THERMODYNAMICAL STUDY AND TAGUCHI OPTIMIZATION OF A TWO-STAGE VAPOR COMPRESSION REFRIGERATION SYSTEM

by

Alaattin Metin KAYA*

Department of Mechanical Engineering, Bursa Uludag University, Bursa, Turkey

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This study's primary purpose is to optimize the multistage refrigeration system with statistical methods. Taguchi optimization and ANOVA methods were applied to statistically determine the effects of components on system performance. The best operational conditions were defined for the maximum COP and exergy efficiency. Critical parameters have been determined to maximize the system's performance. The evaporator temperature was defined as the most vital parameter (46.32%), and it is followed by condenser temperature (32.65%) for the maximum COP. The most important two parameters are determined as evaporator temperature with 29.14% and condenser temperature with 20.34% for maximum exergetic performance. As a result of 27 tests, the highest COP of the system was calculated as 2.67 and exergy efficiency as 55.22%. By using the optimum levels determined by Taguchi, it was ensured that the system's COP was increased to 3.326 and its exergy efficiency to 71.23%. The ANOVA analyses indicate that the results' confidence level is relatively high, to be 99.9%. Another parameter examined in this study is the inter-stage level determination method and its effect on system performance. The method of determining the optimum inter-stage level may vary according to the objective function and system conditions.

Key words: *multistage refrigeration, exergy, COP, Taguchi, ANOVA*

Introduction

Although many international agreements have been made to curb global warming, reaching the desired level has not been achieved yet. Refrigeration systems are critical due to their high energy consumption and the environmental effects of the refrigerants used. This study aims to obtain optimum working conditions in terms of energy and exergy efficiency of a two-stage ideal vapor-compression refrigeration cycle by statistical methods. Studies on vapor compression refrigeration systems have been carried out for years. Baakeerm *et al.* [1] investigated a multistage vapor compression refrigeration system's performance to maximize COP values by varying the parameters' values. In the study where eight different refrigerants were examined, the maximum COP was found using R717. Chopra *et al.* [2] studied eight different refrigerants' energy and exergy analyses in a two-stage vapor compression refrigeration system. They concluded that both the energy and exergy efficiency of refrigerant R134a was lower than R152a and R600. The highest exergy efficiency was achieved for R152a among all refrigerants. They also found that lower irreversibility occurred at higher evaporating temperatures, and the maximum irreversibility arose in the condenser. Seyitoglu and Kilicarslan [3] studied the

* Author's e-mail: alaattinkaya@uludag.edu.tr

Second law analysis of different refrigerants using the EES software program. They conducted their studies at 25 °C ambient temperature and condensation temperature, and evaporation temperature varying from 40-55 °C and –15-0 °C, respectively. They obtained the lowest irreversibility value in the case of refrigerant R141b. Kilicarslan and Hosoz [4] investigated energy and irreversibility analysis for the cascade refrigeration cycle using refrigerant pairs. It is found that the refrigerant pair R717-R23 is the best option for cascade refrigeration systems. Nikolaidis and Probert [5] examined a two-stage vapor compression refrigeration cycle with a flash intercooler with varying condensing and evaporation temperatures. They concluded that the evaporator and condenser's exergy effects are high and should be optimized. Zubair *et al.* [6] examined the individual contribution of system components to system irreversibility. They found that the major loss in the system was due to the low compressor efficiency. The state of the art studies of multistage refrigeration systems focus on either advanced exergy analysis or optimization of the system.

The Taguchi and ANOVA methods have been used to achieve the optimum design with less cost and time. As widely used in engineering applications and calculations, Taguchi method is a simple, effective, and robust method that is mainly employed to determine optimum parameters with fewer experimentations/analyses. The ANOVA methods can reveal each parameter's contribution ratio and estimate the experimental results for the specified design. The Taguchi optimization method is used widely in many areas, from thermal human comfort [7] to cascade refrigeration systems [8]. Motorcu *et al.* [9] investigated the operating temperature of a heat pump using waste heat with Taguchi analysis. they found that the wastewater temperature was the most important factor in the compressor suction and discharge gas temperatures. On the other hand, few studies evaluate exergy analysis of a thermodynamic system using Taguchi and ANOVA methods.

