NUMERICAL STUDY OF THE LAMINAR NATURAL CONVECTION HEAT TRANSFER FROM THREE ATTACHED HORIZONTAL ISOTHERMAL CYLINDERS

by

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Based on the model by Bejan et al., natural convection heat transfer, from three attached horizontal isothermal cylinders immersed in quiescent air in an inverted triangular array, has been numerically investigated over $10 \leq Ra \leq 10^6$. The representative results made up of streamlines and isothermal contours, local and average Nusselt number have been showed. It can be seen there exist several vortexes in the wake region of the downstream cylinders because of the strong interaction of two merging plumes. Additionally, with the effect of the vortex and the preheating among three attached cylinders, the average heat transfer rate for the whole configuration has been reduced by about 38.7~58.5% compared with single cylinder. Finally, average Nusselt number for this configuration has been correlated with Rayleigh number, which provides a valid prediction for engineering calculation including this kind of fundamental configuration.

Keywords*: natural convection, numerical simulation, three attached cylinders, Nusselt number, correlating equation*

Introduction

Natural convection widely exists in our daily life, which is a common flow phenomenon where fluid motion is induced by the buoyancy force generated by both the density gradient and the body force. Like some classic textbooks and research literature about heat transfer, the density gradient is mainly owing to the temperature gradient as well as the body force is due to gravitation [1]. There are many applications relevant to natural convection such as heat exchangers, solar collectors, safe operation about nuclear reactors, as well as high voltage transmission lines and the cooling of electronic devices [2-4].

In the early stage for the research on free convection from cylinder array, most attention was laid to convective heat transfer from an individual cylinder [5-11]. As moderate supplements and modifications to his previous summarizations [6, 9], Morgan [10] further

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reviewed the results from 34 experimental studies on heat transfer from a level isothermal cylinder immersed in air for $10 \leq \mathrm{Ra} \leq 10^7$. Boetcher *et al*. [11] summarized the correlating equations, dating from the late 19$^{\mathrm{th}}$ century, of the average Nusselt number for one horizontal, vertical and inclined cylinder as the function of the Grashof and Prandtl numbers in his book. In addition, the free convection from single cylinder in the confined space is also widely investigated. Recently, Fallah *et al*. [12] used the lattice Boltzmann method to compute the heat transfer of nanofluids in a concentric annulus for $10^3 \leq \mathrm{Ra} \leq 10^5$. Several different influencing factors including annulus gap width ratio and Rayleigh number are discussed. Ansari *et al*. [13] performed an experimental study about the effect of the vertical channel on it. It was reported that $\overline{\mathrm{Nu}}$ of the cylinder was increased when there existed the channel with a width ratio of 2 compared to that without it around the cylinder.

In practical situations, it is more frequently seen that several cylinders are laid out in vertical, inclined and horizontal set. Under these arrangements, heat transfer from every cylinder is mutually affected and that for the whole array cannot be deduced and calculated by simply superposing individual cylinder behavior [14]. Consequently, the fundamental layout with representation of the single flat array of two, or more level cylinders is widely researched, both by experimental investigation and numerical simulation, to have a full understanding of how the temperature field interacts with the buoyant flow field induced by the temperature gradients around the cylinders, and a better prediction for its thermal performance [3, 14-24]. For several horizontal cylinders vertically being aligned, the factors of the number of cylinders, N, the Rayleigh number, and cylinder spacing, S, on natural convection heat transfer are extensively investigated [2-4, 14, 18-20, 22]. Marsters [14] experimentally investigated 3, 5, and 9 cylinders in air for $750 \leq \mathrm{Gr} \leq 2000$ and $2 \leq S/D \leq 20$. Results showed that for closed spaced arrays, $\overline{\mathrm{Nu}}$ for each tube was reduced by as much as 50% compared with single one, while that was increased by up to about 30% for large spacing. The array's $\overline{\mathrm{Nu}}$ with respect to Rayleigh number, S/D, and N was correlated by Sadeghipour and Asheghi [18] for $500 \leq \mathrm{Ra} \leq 700$, $3.5 \leq S/D \leq 27.5$, and $2 \leq N \leq 8$. Chouikh *et al*. [19, 20] numerically and experimentally studied two cylinders for $10^2 \leq \mathrm{Ra} \leq 10^4$ and $2 \leq S/D \leq 6$. It was found that there existed critical cylinder spacing strongly dependent upon the Rayleigh number. Besides, there are some investigations relevant to the effect of the level spacing or both the level and upright spacing on heat transfer [15-17, 21]. The more detailed introduction for these literatures could be referenced to two papers by Liu *et al*. [23, 24]. Additionally, the triangular array with cylinder spacing is also got some attention. Gibbons *et al*. [25] experimentally investigated heat transfer from it in water with three different cylinder spacings (S/D = 2, 3, and 4) for $\mathrm{Ra} = 2 \cdot 10^6$, $4 \cdot 10^6$, and $6 \cdot 10^6$. It was observed that the plume generated from lower cylinders oscillates. O'Shaughnessy *et al*. [26] researched heat transfer of the above configuration by numerical simulation, and found reasonable agreement with the experimental data. Narayan *et al*. [27] used the method of interferometric measurement to study the heat transfer from three cylinders with a triangular configuration for the range of $400 \leq \mathrm{Ra} \leq 625$ and $2.5 \leq S/D \leq 4.25$. Experiments revealed that the heat transfer of top cylinder was strongly affected by the bottom ones.

