MODELLING OF CONVECTION-DOMINATED MELTING IN RECTANGULAR ENCLOSURES

by

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Original scientific paper
https://doi.org/10.2298/TSCI211025074I

A validated CFD by ANSYS model is built for a melting process of PCM (gallium), inside rectangular enclosure heated from one side and cooled from another. This was carried out by enabling the solidification and melting model in addition energy model of ANSYS. Regression model of melting front results from CFD model by ANSYS is utilized and integrated inside numerical approach by MATLAB to generate constants for Nusselt correlations and effective conductivity during different melting stages that can be used to define melting front locations and temperature at grid different locations. In addition, effect of the wall temperature and model aspect ratio variation on melting process have been simulated by ANSYS and analyzed in details. Enthalpy method and conduction equations for 2-D model were used to describe the predicted share of convection in melting process, with trial and error by assuming the constants automatically. Accordingly, melting process can be predicted in a simple and a conservative time way at the same time, the results are mainly indicating that convection heat transfer affects the melting process and melting behavior which is in coherence with previous experimental and analytical literature. Also, the effect of aspect ratio were investigated to assure that the more the aspect ratio, the better is the temperature homogeneity.

Key words: *PCM, convection heat transfer, effective conductivity, melting, Nusselt number*

Introduction

Efforts have been paid off towards studying the effect of convection heat transfer in melting process of PCM by both conducting experiments or through analytical solution and numerical approaches. Experiments proved that the convection is the main engine for the melting process when the liquid phase starts to build up and melting front are directly determined and shaped by convection effect whatever the PCM used or using different boundary or initial conditions.

Webb and Viskanta [1] employed a computational method to verify experimental data for natural-convection in melting of pure metals. The effect of Rayleigh number was investigated regarding changing the parameters of velocity and temperature fields. While, Hale Jr, and Viskanta [2] have firstly applied photographic observation monitor paraffin melting in a rectangular cell heated from side which indicates that the convention played a main role in the melting process. Several experiments are performed [3-6] to have more accurate results and to study effect of different parameters on the melting process as wall temperature variation.

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Main objective was directed to make correlation that can be used to describe the convection share in the whole melting process. At the beginning, the correlation was not satisfactory by Webb and Viskanta [7], but considered as a great step toward formalizing and deducing accurately the melting process through using governing equations.

Deep investigations are carried for understanding how different parameter can affect convection and melting process as subcooling and angle of the model as per Beckerman and Viskanta [8]. They have studied experimentally and numerically effect of subcooling of solid part on melting process and found that it is greatly affect the melting rate. While, rectangular model inclination effect on acid melting experimentally studied by Babak *et al*. [9] and found that changing model from vertical to horizontal position increase and produce more convection current.

A huge effort has been added for better understanding and formalizing of the melting process not only in rectangular model but also in different shapes as cylindrical model by Jones *et al*. [10] through experiment that could be used as benchmark for numerical model validation. Many different model shapes with different materials are also studied and many estimated equations are found by Tehrani *et al*. [11] using effective conductivity model and Nusselt number correlations. Also, many experiments and many computational studies through CFD have been conducted to study the melting process of PCM inside spherical model by Tan *et al*. [12] and validated the results with experimental literature.

The objective is directed to find fast and simplified way to modelling and obtaining reliable results of the melting process itself by different programmable tools. Souayfane *et al*. [13] generate a CFD model by COMSOL Multiphysics program also include radiation effect as well as convection share for fatty acid melting in rectangular model. Also, simplified equation is generated to indicate the same in a faster way. While, Groulx and Murray [14] used also COMSOL Multiphysics program to simulate melting of octadecane material inside rectangular considering both conduction and convection heat transfer and effect of several parameters' variation are studied as thermal conductivity, dynamic viscosity, and fin additions.

Experiments are also extended to different shapes. The HO and Viskanta [15] conducted experimental research to study the melting process of paraffin in a horizontal tube. The experiment indicates a high convection current on the bottom part of tube which melts faster. After being able to simulate PCM melting process using different tools, more parameters are investigated such as mushy zone constant by Kumar and Krishna [16] using ANSYS FLUENT 16 who found that as the constant is higher, more damping of melting process is noticed and melting time is accordingly increased.