This study aims to contribute to reducing global warming by determining the optimum working conditions of a refrigeration system, which has an important share in global warming due to its high energy consumption. The evaluation of a two-stage vapor refrigeration system by thermodynamic approach is limited. Moreover, there is a lack of studies in the literature in which the effects of components on system performance in a two-stage vapor compression refrigeration system are determined statistically. There are hardly any studies in the literature in which exergy optimizations are made using Taguchi and ANOVA in refrigeration systems. Ustaoglu *et al.* [8] investigated a cascade refrigeration system. Refrigerants R717, R134a, R510a used on upper cycle and R744, R410a, and R404a on lower cycle were used. The system was examined at temperatures between -40 °C and 40 °C. It has been found that the most effective parameters in terms of both COP and exergy efficiency are lower cycle evaporator and condenser temperatures. Canbolat *et al.* [10] examined the adsorption refrigeration systems' performance using Taguchi, ANOVA, and gray relation analysis (GRA) methods. They determined the importance order of the adsorption refrigeration system components for COP and eCOP using Taguchi and Anova. Tha GRA was applied to get the highest Cop and eCOP values synchronously. They found that absorber and evaporator are the two most effective components. These both refrigeration systems contain different system elements. In this study, unlike the studies available in the literature, a two-stage ideal vapor compression refrigeration cycle was investigated under different conditions and with different refrigerants. In addition, the effect of the methods of determining the inter-stage pressure in a two-stage refrigeration cycle, together with the exergy analysis, adds an additional innovation the study. The feature that distinguishes this study from other studies in the literature is the statistical analysis of a two-stage vapor compression refrigeration cycle. It is aimed that this study will play a leading role in eliminating this deficiency in the literature. A two-stage vapor compression refrigeration cycle was evaluated with statistical evaluation, Taguchi, and ANOVA methods along with both energetic and exergetic methodology to eliminate this deficiency in literature. The optimum refrigerant, optimum experiment variables, the most influential parameters, and their contribution ratio on the performance of the cycle were decided. Thus, an optimum system design can be achieved with less energy and cost with maximum performance. In other words, it is examined which points should be given more importance so that the system can have higher efficiency or less exergy destruction. This study has aimed to fill a crucial gap in the statistical evaluation of an industrial refrigeration system.

Material and method

Refrigerants

The R290 is one of five less-polluting chemicals or refrigerants approved by the Environmental Protection Agency [11]. In addition the advantage that R290 has a negligible ozone depletion potential, it also has a low GWP [12]. On the other hand, since R290 has very high flammability, additional safety precautions are required, which requires additional costs. [13]. The R290 is considered inexpensive and moderate in terms of pollution, with a good cooling effect. In addition, it has not been included in the Kigali Amendment's progressive reduction program [14].

Although R600, which is in HC class consisting mainly of carbon and hydrogen, is highly flammable, they are inexpensive to manufacture and have outstanding ODP, GWP, and toxicity values. Therefore, they can be preferred among refrigerants [15].

The R717 is 99.98% pure and readily available, inexpensive, and can absorb large amounts of heat during the evaporating process [16], and it is the refrigerant with the highest latent heat [1]. While R717's main advantage is its high thermal capability, it can potentially harm people, especially the eyes and throat, in the gaseous form [17]. These disadvantages of R717 (ammonia), an excellent refrigerant, can also be neutralized with a sound system design [18]. Table 1 shows the properties of the refrigerants.

Table 1. Properties of the refrigerants [19, 20]

Physical				Environmental			Safety		
Refrigerant		T_c [°C]	P_c [MPa]	M [gmol^{-1}]	NBP [°C]	ODP	GWP100 [year]	ALT [year]	Safety group
R290	C_3H_8	96.7	4.248	44.10	–42.1 ±2	<0	3.30	12 ±3	A3
R600	C_4H_{10}	152.0	3.796	58.12	0 ±1	0	4	12 ±3	A3
R717	NH_3	132.4	11.28	17.03	–33.33	0	0	<0.019	B2L

Multistage refrigeration systems

In applications where the difference between the evaporation temperature and the condensation temperature is 40 K or more [21] or where ultra-low temperatures are required [15], performing the refrigeration process in two or more stages is one of the methods preferred. In such cases, cascade refrigeration systems using two or more refrigeration cycles operating in series with each other are preferred. Utilizing different fluids in a lower cycle and upper cycle in cascade refrigeration systems can be helpful. Multistage refrigeration systems are operated by a mixing chamber where heat transfer is better than a heat exchanger between stages [22]. A two-stage system can extend a temperature of about –65 °C [15]. Since the two-stage systems have different usage areas, their analysis has been completed, regardless of the scope of use, rather than just the application, in any case, fig. 1.