Based on the aforementioned research overviews, it could be found the researches on the extreme configuration of several horizontal cylinders, *i.e*. several cylinders come into contact with each other, are relatively few, where the interaction among them is stronger and heat transfer from the overall configuration cannot be predicted by simply superposing that from single cylinder. Besides, such configuration can be usually seen in many gun barrels, *e.g*. three tube shotgun, antiaircraft as well as naval gun [23, 24], but there are no correlations proposed for forecasting the flow and heat transfer features as well as further guiding the structural design and optimum. As always, it is a significant topic in free convection to acquire dimensionless heat transfer correlating equations, either for the single [5-7, 9-11] or multiple cylinder(s) [3, 14, 18, 21, 23, 24, 28-30]. When realizing such kind of lack for researches on the attached cylinders, Liu *et al*. [23, 24] numerically carried out free convection around two in-contact level isothermal cylinders in air under two different configurations, vertically aligned and horizontally aligned, for $10 \le \mathrm{Ra} \le 10^5$. The similar researches were also conducted by Shi *et al*. [28] and Rath and Dash [29, 30]. Considering the configuration with one cylinder below two horizontally touched cylinders, *i.e*. in an inverted triangular array, is one of the most common and fundamental structure in engineering, the research for it is quite significant to investigate the effect of the preheating of upstream cylinder for downstream ones, and the obtained correlating equation has a great guidance to engineering calculation including this fundamental structure. In addition, the computational model used herein is high efficient, resource-saving and greatly precise, which will be explained in next section.

In summary, natural convection heat transfer in free space, for three in-contact horizontal isothermal cylinders immersed in quiescent air in an inverted triangular array, will been numerically investigated over $10 \le \mathrm{Ra} \le 10^6$. The representative results composed of streamlines and isothermal contours, local and average Nusselt number will be illustrated for different Rayleigh numbers. Finally, the correlating equation of $\overline{\mathrm{Nu}}$ with Rayleigh number for whole configuration will be obtained and some comparison will also be made with the previous results of single or multiple cylinder(s).

Problem description and numerical methodology

Think about natural convection heat transfer in the laminar regime from three attached long level cylinders with a cylinder below two horizontally touched cylinders, the surface temperature of which always keeps the constant value T_w and put in the broad and quiescent air with a constant temperature T_∞ ($<T_w$). On account of the existing of temperature gap, the air surrounding these concerned cylinders is heated, which will make its density reduced thereby inducing an upward buoyant flow. Then, natural convection between cylinders' array and air takes place. To analyze the flow and heat transfer appearing from the previous process, these attached cylinders are commonly laid in a sufficiently broad region in order not to make its boundary influence convective heat transfer from these objects. However, such traditional models will make the computational domain very large hence resulting in the great amount of grids. Nevertheless, a new model put forward by Bejan [31] could significantly reduce the domain size and improve the computational efficiency by setting the symmetry boundary for the outer boundary of its domain, as represented in fig. 1.

