub-volumes ΔV_m with the Korcak-scaling law surfaces $\Delta\mathbf{S}_m$, and d$\mathbf{S}$ *is the element of the Korcak-scaling law surface $\mathbf{S}$.

The scaling law curl (29) associated with the Korcak scaling law (2) in a Cartesian co-ordinate system can be expressed:

$$^{\mathrm{KSL}}\nabla^{D}\times\mathbf{H}_{\varpi}=\begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ (-\alpha D)x^{-(D+1)}\,{}^{\mathrm{KSL}}\partial_x^{(1)} & (-\alpha D)y^{-(D+1)}\,{}^{\mathrm{KSL}}\partial_y^{(1)} & (-\alpha D)z^{-(D+1)}\,{}^{\mathrm{KSL}}\partial_z^{(1)}\\ H_{\varpi,x} & H_{\varpi,y} & H_{\varpi,z}\end{vmatrix} \tag{48}$$

where

$$\mathbf{H}_{\varpi}=\mathbf{H}_{\varpi}(x,y,z)=\mathbf{H}(\alpha x^{-D},\alpha y^{-D},\alpha z^{-D})=\mathbf{i}H_{\varpi,x}+\mathbf{j}H_{\varpi,y}+\mathbf{k}H_{\varpi,z}$$

The Green-like theorem for the scaling law vector calculus

The Green-like theorem for the scaling law vector calculus associated with the Korcak scaling law (2) reads:

$$\oint_L\mathbf{T}_{\varpi}\mathrm{d}\mathbf{l}=-(\alpha D)^3\iint_S\left[x^{-(D+1)}\,{}^{\mathrm{KSL}}\partial_x^{(1)}Q_{\varpi}-y^{-(D+1)}\,{}^{\mathrm{KSL}}\partial_y^{(1)}P_{\varpi}\right](xy)^{-(D+1)}\mathrm{d}x\mathrm{d}y \tag{49}$$

or alternatively:

$$\oint_L P_{\varpi}\left[x^{-(D+1)}\,{}^{\mathrm{KSL}}\partial_x^{(1)}\ell_x\right]\mathrm{d}x+Q_{\varpi}\left[y^{-(D+1)}\,{}^{\mathrm{KSL}}\partial_y^{(1)}\ell_y\right]\mathrm{d}y=(\alpha D)^2\cdot$$

$$\cdot\iint_S\left[x^{-(D+1)}\,{}^{\mathrm{KSL}}\partial_x^{(1)}Q_\varpi - y^{-(D+1)}\,{}^{\mathrm{KSL}}\partial_y^{(1)}P_\varpi\right](xy)^{-(D+1)}\mathrm{d}x\mathrm{d}y \tag{50}$$

where

$$\mathbf{T}_\varpi = \mathbf{i}P_\varpi + \mathbf{j}Q_\varpi \tag{51}$$

$$\mathrm{d}\mathbf{l} = (-\alpha D)\left\{\mathbf{i}\left[x^{-(D+1)}\,{}^{KSL}\partial_x^{(1)}\ell_x\right]\mathrm{d}x + \mathbf{j}\left[y^{-(D+1)}\,{}^{KSL}\partial_y^{(1)}\ell_y\right]\mathrm{d}y\right\} \tag{52}$$

$$\mathrm{d}S = \left[(-\alpha D)x^{-(D+1)}\right]\left[(-\alpha D)y^{-(D+1)}\right]\mathrm{d}x\mathrm{d}y = \mathrm{d}o(x)\mathrm{d}p(x) \tag{53}$$

and S is a scaling law domain bounded by a scaling law contour L.

The Gauss-Ostrogradsky-like theorem for the scaling law vector calculus

The Gauss-Ostrogradsky-like theorem for the scaling law vector calculus associated with the Korcak scaling law (2) is represented by:

$$\iiint_\Omega {}^{\mathrm{KSL}}\nabla^D\boldsymbol{\psi}_\varpi\mathrm{d}V = \oint\!\!\oint_{\mathbf{S}}\boldsymbol{\psi}_\varpi\mathrm{d}\mathbf{S} \tag{54}$$

where

$$\boldsymbol{\psi}_\varpi = \boldsymbol{\psi}_\varpi(x,y,z) = \boldsymbol{\psi}(\alpha x^{-D},\alpha y^{-D},\alpha z^{-D}) = \mathbf{i}\,\psi_{\varpi,x} + \mathbf{j}\,\psi_{\varpi,y} + \mathbf{k}\psi_{\varpi,z} \tag{55}$$