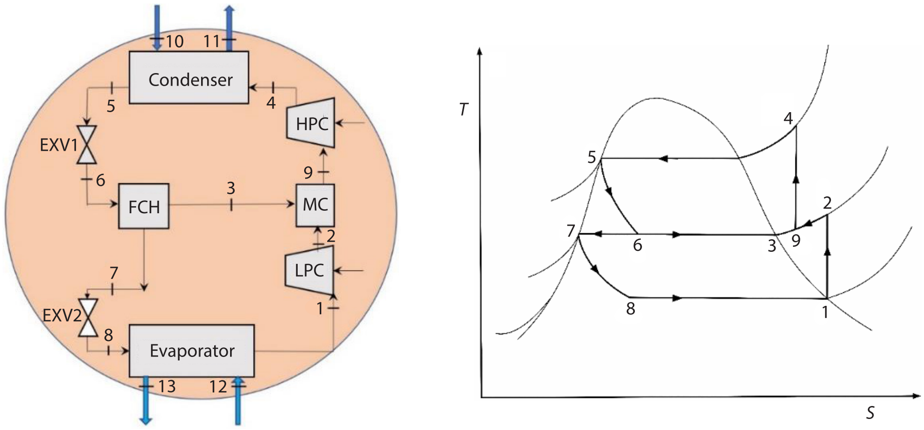

Figure 1. Two-stage vapor compression refrigeration system with *T-s* diagram

Assuming that the system is stable, pressure losses in the system pipes are neglected, and the refrigerants are saturated in the evaporator and condenser. The energy and exergy calculations of the essential elements of the two-stage vapor compression refrigeration cycle are calculated. The energy and exergy balance equations are:

– for low pressure compressor (LPC)

$$\dot{W}_{\text{LPC}} + \dot{m}_1 h_1 = \dot{m}_2 h_2, \ \dot{W}_{\text{LPC}} + \dot{E}x_1 = \dot{E}x_2 + \dot{E}x_{\text{dest,LPC}} \tag{1}$$

– for high pressure compressor (HPC)

$$\dot{W}_{\text{HPC}} + \dot{m}_9 h_9 = \dot{m}_4 h_4, \ \dot{W}_{\text{HPC}} + \dot{E}x_9 = \dot{E}x_4 + \dot{E}x_{\text{dest,HPC}} \tag{2}$$

– for condenser

$$\dot{m}_4 h_4 + \dot{m}_{10} h_{10} = \dot{m}_5 h_5 + \dot{m}_{11} h_{11}, \ \dot{E}x_4 + \dot{E}x_{10} = \dot{E}x_5 + \dot{E}x_{11} + \dot{E}x_{\text{dest,CON}} \tag{3}$$

– for evaporator

$$\dot{m}_8 h_8 + \dot{m}_{12} h_{12} = \dot{m}_1 h_1 + \dot{m}_{13} h_{13}, \ \dot{E}x_8 + \dot{E}x_{12} = \dot{E}x_1 + \dot{E}x_{13} + \dot{E}x_{\text{dest,EVA}} \tag{4}$$

Expansion Valves 1 and 2, flash chamber, and mixing chamber's energy and exergy balance equations are stated in the following equations, respectively:

$$\dot{m}_5 h_5 = \dot{m}_6 h_6, \ \dot{E}x_5 = \dot{E}x_6 + \dot{E}x_{\text{dest,EXV}_1} \tag{5}$$

$$\dot{m}_7 h_7 = \dot{m}_8 h_8, \ \dot{E}x_7 = \dot{E}x_8 + \dot{E}x_{\text{dest,EXV}_2} \tag{6}$$

$$\dot{m}_6 h_6 = \dot{m}_7 h_7 + \dot{m}_3 h_3, \ \dot{E}x_6 = \dot{E}x_7 + \dot{E}x_3 + \dot{E}x_{\text{dest,FCH}} \tag{7}$$

$$\dot{m}_9 h_9 = \dot{m}_2 h_2 + \dot{m}_3 h_3, \ \dot{E}x_3 + \dot{E}x_2 = \dot{E}x_9 + \dot{E}x_{\text{dest,MC}} \tag{8}$$

The COP and exergy efficiency of two-stage vapor compression refrigeration cycle:

$$COP = \frac{\dot{Q}_L}{\dot{W}_{\text{LPC}} + \dot{W}_{\text{HPC}}} \tag{9}$$

$$\eta_{\text{ex}} = \left[1 - \frac{\sum \dot{E}x_{\text{dest}}}{\dot{W}_{\text{LPC}} + \dot{W}_{\text{HPC}}} \right] \cdot 100 \tag{10}$$