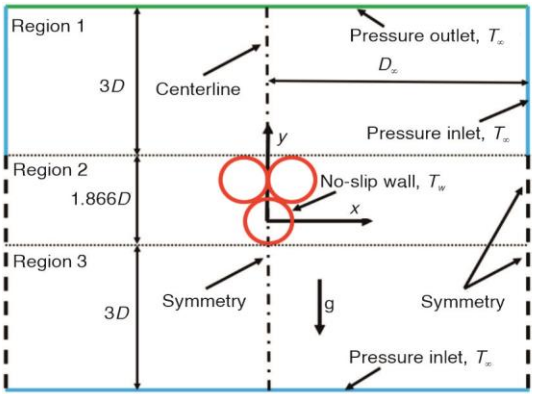

Figure 1. Schematic representation of the computational domain

In fig. 1, the origin of Cartesian co-ordinate system (x, y) is at the center of the bottom cylinder, and the gravitation is in the direction of negative y-axis. The thermophysical properties of air is evaluated at the constant film temperature $T_m = (T_w + T_\infty)/2$, except the density in the y-direction of momentum equation. Its variation with temperature is obtained by the Boussinesq approximation [1], *i.e.*:

$$\rho_\infty - \rho = \rho\beta\left(T - T_\infty\right) \tag{1}$$

For ensuring the validity of Boussinesq approximation and obtaining the precise results, the temperature differences, *i.e.* $\Delta T = T_w - T_\infty$, are set as around 1 K for all cases. Therefore, the thermal expansion coefficient, β, is able to be approximated as $\beta = 1/T_m$ by the ideal gas law. Meanwhile, the fluid is assumed to be laminar, incompressible and steady-state. Hence, the viscous dissipation in energy equation could be ignored. Correspondingly, the simplification form of the governing equations could be expressed [23, 24]:

Continuity equation:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{2}$$

Momentum equations:

$$x\text{-direction} \quad u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\nabla^2 u \tag{3}$$

$$y\text{-direction} \quad u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial y} + \nu\nabla^2 v + \mathrm{g}\beta\left(T - T_\infty\right) \tag{4}$$

Energy equation:

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\nabla^2 T \tag{5}$$

On account of the symmetry of flow, only right half computational domain is chosen to calculate. The boundary conditions for it have been demonstrated in fig. 1, and for the corresponding mathematic descriptions of it, please refer to these two literatures [23, 24].

The aforementioned governing equations satisfying the given boundary conditions, are iterately solved after being discretized by the finite volume method. The SIMPLE algorithm is applied to the pressure-velocity coupling. With regard to the spatial discretization, the least squares cell based method and second order scheme are used for the gradient and pressure terms. the high-precision second order upwind scheme is adopted to deal with the convection terms in the momentum and energy equations. Additionally, the convergence criterions of governing equations are set to the order of 10^{-6}. Meanwhile, several key parameters such as $\overline{\mathrm{Nu}}$, C_P, C_D are monitored to judge the convergence of numerical iterative solutions.

In the post-processing stage, the variables of position and temperature showing in the result section are non-dimensionalized as:

$$X = \frac{x}{D},\ Y = \frac{y}{D},\ T^* = \frac{T - T_\infty}{T_w - T_\infty} \tag{6}$$

For easily understanding, several relevant dimensionless numbers are defined briefly as:

– Rayleigh number:

$$\mathrm{Ra} = \mathrm{GrPr} = \frac{\mathrm{g}\beta\Delta T D^3}{\nu\alpha} \tag{7}$$

where the Grashof number is equal to $(\mathrm{g}\beta\Delta T D^3)/\nu^2$ which is the ratio of the buoyancy forces and viscous forces on the fluid. The Prandtl number is defined as $\mathrm{Pr} = \nu/\alpha$ which is the ratio of the momentum and thermal diffusivities, and its value is 0.72 herein for air. The product of both, *i.e.* Rayleigh number, is used to measure the strength of flow and heat transfer in natural convection.

– The local and average Nusselt numbers:

$$\mathrm{Nu}_\theta = \frac{hD}{k} = \left(\frac{\partial T^*}{\partial \vec{n}}\right)_s \tag{8}$$

$$\overline{\mathrm{Nu}} = \frac{1}{s}\int_s \mathrm{Nu}_\theta \mathrm{d}s \tag{9}$$

The local Nusselt number reflects the capacity of convective heat transfer at a given point on the cylinders' surface. The average Nusselt number is obtained by integrating local Nusselt number along the cylinders' surface. Due to its wide use in engineering application, it is our main aim to compute its value as well as getting related correlation.