$$\mathrm{d}V = \mathrm{d}o(x)\mathrm{d}p(x)\mathrm{d}q(x) = -(\alpha D)^3(xyz)^{-(D+1)}\mathrm{d}x\mathrm{d}y\mathrm{d}z \tag{56}$$

and

$$\mathrm{d}\mathbf{S} = \mathrm{d}p(y)\mathrm{d}q(z)\mathbf{i} + \mathrm{d}o(x)\mathrm{d}q(z)\mathbf{j} + \mathrm{d}o(x)\mathrm{d}p(y)\mathbf{k} =$$

$$= (\alpha D)^2\left[\mathbf{i}y^{-(D+1)}z^{-(D+1)}\mathrm{d}y\mathrm{d}z + \mathbf{j}x^{-(D+1)}z^{-(D+1)}\mathrm{d}x\mathrm{d}z + \mathbf{k}x^{-(D+1)}y^{-(D+1)}\mathrm{d}x\mathrm{d}y\right] \tag{57}$$

The Stokes like theorem for the scaling law vector calculus

The Stokes like theorem for the scaling law vector calculus associated with the Korcak scaling law (2) is expressed:

$$\iint_{\mathbf{S}}\left({}^{\mathrm{KSL}}\nabla^D\times\boldsymbol{\psi}_\varpi\right)\mathrm{d}\mathbf{S} = \oint_L\boldsymbol{\psi}_\varpi\mathrm{d}\mathbf{l} \tag{58}$$

where

$$\mathrm{d}\mathbf{S} = \mathrm{d}p(y)\mathrm{d}q(z)\mathbf{i} + \mathrm{d}o(x)\mathrm{d}q(z)\mathbf{j} + \mathrm{d}o(x)\mathrm{d}p(y)\mathbf{k} =$$

$$= (\alpha D)^2\left[\mathbf{i}y^{-(D+1)}z^{-(D+1)}\mathrm{d}y\mathrm{d}z + \mathbf{j}x^{-(D+1)}z^{-(D+1)}\mathrm{d}x\mathrm{d}z + \mathbf{k}x^{-(D+1)}y^{-(D+1)}\mathrm{d}x\mathrm{d}y\right] \tag{59}$$

$$\mathrm{d}\mathbf{l}=\mathbf{i}(-\alpha D)\left[x^{-(D+1)}\,{}^{\mathrm{KSL}}\partial_x^{(1)}\ell_x\right]\mathrm{d}x+\mathbf{j}(-\alpha D)\left[y^{-(D+1)}\,{}^{\mathrm{KSL}}\partial_y^{(1)}\ell_y\right]\mathrm{d}y+$$
$$\mathbf{k}(-\alpha D)\left[z^{-(D+1)}\,{}^{\mathrm{KSL}}\partial_z^{(1)}\ell_z\right]\mathrm{d}z \tag{60}$$

and

$$\boldsymbol{\psi}_\varpi=\boldsymbol{\psi}_\varpi(x,y,z)=\boldsymbol{\psi}(\alpha x^{-D},\alpha y^{-D},\alpha z^{-D})=\mathbf{i}\,\psi_{\varpi,x}+\mathbf{j}\,\psi_{\varpi,y}+\mathbf{k}\,\psi_{\varpi,z} \tag{61}$$

The Green like identities

Taking $\Phi_\varpi=\Theta_\varpi\,{}^{\mathrm{KSL}}\nabla^D\Phi_\varpi$ such that:

$${}^{\mathrm{KSL}}\nabla^D(\Theta_\varpi\,{}^{\mathrm{KSL}}\nabla^D\Phi_\varpi)=\Theta_\varpi\,{}^{\mathrm{KSL}}\Delta^D\Phi_\varpi+{}^{\mathrm{KSL}}\nabla^D\Phi_\varpi\,{}^{\mathrm{KSL}}\nabla^D\Theta_\varpi \tag{62}$$

and

$${}^{\mathrm{KSL}}\nabla^D(\Phi_\varpi\,{}^{\mathrm{KSL}}\nabla^D\Theta_\varpi)=\Phi_\varpi\,{}^{\mathrm{KSL}}\Delta^D\Theta_\varpi+{}^{\mathrm{KSL}}\nabla^D\Phi_\varpi\,{}^{\mathrm{KSL}}\nabla^D\Theta_\varpi \tag{63}$$

where

$$\Phi_\varpi=\Phi_\varpi(x,y,z)=\Phi(\alpha x^{-D},\alpha y^{-D},\alpha z^{-D}) \tag{64}$$

and

$$\Theta_\varpi=\Theta_\varpi(x,y,z)=\Theta(\alpha x^{-D},\alpha y^{-D},\alpha z^{-D}) \tag{65}$$