Inter-stage level

Parameters such as the type of refrigerant used in the system, evaporator and condenser temperatures, compressor efficiency significantly determine the COP and exergy destruction values of the system. Another critical parameter that needs to be evaluated is the inter-stage level, as it significantly impacts the economics of two-stage refrigeration systems [21]. Prasad [23] developed a computer program to optimize an n-stage vapor compression refrigeration cycle with a minimum total cost approach and found that the inter-stage pressure should be 15% higher than the geometric mean pressure. Gosney [24] argued that the optimum inter-stage temperature in a two-stage refrigeration cycle should be 5 K more than the saturation temperature corresponding to the pressure achieved with equal pressure ratios. De Lepeleire [25] found that in a refrigeration system using R-22 refrigerant, the optimum inter-stage pressure should be 0.35 bar higher than the geometric mean of the evaporation and condensation pressures [25]. Torrella *et al.* [21] experimentally investigated the inter-stage conditions of a two-stage refrigeration cycle. In their experimental studies, the researchers completed the task using refrigerant R404a at evaporating temperatures ranging from –36 to –20 °C, and condensing temperatures ranging from 30-47 °C, the pressure in systems using intermediate stage pressure is always higher than in systems without the intermediate stage. It was found that the stage pressure level cannot be preselected as it depends on the cycle conditions and operating conditions. Prasad [26] determined the most suitable inter-stage pressure for R12 refrigerant to give the highest COP in two-stage refrigeration cycles. As a result, he found that optimum inter-stage pressure is the geometric mean of evaporation and condensation pressures. Zubair and Khan [27] also did similar work. Zubair *et al.* [6] showed that the most suitable inter-stage pressure to give the highest COP value for R134a is close to the arithmetic mean of evaporation and condensation pressures. Ratts and Brown [28] determined the optimal inter-stage temperature for the two-stage vapor compression refrigeration cycle using the entropy generation minimization method when using R134a. They concluded that this method gives better results than the inter-stage temperature found by geometric mean compared to R502. Optimization of inter-stage pressure can be achieved using thermo-economic criteria according to investment and energy costs [29]. This method gives the geometric mean of condensation and evaporation pressures as inter-stage pressure. It is not sensitive enough because the refrigerants cannot be considered ideal gas due to the phase changes in the system. The temperatures in the aspiration in the two-stages are different [30]. Researchers have used various methods to determine the inter-stage level. While some researchers [26] calculate the optimum intermediate stage pressure for the system's highest performance coefficient value, other researchers [28] have considered minimizing the power consumed for cooling.

One of this study's goals is to determine how the inter-stage level affects energy and exergy efficiency performance. For this purpose, three different inter-stage level determination methods used in the literature were selected, and their effects on system performance were determined. These methods are:
- geometric mean [26],
- the 0.35 bar higher than the geometric mean [25], and
- the 5 K above the saturation temperature corresponding to the pressure achieved with geometric mean [24].

Taguchi method

The Taguchi method uses the standard orthogonal array when analyzing many factors with a few experiments. This sequence matches the level of a factor with different levels of the same number of other factors. There are many standard orthogonal arrays in the Taguchi method for constructing the experimental plan. The array can be selected according to the number of parameters and levels. In this study, L27 design was preferred instead of L9 because the parameters were more than four. Thus, more sensitive results can be obtained with the increase in the number of tests. Figure 2 shows the stages for the progression of the Taguchi method and system parameters, and the parameters' levels are presented in tab. 2. Taguchi usually uses a two-step optimization process by identifying the signal-to-noise (S/N) ratio in Step 1 to identify control factors that reduce variability. In Step 2, it identifies control factors that move the mean to target and have little or no effect on the S/N ratio. The S/N ratio indicates how the response varies with the nominal or target value under different conditions. Different S/N ratio types can be selected according to the target.

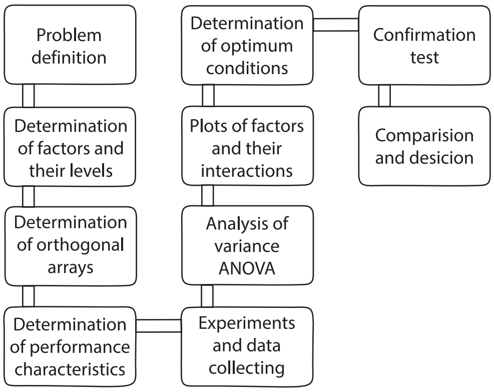

Figure 2. Process steps of Taguchi design methods

Table 2. Factors and levels

	Factor	Levels		
		1	2	3
A	T_{EVA} [°C]	–20	–30	–40
B	T_{CON} [°C]	40	50	60
C	$\dot{Q}_L$ [kW]	1	10	20
D	η_{COMP}	0.7	0.8	0.9
E	T_H [°C]	25	30	35
F	T_L [°C]	–5	–10	–15
G	Inter-stage level	L_1	L_2	L_3
H	Refrigerants	R290	R600	R717

* L_1, L_2, and L_3 stand for the inter-stage level defining methods, respectively.

In this section, S/N ratios are calculated according to the *larger is better* quality characteristic in COP and exergy efficiency:

$$\frac{S}{N} = -10\log\frac{1}{n}\left(\sum_{i=1}^{n}\frac{1}{yi^2}\right) \tag{11}$$

where *n* is the number of tests and *yi* – the value of the analyze result of the ith test.