– Drag coefficient:

$$C_D = C_F + C_P \tag{10}$$

The drag coefficient, C_D, indicates the drag of three attached cylinders in air. It is equal to the friction drag coefficient, C_F, plus pressure drag coefficient, C_P. Both are expressed as:

$$C_F = \frac{1}{0.5\rho u_{\mathrm{ref}}^2 D}\int_s \tau\left(\vec{t}\cdot\vec{i}\right)\mathrm{d}s \tag{11}$$

$$C_P = \frac{1}{0.5\rho u_{\mathrm{ref}}^2 D}\int_s \left(p - p_\infty\right)\left(\vec{n}\cdot\vec{i}\right)\mathrm{d}s \tag{12}$$

Before performing the simulations in this work, the domain grid independence test is necessary. According to the investigation by Bejan [31], it is viable to set the lengths of the inlet and outlet part as 3D for all Rayleigh number studied here. The width of computational domain, D_∞, is validated at the smallest Rayleigh number (Ra = 10) because the velocity and thermal boundary-layers are the thickest, and the affected region is the largest therefore. As listed in tab. 1, when the domain width is doubled, the relative errors of C_P, C_D, and $\overline{\mathrm{Nu}}$ are all less than 1.6%, showing that the width D_∞ = 10D meets the requirements. It is worth mentioning that the area of this domain is around 79D^2(≈7.9D × 10D), while in the traditional models, it is about 628D^2(≈π/2 × 20D × 20D) [8], even up to 565486D^2(≈π/2 × 600D × 600D) [32], which suggests the new model can in a large extent reduce the computational efforts.

Table 1. Domain independence test at Ra = 10

Domain width			C_P	C_D	$\overline{\mathrm{Nu}}$
Inlet section	Outlet section	D_∞			
3D	3D	10D	4.4933	7.4829	0.7349
6D	6D	20D	4.5702	7.5650	0.7293

Meanwhile, the grid independence test is also indispensable. It is clear that the interaction between these cylinders and surrounding plume is strong, and the velocity as well as temperature gradients are much steep near the cylinders' array. Hence, a fine grid is needed to obtain the accurate result. However, the interaction becomes gentle for the place far away from the concerned cylinders, thus using a relative coarse grid is enough for avoiding waste of computing resources. Therefore, the whole area is consisted of two regions: Region I and II, as schematically shown in fig. 2. Generally, the thicknesses of the velocity and temperature boundary-layers both decrease with the increasing Rayleigh number. Correspondingly, the Region I corresponds to the area which is widened a distance of $2D$ for Ra = 10 and 10^2, $1.5D$ for Ra = 10^3 and 10^4 as well as $1.0D$ for Ra = 10^5 and 10^6 in the level and upright directions from the cylinder boundary, respectively. In this region, the momentum as well as thermal boundary-layers is well covered, and a fine mesh is used. The rest part is Region II where a relative coarse grid is meshed. Contrary to the domain independence test, the grid independence one is carried out at the largest Rayleigh number (Ra = 10^6) since the momentum as well as thermal boundary-layers are thinnest and a finer mesh is required to better capture the stronger variations of velocity and temperature. As represented in tab. 2, when the smallest cell size δ/D halves and the total number of cells doubles, the relative derivatives of C_P, C_D, $\overline{\text{Nu}}$ are all less than 0.9%. Hence, it is feasible to use the grid arrangement of Grid 1 whose overall number of cells and the ratio δ/D is 652044 and 0.0025, respectively. Similarly, it could be applied to the other Rayleigh numbers investigated in this paper.

Table 2. Grid independence test at Ra = 10^6

Grid	Total number of cells	Smallest cell size, δ/D	C_P	C_D	$\overline{\text{Nu}}$
G1	652044	0.0025	0.4015	0.4953	8.6275
G2	1396518	0.0010	0.4013	0.4948	8.6130

Results and discussions

Validation of results

The research model put forward by Bejan *et al*. [31] is utilized herein to reduce the computational efforts and further widen the range of Rayleigh number to be investigated. It is significantly necessary to validate the numerical model and method before performing the simulation relevant to this work. As we all know, it is the most straightforward and convincing to compare with the existing experimentally or numerically obtained results for the configuration to be studied herein. However, there are unavailable or no accurate enough and reliable results used for directly comparing so far according to the above introduction. What has been known is that there are large number of results for single cylinder especially represented by the correlating equations by Morgan [6] and Churchill and Chu [5], which have been widely applied to engineering and written to the classic heat transfer books [1, 33] due to its high accuracy and applicability to the quite wide Rayleigh number. Therefore, the results obtained by numerical model is validated by comparison with that by the aforementioned two researchers [5, 6] for $10 \leq \text{Ra} \leq 10^6$. As seen in fig. 3, the results are in good agreement with both, and