Let us denote:

$${}^{\mathrm{KSL}}\nabla^D\Phi_\varpi\mathbf{n}=\partial_n^{(D)}\Phi_\varpi \tag{66}$$

With the aid of (54) and (62), the Green like identity of first type can be given:

$$\iiint\limits_\Omega {}^{\mathrm{KSL}}\nabla^D(\Theta_\varpi\,{}^{\mathrm{KSL}}\Delta^D\Phi_\varpi+{}^{\mathrm{KSL}}\nabla^D\Phi_\varpi\,{}^{\mathrm{KSL}}\nabla^D\Theta_\varpi)\mathrm{d}V=\oint\!\!\!\oint\limits_{\mathbf{S}}\Theta_\varpi({}^{\mathrm{KSL}}\nabla^D\Phi_\varpi\mathbf{n})\mathrm{d}S=$$
$$=\oint\!\!\!\oint\limits_{\mathbf{S}}\Theta_\varpi\partial_n^{(D)}\Phi_\varpi\mathrm{d}S \tag{67}$$

In a similar way of (66), we obtain:

$$\iiint\limits_\Omega {}^{\mathrm{KSL}}\nabla^D(\Phi_\varpi\,{}^{\mathrm{KSL}}\Delta^D\Theta_\varpi+{}^{\mathrm{KSL}}\nabla^D\Phi_\varpi\,{}^{\mathrm{KSL}}\nabla^D\Theta_\varpi)\mathrm{d}V=\oint\!\!\!\oint\limits_{\mathbf{S}}\Phi_\varpi({}^{\mathrm{KSL}}\nabla^D\Theta_\varpi\mathbf{n})\mathrm{d}S=$$
$$=\oint\!\!\!\oint\limits_{\mathbf{S}}\Phi_\varpi\partial_n^{(D)}\Theta_\varpi\mathrm{d}S \tag{68}$$

which reduces to the Green like identity of second type:

$$\iiint\limits_\Omega {}^{\mathrm{KSL}}\nabla^D(\Theta_\varpi\,{}^{\mathrm{KSL}}\Delta^D\Phi_\varpi-\Phi_\varpi\,{}^{\mathrm{KSL}}\Delta^D\Theta_\varpi)\mathrm{d}V=\oint\!\!\!\oint\limits_{\mathbf{S}}\left[\Theta_\varpi\partial_n^{(D)}\Phi_\varpi-\Phi_\varpi\partial_n^{(D)}\Theta_\varpi\right]\mathbf{n}\mathrm{d}S \tag{69}$$

or alternatively:

$$\iiint\limits_{\Omega} {}^{\mathrm{KSL}}\nabla^{D}(\Theta_{\varpi}\,{}^{\mathrm{KSL}}\Delta^{D}\Phi_{\varpi}-\Phi_{\varpi}\,{}^{\mathrm{KSL}}\Delta^{D}\Theta_{\varpi})\mathrm{d}V=\oiint\limits_{\mathbf{S}}(\Theta_{\varpi}\,{}^{\mathrm{KSL}}\nabla^{D}\Phi_{\varpi}-\Phi_{\varpi}\,{}^{\mathrm{KSL}}\nabla^{D}\Theta_{\varpi})\mathrm{d}\mathbf{S} \quad (70)$$

Modelling a scaling law heat conduction problem

Suppose that $T(x, y, z, t)$ is the scaling law temperature field.

The First law of thermodynamics reads:

$$\mathbb{F}_1(x,y,z,t)+\mathbb{F}_2(x,y,z,t)=0 \quad (71)$$

in which

$$\mathbb{F}_1(x,y,z,t)=\oiint\limits_{S}\mathbf{q}(x,y,z,t)\mathrm{d}\mathbf{S} \quad (72)$$

is the heat entering unit time, and:

$$\mathbb{F}_2(x,y,z,t)=\iiint\limits_{V}\rho c\,{}^{\mathrm{MSL}}\partial_t^{(1)}T(x,y,z,t)\mathrm{d}V \quad (73)$$

is the change unit time in storage energy, where ρ is the density and c is the specific heat.

We now define the non-Fourier law, given by:

$$\mathbf{q}(x,y,z,t)=-\mathfrak{A}\,{}^{\mathrm{KSL}}\nabla^{D}T(x,y,z,t) \quad (74)$$

where $\mathfrak{A}$ is the thermal conductivity.