Analysis of variance

The ANOVA is one of the preferred statistical analysis methods to determine the effects of all variables involved in any experimental design on desired outcomes. The impact of the parameters can be specified as individuals or their interactions in this analysis. Thermodynamically analyzes results obtained for analysis in ANOVA are converted to S/N ratios. An ANOVA table is created with parameters and/or their interactions, degree of freedom, DoF, the total sum of squares (Adj SS), variance (Adj MS), *F*-value of the parameter, *F* table value, and the percentage contribution (*P*%).

Results and discussion

Validation generated code

Before moving on statistical calculations, confirming that the code used in thermodynamic calculations gives correct results is of high importance for the reliability of the study. Analyzing without checking the accuracy of the calculation methods used in this study may provide erroneous results, and it would be better to compare it with a study available in the literature. In this context, when the literature is examined since there is no study under the same conditions, the research available in [2], whose conditions and results are given clearly, was selected. The study was repeated with our calculations in its conditions. The comparisons obtained from repetitive operation for refrigerant R290 under the same conditions, the condensing temperature of 45 °C, the evaporating temperature varying –50 °C to 5 °C, and isentropic compressor efficiency 80%, are presented in fig. 3. As shown in fig. 3, the calculations used in this study yield similar results to those in another study available in the literature.

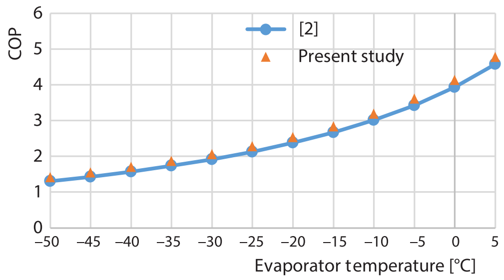

Figure 3. The COP *vs.* evaporator temperature for R290

Taguchi and ANOVA results

After calculating the system's COP and exergy efficiency values thermodynamically employing the parameters given in tab. 2, the values obtained were analyzed statistically. The S/N ratios for COP and exergy efficiency were acquired. The results, both thermodynamically and statistically, are presented in tab. 3.

The S/N ratios obtained for each level of analysis variables are called the *response table*. Tables 4 and 5 show the response table values for COP and exergy efficiency. When these tables are examined, the most influential parameters on the analysis outputs are listed in rank from 1-8 by comparing delta values.

The main effect plots for S/N ratios were shown in figs. 4 and 5 for COP and exergy efficiency, respectively. When these plots are examined, the highest *S*/*N* ratio allows us to understand the optimum test variables quickly.

The optimum variables' levels obtained from the S/N ratios were marked on the plots with a circle. According to fig. 4, optimum test levels were obtained as $A_1B_1C_1D_3E_1F_1G_3H_2$. In brief, the maximum COP was obtained in conditions at –20 °C evaporator temperature, 40 °C condenser temperature, 1 kW cooling capacity, 0.9 isentropic efficiencies of compressors, T_H = 25 °C, T_L = –5 °C, and L_3 combination for inter-stage level with R600 refrigerant. When fig. 5 was examined, the levels of the optimum variables were determined as $A_1B_1C_1D_3E_3F_3G_2H_2$. To obtain maximum exergy efficiency, –20 °C evaporator temperature, 40 °C condenser temperature, 1 kW cooling capacity, 0.9 isentropic efficiencies of compressors, T_H = 35 °C, T_L = –15 °C, and L_2 combination for inter-stage level combinations should be used with R600 refrigerant.