Efforts are continusily paid and recent studies have been conducted to report the effect of the convection currents experimentally and numerically by MATLAB and COMSOL on the melting of PCM [17].

Efforts are also directed to take advantage of research role is practical engineering specifically in renewable energy field. Regin *et al*. [18], that define the requirements that shall be considered in designing of storage system and found that the most important is PCM selection which have less damping to convection current and accordingly smaller storage cane be applied.

Methodology

As shown in fig. 1, the model used in both CFD model and numerical approach by MATLAB are rectangular shape with aspect ratio, Ar, of 0.5 filled with PCM (gallium, tab. 1) and heated from the left side at elevated temperature, $T_{\mathrm{wall}} = 311.31$ K and the right side is kept at temperature, $T_{\mathrm{cold}} = 301.3$ K and the initial temperature of the entire model is maintained at the melting temperature, $T_{\mathrm{m}} = 302.78$ K.

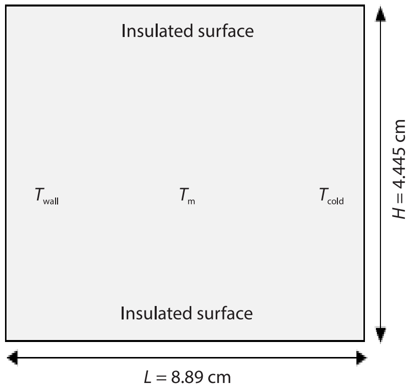

Figure 1. Rectangular model

Table 1. Gallium properties

Property	Value
Conductivity, K	31 W/mK
Specific heat, C_p	381.5 J/kgK
Density, ρ	6093 kg/m^3
Viscosity, μ	0.00181 kg/ms
Melting temperature, T_m	302.78 K
Latent melting heat, h_{melt}	80160 J/kg

The model is divided into 2401 cells and mushy zone parameter is 40000, by dividing the height, H, and width, L, into 49 parts, used in the solution for the CFD model and numerical approach of MATLAB. The time step used is 0.05 second and maximum number of iterations per time step is 100 for CFD model by ANSYS. The CFD model is solved by enabling the solidification and melting model of the ANSYS with following governing equiations:

– Second order energy equation.
– Second order momentum equation.
– Continuity equation.
– Pressure velocity coupling scheme is considered simple.

Meanwhile the numerical approach by MATLAB is based mainly on the conduction equation on 2-D and enthalpy method [19] with proper modifications as indicated in the section related to results.

The current method depends on splitting the model into small cells and every cell has its effective conductivity, K_{eff}, which taking into consideration the normal conductivity, K, of PCM and convection currents affects based on the different density in the model that build up due to temperature grade in the model with time and cells effect on each other's as per fig. 2.

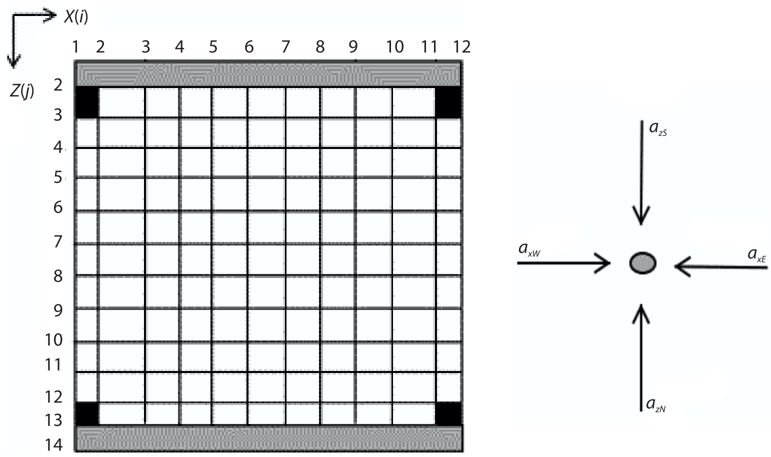

Figure 2. Model basics

For numerical solution be MATLAB, The following governing equations are explaining the basis for operating the model:

$$a_{\mathrm{Po}}(i,j) = a_{\mathrm{xE}} + a_{\mathrm{xW}} + a_{\mathrm{zN}} + a_{\mathrm{zS}} \tag{1}$$

$$a(x \text{ or } z) = 2K_{\mathrm{eff}} A(x \text{ or } z)\left[N_{(x \text{ or } z)} + -1\right]\frac{\mathrm{d}t}{\mathrm{d}x} \tag{2}$$

$$K_{\mathrm{eff}} = K(\mathrm{Nu}+1) \tag{3}$$

$$\mathrm{Nu} = C\,\mathrm{Ra}^{m} \tag{4}$$

$$\mathrm{Ra} = \frac{\mathrm{g}\beta X_{\mathrm{melt}}^{3}\left(T_{\mathrm{wall}} - T_{\mathrm{m}}\right)}{\vartheta\alpha} \tag{5}$$

These equations will lead to have a high effective conductivity in the parts where convections currents are active and accordingly higher melting process than other parts where convection currents are weak. The gap here is how to determine constant *C, m* in a detailed independent explained method with high accuracy with literature results.

In the current study, three wall temperatures were examined 308 K, 311.31 K, and 315 K. While, the investigated *Ar* were 1.0, 0.5, and 2.0. the main objective now is determining constants *C* and *m* in both mixed convection and conduction stage (2nd stage) and the convection stage (3rd stage). In fig. 3, flow chart with steps is detailed for the method used for building the numerical approach of MATLAB to solve this specific problem. The below describes in steps, how the constants are found in the numerical approach by MATLAB:

– Regression equation is estimated for validated ANSYS model. This regression equation estimates the X_{melt} as function of time and *Y*.
– By using try and error, the constants are assumed to be specific value in range from 0.05-10 with increased value by 0.05 for each try, it means that 200 × 200 try had been performed.
– For each try, X_{melt} as function of time and *Y* is estimated by MATLAB as described in flowchart fig. 4 and compared with the regression model.
– Error between the regression model and numerical approach for each time step and *Y* is calculated.

$$Err = \frac{\left(X_{\text{melt av ANSZYS}} - X_{\text{melt av MATLAB}}\right)}{\left(X_{\text{melt av ANZZYS}}\right)} \tag{6}$$

– Mean error is estimated for each try:

$$M.Err = \frac{\sum_{\tau=\tau_{\mathrm{end}}}^{\tau=0} Err}{\text{no. of time steps}} \tag{7}$$

– The try with minimum mean error from 200 × 200 try has the correct constants *C* and *m*. The same is used for determining *C* and *m* in 2nd stage and 3rd stage.

Results and discussions

Model validation

In order to verity the current model, a comparison has been performed with results by Webb and Viskanta [1]. As shown on below fig. 5, results of the melting front position with time from the conducted CFD model by ANSYS is validated against analytical solution performed by Webb and Viskanta [1].