The First law of thermodynamics implies that (71) is rewritten:

$$\iiint\limits_{V}\rho c\,{}^{\mathrm{MSL}}\partial_t^{(1)}T(x,y,z,t)\mathrm{d}V+\oiint\limits_{S}\mathbf{q}(x,y,z,t)\mathrm{d}\mathbf{S}=0 \quad (75)$$

By using (71), we find that:

$$\begin{aligned}\oiint\limits_{S}\mathbf{q}(x,y,z,t)\mathrm{d}\mathbf{S}&=-\oiint\limits_{S}\mathfrak{A}\,{}^{\mathrm{KSL}}\nabla^{D}T(x,y,z,t)\mathrm{d}\mathbf{S}\\&=-\oiint\limits_{S}\mathfrak{A}\,{}^{\mathrm{KSL}}\nabla^{D}T(x,y,z,t)\mathrm{d}\mathbf{S}\\&=-\iiint\limits_{V}{}^{\mathrm{KSL}}\nabla^{D}\left[\mathfrak{A}\,{}^{\mathrm{KSL}}\nabla^{D}T(x,y,z,t)\right]\mathrm{d}V\end{aligned} \quad (76)$$

Collecting (75) and (76), we have:

$$\iiint\limits_{V}\rho c\,{}^{\mathrm{MSL}}\partial_t^{(1)}T(x,y,z,t)\mathrm{d}V-\iiint\limits_{V}{}^{\mathrm{KSL}}\nabla^{D}\left[\mathfrak{A}\,{}^{\mathrm{KSL}}\nabla^{D}T(x,y,z,t)\right]\mathrm{d}V=0 \quad (77)$$

which leads to:

$$\iiint\limits_{V}\left[\rho c\,{}^{\mathrm{MSL}}\partial_t^{(1)}T(x,y,z,t)-{}^{\mathrm{KSL}}\nabla^{D}\left[\mathfrak{A}\,{}^{\mathrm{KSL}}\nabla^{D}T(x,y,z,t)\right]\right]\mathrm{d}V=0 \quad (78)$$

This implies that:

$$\rho c\, {}^{\mathrm{MSL}}\partial_t^{(1)} T(x,y,z,t) - {}^{\mathrm{KSL}}\nabla^D \left[\mathfrak{A}\, {}^{\mathrm{KSL}}\nabla^D T(x,y,z,t) \right] = 0 \tag{79}$$

When $\mathfrak{A}$ is a constant, we get:

$$\rho c\, {}^{\mathrm{MSL}}\partial_t^{(1)} T(x,y,z,t) - \mathfrak{A}({}^{\mathrm{KSL}}\nabla^D\, {}^{\mathrm{KSL}}\nabla^D) T(x,y,z,t) = 0 \tag{80}$$

or alternatively:

$$\rho c\, {}^{\mathrm{MSL}}\partial_t^{(1)} T(x,y,z,t) - \mathfrak{A}\, {}^{\mathrm{KSL}}\nabla^{2D} T(x,y,z,t) = 0 \tag{81}$$

Thus:

$${}^{\mathrm{MSL}}\partial_t^{(1)} T(x,y,z,t) = \frac{\mathfrak{A}}{\rho c}\, {}^{\mathrm{KSL}}\nabla^{2D} T(x,y,z,t) \;\; (x,y,z,t) \tag{82}$$

From (82), we obtain:

$${}^{\mathrm{MSL}}\partial_t^{(1)} T(x,t) = \frac{\mathfrak{A}}{\rho c}\, {}^{\mathrm{KSL}}\partial_x^{(1)} \left[{}^{\mathrm{KSL}}\partial_x^{(1)} T(x,t) \right] \tag{83}$$

subject to the initial condition:

$$T(0,t) = J_1(t) \tag{84}$$

and boundary conditions:

$$T(0,t) = J_2(t) \tag{85}$$

And:

$$\lim_{x\to\infty} T(x,t) = 0 \tag{86}$$

Conclusion

In the present work we have proposed the scaling law vector calculus associated with the Korcak scaling law. The scaling law heat conduction equation associated with the Korcak scaling law was suggested in detail. The obtained result is proposed as a efficiently mathematical tool to give the scaling law behaviors of the solid meterials in complex media.

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Nomenclature

c – specific heat capacity, [Jkg^{-1}K^{-1}]
$\mathbf{q}(x, y, z, t)$ – local heat flux density, [W]
t – time, [s]
x, y, z – co-ordinates, [m]
$T(x, y, z, t)$ – temperature field, [K]
ρ – density, [kgm^{-3}]
$\mathfrak{A}$ – heat conductivity, [Wm^{-1}K^{-1}]

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