Table 3. The COP and exergy efficiency for Taguchi L_{27} orthogonal array

	A	B	C	D	E	F	G	H	Results			
No.	Levels								COP	S/N COP	Exergy efficiency	S/N exergy efficiency
1	1	1	1	1	1	1	1	1	2.43	7.715699	31.52	29.97172418
2	1	1	1	1	2	2	2	2	2.55	8.127397	44.26	32.92022819
3	1	1	1	1	3	3	3	3	2.37	7.480295	51.83	34.29162418
4	1	2	2	2	1	1	1	2	2.43	7.70855	31.49	29.96345321
5	1	2	2	2	2	2	2	3	2.29	7.200502	40.01	32.04337103
6	1	2	2	2	3	3	3	1	2.31	7.257186	50.60	34.08301034
7	1	3	3	3	1	1	1	3	2.22	6.942696	28.89	29.21495084
8	1	3	3	3	2	2	2	1	2.15	6.656845	37.72	31.53143368
9	1	3	3	3	3	3	3	2	2.33	7.358295	51.17	34.18030834
10	2	1	2	3	1	2	3	1	2.57	8.205419	38.83	31.78334781
11	2	1	2	3	2	3	1	2	2.67	8.533478	52.22	34.35673735
12	2	1	2	3	3	1	2	3	2.50	7.951849	43.58	32.78574452
13	2	2	3	1	1	2	3	2	1.74	4.78599	26.40	28.43207854
14	2	2	3	1	2	3	1	3	1.58	3.98413	31.87	30.06764127
15	2	2	3	1	3	1	2	1	1.63	4.238422	29.76	29.47265854
16	2	3	1	2	1	2	3	3	1.60	4.0824	24.40	27.74779653
17	2	3	1	2	2	3	1	1	1.56	3.884735	31.53	29.97447941
18	2	3	1	2	3	1	2	2	1.71	4.680216	31.12	29.86079177
19	3	1	3	2	1	3	2	1	1.85	5.35282	32.27	30.17597931
20	3	1	3	2	2	1	3	2	1.94	5.756035	30.08	29.56555664
21	3	1	3	2	3	2	1	3	1.75	4.875638	35.49	31.00211999
22	3	2	1	3	1	3	2	2	1.89	5.547599	32.99	30.36764631
23	3	2	1	3	2	1	3	3	1.74	4.825948	27.25	28.70733013
24	3	2	1	3	3	2	1	1	1.77	4.934894	35.71	31.055797
25	3	3	2	1	1	3	2	3	1.13	1.030768	19.88	25.9683276
26	3	3	2	1	2	1	3	1	1.15	1.183692	18.71	25.44147575
27	3	3	2	1	3	2	1	2	1.23	1.783967	26.02	28.30614584

Table 4. Response table for mean S/N ratios of COP

Level	A	B	C	D	E	F	G	H
1	7.383	7.111	5.698	4.481	5.708	5.667	5.596	5.492
2	5.594	5.609	5.651	5.644	5.573	5.628	5.643	6.031
3	3.921	4.178	5.550	6.773	5.618	5.603	5.659	5.375
Delta	3.462	2.933	0.148	2.292	0.135	0.064	0.063	0.656
Rank	1	2	5	3	6	7	8	4

Table 5. Response table for mean S/N ratios of exergy efficiency

Level	A	B	C	D	E	F	G	H
1	32.02	31.87	30.54	29.43	29.29	29.44	30.43	30.39
2	30.50	30.47	30.53	30.49	30.51	30.54	30.57	30.88
3	28.95	29.14	30.40	31.55	31.67	31.50	30.47	30.20
Delta	3.07	2.74	0.14	2.12	2.38	2.05	0.13	0.68
Rank	1	2	7	4	3	5	8	6

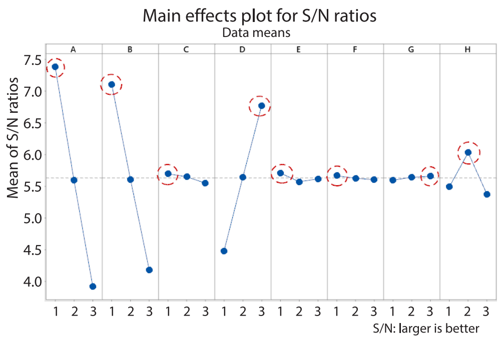

Figure 4. Optimum levels of variables for COP

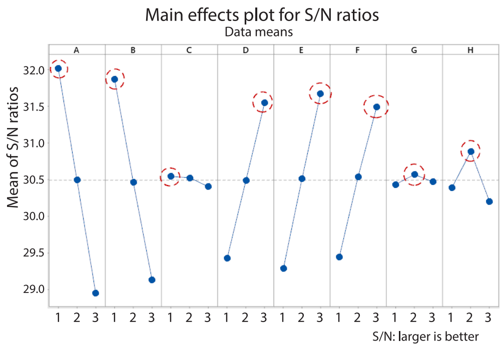

Figure 5. Optimum levels of variables for exergy efficiency

As a result of thermodynamic calculations made under these specified conditions, the highest COP of 3.326 and exergy efficiency of 71.23 was achieved. These values are quite larger than the COP and exergy efficiency results of 27 different test designs.

In the ANOVA analysis, when the *F*-factor value is greater than 5, it becomes physically and statistically significant [11]. The ANOVA tables obtained for COP and exergy efficiency are presented in tabs. 6 and 7. The *F*-factor of T_H, T_L, and inter-stage pressure values are less than 5, tab. 6. It indicates that the significance level of these parameters is low.