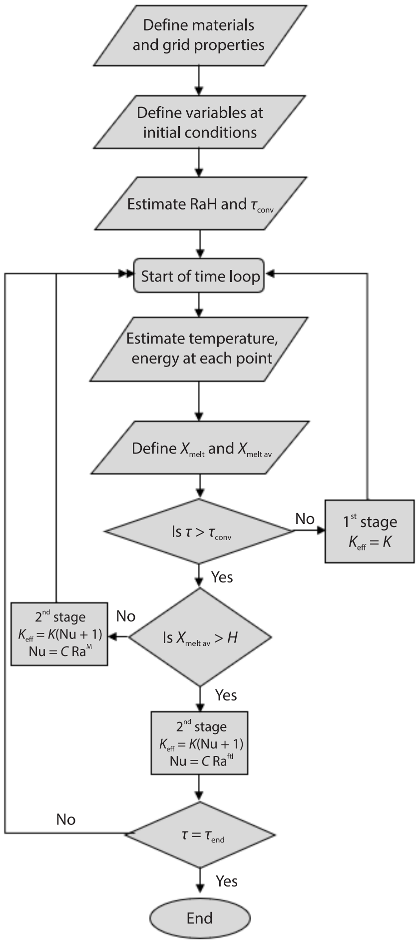

Figure 3. Flow chart explaining detailed steps in MATLAB

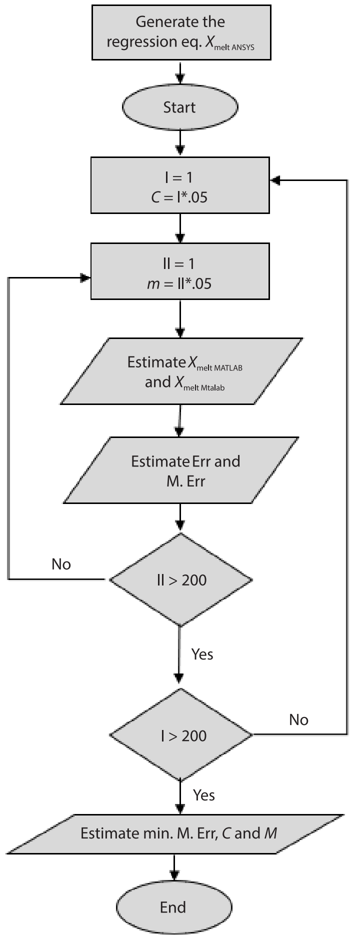

Figure 4. Flow chart explaining detailed steps in MATLAB for estimating equation constants

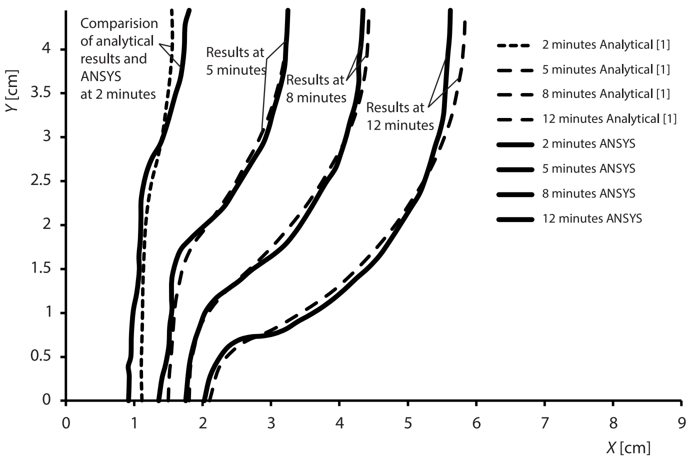

Figure 5. Current ANSYS results validation

Accordingly, the results from CFD model can be used with high accuracy and integrity to validate the results from our numerical approach by MATLAB as shown below for the melting front position with time in fig. 6.

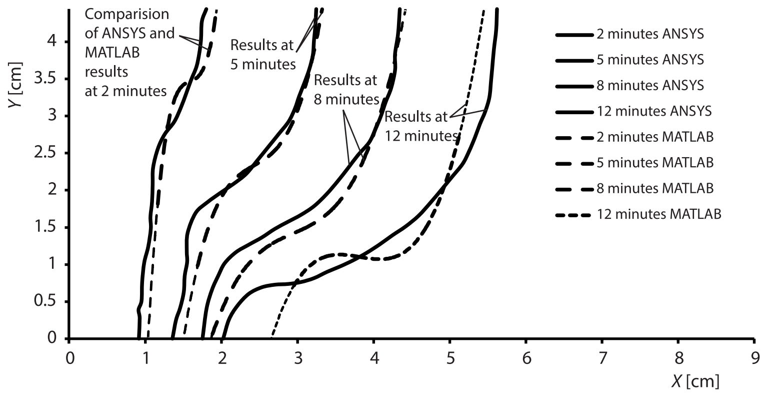

Figure 6. Current MATLAB results *vs.* ANSYS results

Numerical approach by MATLAB used can generate the results in time less than 10 seconds with mean error less than 5.5% from the CFD model when the constants are defined.

Both results from CFD model and numerical approach by MATLAB are complied with melting scenario illustrated by Jany and Bejan [20] that the melting process is passing through three mainly stages can be detailed below with detailed equation used during each stage.