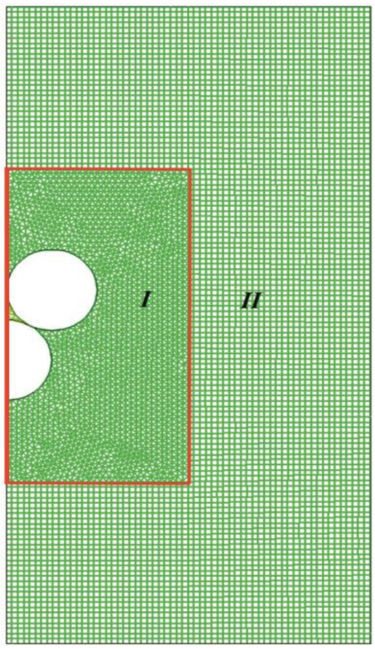

Figure 2. Schematic representation of computational grid

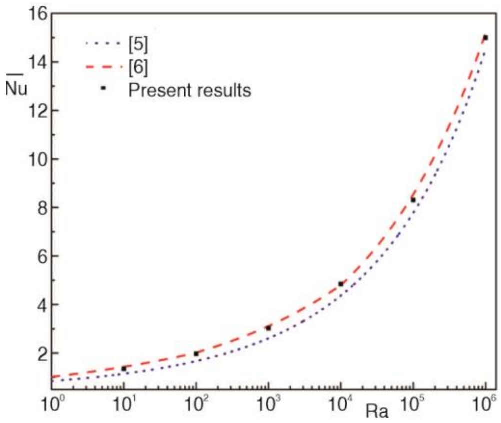

Figure 3. Comparison of average Nusselt numbers for single cylinder

the deviations about $\overline{\mathrm{Nu}}$ are less than 5% over the entire Rayleigh number range investigated. Hence, it could be concluded that the numerical model and method used herein are capable of providing the reliability and accuracy for the following simulation in this work.

Streamline and isothermal contours

The isothermal and streamline contours (on left and right half, respectively) around three attached cylinders over $10 \leq \mathrm{Ra} \leq 10^6$ have been shown in fig. 4. Due to $T_w > T_\infty$ the heat from three cylinders is transferred to the surrounding fluid, making its temperature increase. Correspondingly, its density will be lowered as a result of the thermal expansion. Under the action of buoyancy force due to the occurrence of density gradient, the fluid around the cylinders' array starts slowly rising, gradually forming a plume flowing upward. Generally speaking, as Rayleigh number increases, the buoyant force gradually enhances, making heat transfer dominated by conduction progressively transformed to that by convection. Meanwhile, the thermal and velocity boundary-layers become thinner as shown in fig. 4.

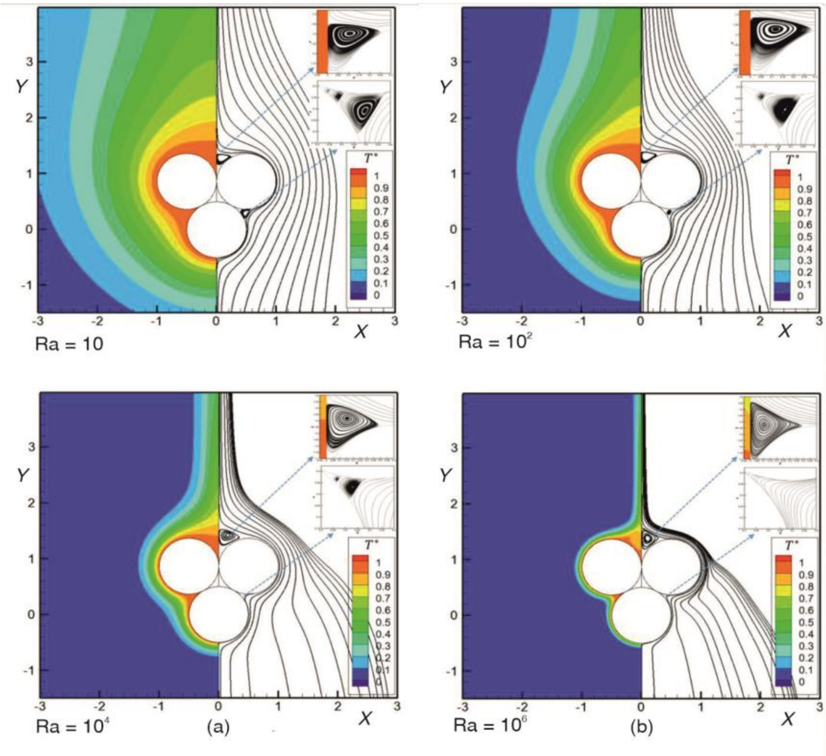

Figure 4. Isotherm contours (a) and streamline (b) in the vicinity of three attached cylinders for $10 \leq \mathrm{Ra} \leq 10^6$

The fluid flowing upwards along the upper cylinders' surface is accelerated first under the favorable pressure gradient and then progressively decelerated downstream with the appearance of the unfavorable one. Subsequently, a position termed as separation point at which the velocity gradient at surface is equal to zero occurs. Meanwhile, the fluid momentum near the cylindrical surface is insufficient to get over the adverse pressure gradient. As a result, the boundary-layer is separated and a wake is formed downstream. Two plumes around two in-contact upper cylinders then begin to merge at the centerline in the wake region, accompanying by the formation of vortexes.