Table 6. The ANOVA table for COP

Parameters	DoF	Adj SS	Adj MS	F_{factor}	F_{table}	P%
T_{EVA} [°C)]	2	2.44793	1.22397	1686.54	14.91*	46.32
T_{CON} [°C]	2	1.72563	0.86281	1188.90	14.91*	32.65
$\dot{Q}_L$ [kW]	2	0.06411	0.03205	44.17	14.91*	1.21
η_{COMP}	2	0.92606	0.46303	638.02	14.91*	17.52
T_H [°C]	2	0.00467	0.00234	3.22		
T_L [°C]	2	0.00066	0.00033	0.46		
Inter-stage level	2	0.00050	0.00025	0.34		
Refrigerants	2	0.10847	0.05423	74.73	14.91*	2.05
Error	10	0.00726	0.00073			
Total	26	5.28529				

*99.9% confidence level

On the other hand, the other parameters are pretty larger than 5. Evaporator temperature, condenser temperature, and isentropic efficiency are the most significant components of the system's performance. It was determined that the contribution ratio of the evaporator temperature is about 46.32% for COP, tab. 6. It is followed by the condenser temperature with a contribution ratio of 32.65% and the isentropic efficiency of compressors with 17.52%. It was observed that the most effective test variable is evaporator temperature with a 29.14% contribution ratio for exergy efficiency, tab. 7. Besides, the condenser temperature variable with a 20.34% contribution had an essential impact. Finally, the variables that contribute to exergy efficiency are the T_H variable with 19.23%, the T_L variable with 16.50%, and the isentropic efficiency of the compressors variable with 11.52%. The ANOVA analyses indicate that the results' confidence level is relatively high, with 99.9% for COP. In exergy efficiency, the cooling capacity and inter-stage level have comparably less confidence with 97.5% and 95%. However, these results are pretty reasonable.

Table 7. The ANOVA table for exergy efficiency

Parameters	DoF	Adj SS	Adj MS	F_{factor}	F_{table}	P%
T_{EVA} [°C]	2	661.92	330.961	271.68	14.91[a]	29.14
T_{CON} [°C]	2	462.00	231.002	189.63	14.91[a]	20.34
$\dot{Q}_L$ [kW]	2	17.65	8.824	7.24	5.46[b]	0.78
ηCOMP	2	261.68	130.842	107.41	14.91[a]	11.52
TH [°C]	2	436.74	218.369	179.26	14.91[a]	19.23
TL [°C]	2	374.72	187.359	153.80	14.91[a]	16.50
Inter-stage level	2	11.74	5.871	4.82	4.10[c]	0.52
Refrigerants	2	32.79	16.393	13.46	14.91[a]	1.44
Error	10	12.18	1.218			
Total	26	2271.42				

[a] 99.9% confidence level, [b] 97.5% confidence level, 95% confidence level.

Regression analysis

Regression analysis is an analysis method used to measure the relationship between experiment outputs and experiment variables. In this study, mathematical equations related to COP and exergy efficiency and test variables, which are test outputs, were obtained using multi-linear regression analysis.

Regression equation for COP:

$$COP = 10.091 - 1.7309T_{\text{EVA}} - 1.4664T_{\text{CON}} - 0.0738\dot{Q}_{\text{L}} + 1.1459\eta_{\text{COMP}} - $$
$$-0.0451T_{\text{H}} - 0.0319T_{\text{L}} + 0.0317 \times \text{inter-stage level} - 0.0586 \times \text{refrigerants}, \ R^2 = 098 \quad (12)$$

Regression equation for exergy efficiency:

$$\eta_{\text{ex}} = 30.028 - 1.5339T_{\text{EVA}} - 1.3682T_{\text{CON}} - 0.06977\dot{Q}_{\text{L}} + 1.0617\eta_{\text{COMP}} +$$
$$+1.1896T_{\text{H}} + 1.0268T_{\text{L}} + 0.0177 \times \text{inter-stage level} - 0.0923 \times \text{refrigerants}, \ R^2 = 0.9827 \quad (13)$$

where R^2 is the regression coefficient in multiple linear regression analysis, which should be more than 0.8 for obtained mathematical models [31].

Verification test

The final stage of the Taguchi method is the verification stage. This last step is proposed to determine the accuracy of optimum test levels determined by the Taguchi method. In this study, the levels of the optimum test variables defined for COP and exergy efficiency were determined from figs. 4 and 5. Accordingly, optimum levels are $A_1B_1C_1D_3E_1F_1G_3H_2$ and $A_1B_1C_1D_3E_3F_3G_2H_2$ for COP and exergy efficiency, respectively. The estimated S/N ratios can be calculated by eqs. (14) and (15):

$$\eta_{\text{COP}} = \overline{A_1} + \overline{B_1} + \overline{C_1} + \overline{D_3} + \overline{E_1} + \overline{F_1} + \overline{G_3} + \overline{H_2} - 7\overline{T} \tag{14}$$

$$\eta_{\text{exergy efficiency}} = \overline{A_1} + \overline{B_1} + \overline{C_1} + \overline{D_3} + \overline{E_3} + \overline{F_3} + \overline{G_2} + \overline{H_2} - 7\overline{T} \tag{15}$$

The verification and estimated test results performed according to the optimum test conditions are shown in tab. 8. The *F*-table values were taken according to a 99.5% confidence interval.