Meting stages

Pure conduction stage analysis:

Where heat transfer by conduction is the main heat transfer mode and the liquid part (convection current) is not enough to play a major role in the heat transfer process during stage.

It is found that this stage duration is greatly affected and controlled by Rayleigh number based on model height, RaH:

$$\tau_{\mathrm{conv}} = 8\ \mathrm{Ra}H^{-1/2} \tag{8}$$

The effective conductivity during this stage is mainly same as the normal conductivity of the PCM material:

$$K_{\mathrm{eff}} = K \tag{9}$$

Mixed conduction and convection stage analysis

At this stage, convection starts sharing in the heat transfer and effect the melting rate and the shape of the melting front in which the speed of the melting front in the top part of model is higher than the below part due to the presence of convection currents. The effective conductivity during this stage is mainly same as the normal conductivity of the PCM material plus the intensity of the convection currents in the top liquid side of the model:

$$K_{\mathrm{eff}} = K\left(\mathrm{Nu}+1\right) \tag{10}$$

The intensity of the convection currents found to be calculated by using the below formula and has a high accuracy as shown in fig. 7:

$$\mathrm{Nu} = 0.1\ \mathrm{Ra}^{0.6} \tag{11}$$

$$\text{Ra} = \frac{g\beta X_{\text{melt}}^3 \left(T_{\text{wall}} - T_{\text{m}}\right)}{\vartheta\alpha} \tag{12}$$

The L_{melt} indicated here is basically changed with every time step and different with each model position in height based on the melting front position.

For the bottom part of the model, the conductivity is the main heat transfer mode and the effective conductivity in this part is mainly same as the normal conductivity of the PCM material.

$$K_{\text{eff}} = K \tag{13}$$

During calculation, limit between the top and bottom part considered during the calculation is taken as height at average position of X_{melt} (time, Y) as explained in fig. 7.

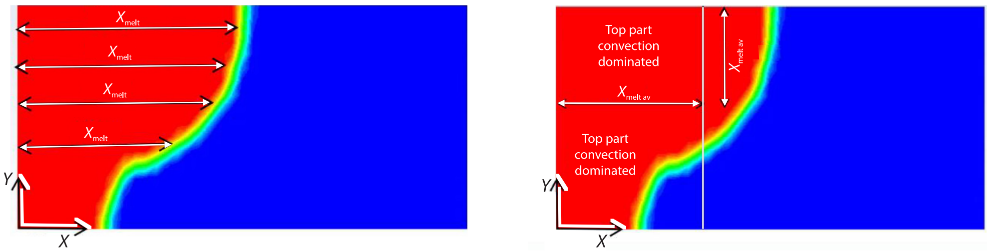

Figure 7. Convections currents effect

Pure convection stage analysis:

At this stage, conduction effect can be neglect and the convection is the main controller for the heat transfer of the liquid part of the model in the entire model height, H:

$$K_{\text{eff}} = K\left(\text{Nu} + 1\right) \tag{14}$$

The intensity of the convection currents found to be calculated by using the below formula and has a high a accuracy as shown in fig. 7:

$$\text{Nu} = 0.1\ \text{Ra}^{0.4} \tag{15}$$

$$\text{Ra} = \frac{g\beta X_{\text{melt}}^3 \left(T_{\text{wall}} - T_{\text{m}}\right)}{\vartheta\alpha} \tag{16}$$

where X_{melt} (time, Y) is basically changed with every time step based on the melting front height position as indicated in fig. 7., C and m are like in tab. 2, for each stage.