As displayed in fig. 4, there also exist several vortexes near the contact region between the lower and upper cylinders for small Rayleigh number. This is caused by that subject to the buoyant force applied by the upper cylinders, the fluid that gradually flows along the down cylinder surface is pre-separated from its surface. With the increase of Rayleigh number, the inertia of fluid gradually becomes strong, therefore, making the buoyance force exerted by the upper cylinders could be overcome. Subsequently, the onset of flow separation is delayed, and the region with vortexes gradually reduces and vanishes further.

Local Nusselt number distribution

As a result of the flow symmetry over the range of Rayleigh number studied here, only along the right half of three cylinders' surface, *i.e*. the bold curve $\overset{\frown}{AB}$ plus $\overset{\frown}{BC}$ in fig. 5, is the Nu_θ distribution given for several Rayleigh numbers. It is noteworthy that the radian θ, indicating the specific position on its surface, is defined as the ratio of surface arc length from point A, bottom of the lower cylinder, to its radius, *R*. Hence, as illustrated in fig. 5, the point A and B correspond to $\theta = 0$ and $5\pi/6$, respectively, and the point C $\theta = 5\pi/2$ which is the ratio of the length of curve $\overset{\frown}{AB}$ plus $\overset{\frown}{BC}$ to radius *R*. Generally speaking, for small Rayleigh number, the viscous force is relatively large thereby resulting in the thick boundary-layer and heat transfer by conduction is predominant. It could be seen Nu_θ have little dependence on radian θ. With the growing Rayleigh number, the buoyancy-driven flow is enhanced and heat transfer is progressively dominant by convective, and the fluctuation of Nu_θ with radian θ is sharper. The place of the maximum Nu_θ locates at the forward stagnation point of lower cylinder, *i.e*. point A, which is in consistent with single cylinder. For the region $0 \leq \theta \leq \pi/2$, heat transfer is much similar to single cylinder with all else being equal, as represented in fig. 5. However, in the region of $\pi/2 \leq \theta \leq 3\pi/4$ Nu_θ begins to decrease greatly. This is mainly caused by the preheating of the upper cylinders to the bottom one, which considerably reduces temperature gap between the neighboring fluid and lower cylinder thus fairly leading to the weakening of heat transfer rate. Otherwise, the formation of several vortexes accompanying by flow separation makes heat transfer further deteriorated. Moreover, as Rayleigh number increases, Nu_θ decreases more strongly with increasing radian θ. When θ is in the range of $3\pi/4 \leq \theta \leq 15\pi/16$, the enhancement of the preheating effect and deterioration of heat transfer due to several vortexes both make Nu_θ almost zero.

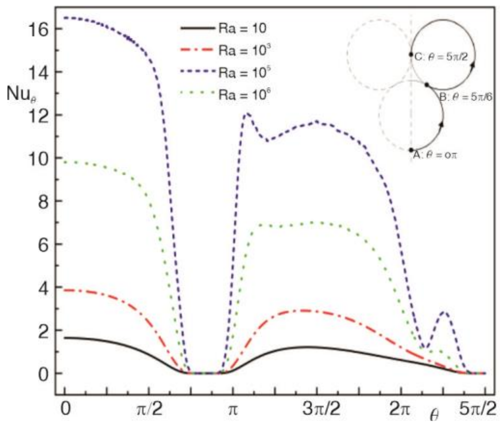

Figure 5. Distribution of local Nusselt number along cylinders' surface for $10 \leq Ra \leq 10^6$