Table 8. Verification test results

		Optimum variables levels		
		Prediction results	Test results	CI [dB]
Level	COP Exergy efficiency	$A_1B_1C_1D_3E_1F_1G_3H_2$	$A_1B_1C_1D_3E_1F_1G_3H_2$	
		$A_1B_1C_1D_3E_3F_3G_2H_2$	$A_1B_1C_1D_3E_3F_3G_2H_2$	
S/N ratio	COP	10.59	10.43	±0.098
	Exergy efficiency	37.17	37.05	±4.017

Conclusions

Energetic and exergetic performances of a two-stage vapor compression refrigeration system were thermodynamically and statistically examined. After obtaining the optimum conditions in parametric examinations performed under specified conditions, the components' effect ratios and their importance levels on the system's energetic and exergetic performance were determined. The optimum refrigerant, optimum experiment variables, the most influential parameters, and their contribution ratio on the performance of the cycle were decided. Thus, an optimum system design can be achieved with less energy and cost with maximum performance.

The evaporator temperature of –20 °C, the condenser temperature of 40 °C, the cooling capacity of 1 kW, and refrigerant R600 were determined as the optimum operating conditions by Taguchi to reach the maximum COP and exergy efficiency inside of the specified functional array. The T_H and T_L temperatures are defined as 25 °C and 5 °C, respectively, in the maximum COP calculations. They do not affect the COP while they affect the exergetic performance. For the case of maximum exergy efficiency, T_H and T_L were 35 °C and –15 °C, respectively.

While the optimum levels for the maximum energetic performance (COP) were found as $A_1B_1C_1D_3E_1F_1G_3H_2$, the maximum exergetic performance was observed under $A_1B_1C_1D_3E_3F_3G_2H_2$ conditions. As a result of a thermodynamic examination, the highest COP was 2.67 in 27 tests. After the statistical determination, the largest COP of 3.326 was achieved, and it is approximately one quarter higher than the highest value in all test patterns. Similarly, among all test patterns, the highest exergy efficiency was obtained to be 52.22 %, and it reached 71.23%, more than one-third of the highest value of the tests for the optimum operating conditions. After the statistical analysis for the maximum COP, it was observed that the predominant factor was the evaporator temperature with an effective rate of 46.32%. The effective rate of the condenser temperature was 32.65 %, and that of isentropic compressor efficiency was 17.52%. The evaporator temperature is the most influential parameter for maximum exergy efficiency. Its effect was 29.14%. The condenser temperature and T_H have an effect of 20.34% and 19.23%, respectively.

Another parameter examined in the study is the inter-stage level determination method. It is observed that although there is no significant difference between the methods, it may affect both COP and exergy efficiency. The method should be selected according to the objective function.

Statistically obtained results have a margin of error, albeit small. They provide critical information about the system's performance. Nevertheless, they are evaluated together with thermodynamic analysis. As a result of the evaluation, it is concluded that cooling capacity does not affect the cycle's COP or exergy efficiency.

Nomenclature

$\dot{E}x$ – exergy, [kW]
F – F-statistics
h – specific enthalpy, [kJkg^{-1}]
$\dot{m}$ – mass-flow rate, [kgs^{-1}]
n – number of tests
P% – percentage contribution
P_c – critical pressure, [kPa]
$\dot{Q}_L$ – cooling capacity, [kW]
s – specific entropy, [kJkg^{-1}K^{-1}]
T – temperature, [ºC]
T_c – critical-temperature, [ºC]
$\dot{W}$ – power, [kW]
y – value of the analyze result

Greek symbols

η_{ex} – exergy efficiency
η_{COMP} – Compressor efficiency

Subscripts

dest – destruction
H – high
L – low
c – critic

Acronyms

Adj SS – adjusted sum of squares
Adj MS – adjusted mean squares
ALT – atmospheric-lifetime
CON – condenser
COMP – compressor
DoF – degree of freedom
eCOP – exergetic coefficient of performance
EXV – expansion-valve
EVA – evaporator
FCH – flash chamber
GWP – global-warming-potential
GRA – gray relation analysis
HFC – hydrofluorocarbon
HPC – high pressure compressor
LPC – low pressure compressor
M – molecular-mass
MC – mixing chamber
NBP – normal-boiling-point
ODP – ozone-depletion-potential
S/N – signal to noise

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