When θ is located in the region of upper cylinders $5\pi/16 \leq \theta \leq 5\pi/2$, Nu_θ increases first and then decreases, and further approaches zero at $\theta = 5\pi/2$. The preheating of the lower cylinder to fluid around upper ones could have gradually decreased temperature difference thereby diminishing heat transfer, whereas the increasing Nu_θ in that area should be mainly due to the incoming fluid entraining some distant cold fluid. Furthermore, with the gradual increase of θ, the increase in entrainment increases temperature gradient near the upper cylinders thereby enhancing heat transfer. Meanwhile, with the growing Rayleigh number, the extent of entrainment becomes stronger. Consequently, the variation of Nu_θ with regard to radian θ is steeper at higher Rayleigh number. However, the developing boundary-layer causes its thickness to increase progressively along the upper cylinders' surface for some Rayleigh number, which correspondingly reduces temperature gradient. Therefore, Nu_θ starts to reduce. With the occurrence of flow separation and vortexes, heat transfer is further worsen, resulting in steeper variation in Nu_θ

Average Nusselt number

The distribution of $\overline{\mathrm{Nu}}$ with regard to Rayleigh numberis presented in fig. 6. As is expected, $\overline{\mathrm{Nu}}$ has a positive correlation with Rayleigh number because of the enhancement of convective flow arising from the increasing Rayleigh number. Meanwhile, $\overline{\mathrm{Nu}}$ obtained herein over $10 \leq \mathrm{Ra} \leq 10^6$ could be correlated:

$$\overline{\mathrm{Nu}} = 0.3493\mathrm{Ra}^{0.2314} \tag{13}$$

The determination coefficient R^2 for it is 0.9992, and the average error between eq. (13) and numerical results is 4.7%, which shows the previous correlating equation could provide the precise enough results for the involved engineering applications. The correlations of $\overline{\mathrm{Nu}}$ with regard to Rayleigh number for the lower and upper cylinder(s) are also attained:

$$\text{For the lower cylinder} \quad \overline{\mathrm{Nu}}_L = 0.4513\mathrm{Ra}^{0.2366} \tag{14}$$

$$\text{For two upper cylinders} \quad \overline{\mathrm{Nu}}_U = 0.2996\mathrm{Ra}^{0.2272} \tag{15}$$

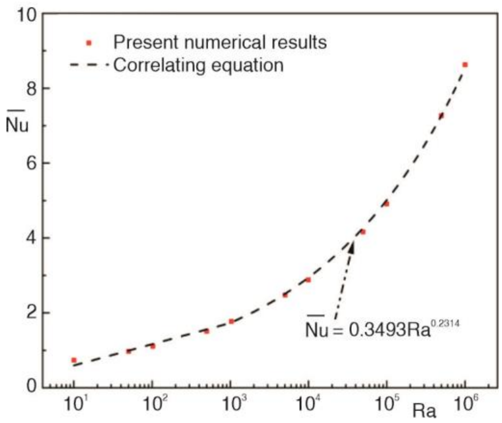

Figure 6. Correlation for the present average Nusselt number with Rayleigh number

It is obviously observed that $\overline{\mathrm{Nu}}_U$ is degraded more than $\overline{\mathrm{Nu}}_L$, which is mainly because of the preheating of the lower cylinder for the upper ones.

Additionally, the comparison of average Nusselt number for the present array is made in tab. 3 with the previous results for single or multiple horizontal cylinder(s) based on the known correlating equations. It could be seen $\overline{\mathrm{Nu}}_U$ for the present array is smaller than that for single cylinder by Morgan [6] at the identical Rayleigh number. This is mainly because that the boundary-layer of three attached cylinders interacts with each other, and the preheating effect of each cylinder on the other two makes heat transfer worsen. What's more, the difference between both on heat transfer is more obvious with gradually growing Rayleigh number. The $\overline{\mathrm{Nu}}$ from the whole array has been reduced by about 38.7~58.5% compared with single cylinder, which suggests the irrationality and inaccuracy of estimating heat transfer rate from three touched cylinders by using the empirical correlations for single cylinder. Meanwhile, the present results are also compared with two horizontally attached cylinders by Liu [23] and three attached cylinders with an upright triangular array by Shi [28] in tab. 3. It is found that the maximum relative deviation of $\overline{\mathrm{Nu}}$ for current configuration with two horizontally attached cylinders is 21.7%. However, that's only 4.6% compared with three attached cylinders with an upright triangular array, showing the effect of gravitational direction, which is opposite for the inverted and regular arrays, on heat transfer for present array could be ignored in actual application. Consequently, it is fairly necessary to perform the simulations in this paper for better predicting heat transfer rate of the fundamental structure of three attached cylinders especially considering the much large discrepancies with single or two cylinder(s).

Table 3. Comparison of average Nusselt number for the present array with the previous results for single or multiple horizontal cylinder(s) based on the known correlating equations

Ra	Present array	Single cylinder [6]		Two horizontally attached cylinders [23]		Three attached cylinders with an upright triangular array [28]	
	$\overline{\mathrm{Nu}}$	$\overline{\mathrm{Nu}}$	Relative deviation with present result	$\overline{\mathrm{Nu}}$	Relative deviation with present result	$\overline{\mathrm{Nu}}$	Relative deviation with present result
10	0.5951	1.4342	58.5%	0.7596	21.7%	0.6236	4.6%
10^2	1.0139	2.0165	49.7%	1.2033	15.7%	1.0441	2.9%
10^3	1.7274	3.1147	44.5%	1.9063	9.4%	1.7479	1.2%
10^4	2.9431	4.8020	38.7%	3.0199	2.5%	2.9264	0.6%
10^5	5.0142	8.5357	41.3%	4.7840	4.8%	4.8992	2.3%
10^6	8.5428	15.1789	43.7%	–	–	8.2021	4.2%

Conclusion

In this work, based on the numerical model put forward by Bejan *et al.*, [31] natural convection heat transfer from three in-contact horizontal isothermal cylinders in air with an inverted triangular array, has been investigated over $10 \leq \mathrm{Ra} \leq 10^6$. Some representative results have been showed for different Rayleigh number. From these results, the following conclusions are obtained.

- Accompanying by the separation of boundary-layer with the presence of the adverse pressure gradient, there exist several vortexes in the wake region of the downstream cylinders because of the strong interaction between two merging plumes.
- The Nu_θ shows positive dependence on Rayleigh number. The maximum Nu_θ locates at the forward stagnation point of the lower cylinder, which keeps in consistent with single cylinder. For the region of $3\pi/4 \leq \theta \leq 15\pi/16$, the enhancement of preheating effect and deterioration of heat transfer due to several vortexes both make Nu_θ almost zero.
- With the effect of vortexes and preheating among three attached cylinders, the heat transfer rate on average from the overall configuration has been reduced by about 38.7~58.5% compared with single cylinder. Besides, the correlating equations for them are obtained.

Acknowledgment

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Nomenclature

C_D – total drag coefficient, [–]
C_F –friction drag coefficient, [–]
C_P – pressure drag oefficient, [–]
D – diameter of each cylinder, [m]
D_∞ – width of computational domain, [m]
g – gravitational acceleration, [$\mathrm{ms^{-2}}$]
Gr – Grashof number, [–]
h – convection heat transfer coefficient, [$\mathrm{Wm^{-2}K^{-1}}$]
$\vec{i}$ – unit vector in the flow direction, [–]
k – thermal conductivity, [$\mathrm{Wm^{-1}K^{-1}}$]
$\vec{n}$ – unit inward normal vector to the surface d*s*, [–]
Nu_θ – local Nusselt number, [–]
$\overline{\mathrm{Nu}}$ – average Nusselt number for whole array, [–]
$\overline{\mathrm{Nu}}_L$ – average Nusselt number for lower cylinder, [–]
$\overline{\mathrm{Nu}}_U$ – average Nusselt number for upper cylinders, [–]
Pr – Prandtl number, [–]
p – gauge pressure, [Pa]
p_∞ – ambient fluid pressure, [Pa]
R – radius of each cylinder, [m]
Ra – Rayleigh number, [–]
S – cylinder spacing, [m]
s – surface area of array, [$\mathrm{m^2}$]
T – temperature of fluid, [K]
T^* – temperature of fluid, [–]
T_m – film temperature of fluid, [K]
T_w – temperature of the cylinders' surface, [K]
T_∞ – temperature of ambient fluid, [K]
ΔT – temperature difference, [K]
$\vec{t}$ – unit vector in the direction of shear stress on surface ds, [–]
u,v – *x*- and *y*- components of the velocity, [$\mathrm{ms^{-1}}$]
u_{ref} – reference velocity [= $(g\beta\Delta TD)^{1/2}$], [$\mathrm{ms^{-1}}$]
X,Y – Cartesian coordinates, [m]
x,y – Cartesian coordinates, [m]

Greek symbols

α – thermal diffusive coefficient, [$\mathrm{m^2s^{-1}}$]
β – coefficient of thermal expansion, [$\mathrm{K^{-1}}$]
δ – distance between two grid points on the surface of cylinder, [m]
θ – equivalent radian, [–]
ν – kinematic viscosity, [$\mathrm{m^2s^{-1}}$]
ρ – density of the fluid, [$\mathrm{kgm^{-3}}$]
ρ_∞ – density of ambient fluid, [$\mathrm{kgm^{-3}}$]
τ – shear stress acting on the surface ds, [Pa]

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