Table 2. The *C* and *m* constants values at different melting stages

Stage	*C*	*M*
Pure conduction stage	$K_{\text{eff}} = K$	
Mixed conduction and convection stage	0.1	0.6
Pure convection stage	0.1	0.4

Temperature profiles

Additionally, temperature contours and profiles were investigated for different wall temperatures and *Ar*. Three wall temperatures were examined 308 K, 311.31 K, and 315 K, where Rayleigh number is maintained in laminar range. While, the investigated *Ar* were 1.0,

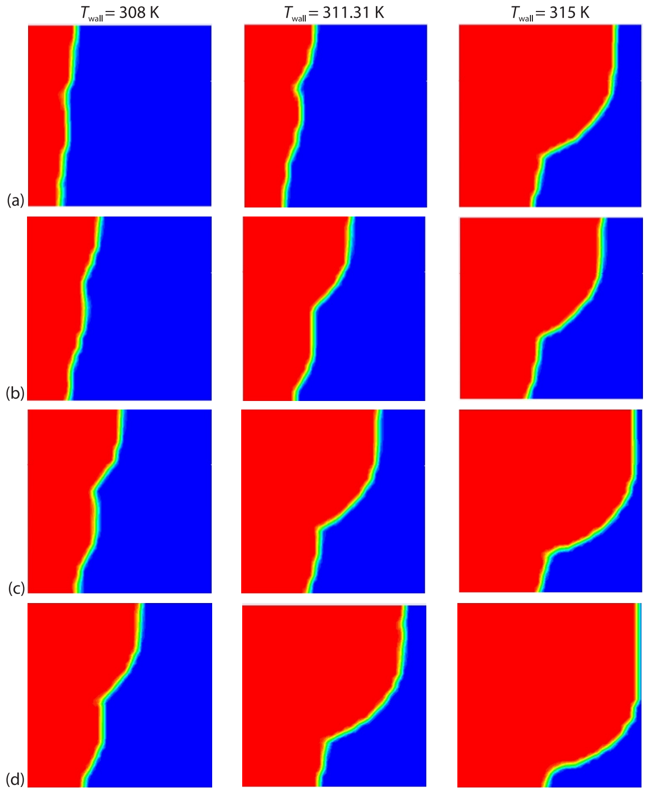

Figure 8. Melting front location for model Ar = 1 at different wall temperature and same time steps; (a) time 2 minutes, (b) time 5 minutes, (c) time 8 minutes, and (d) time 12 minutes

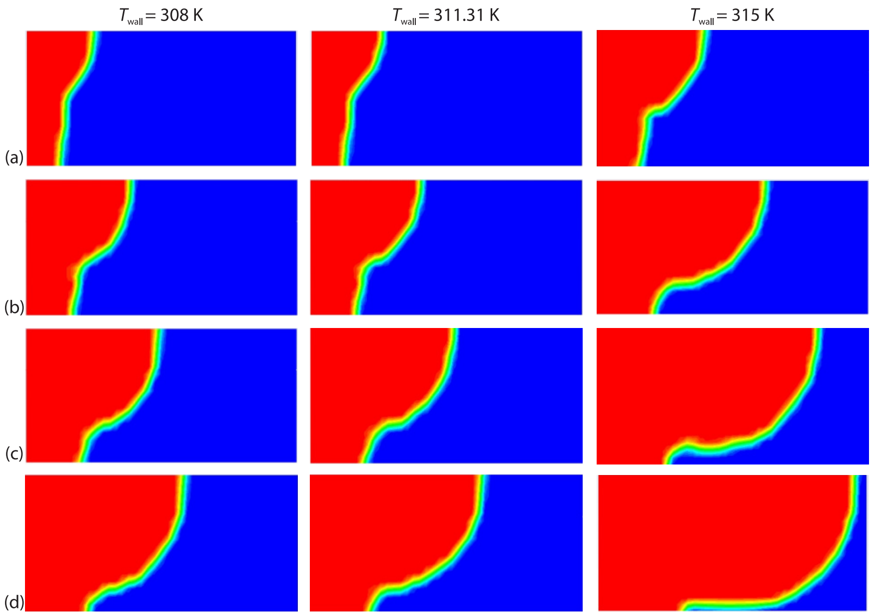

Figure 9. Melting front location for model Ar = 0.5 at different wall temperature and same time steps; (a) time 2 minutes, (b) time 5 minutes, (c) time 8 minutes, and (d) time 12 minutes

0.5, and 2.0. The following CFD simulations shown in figs. 8-10 have been conducted to show the effect of the aspect ratio and wall temperature on the melting process and melting front location respect to time.

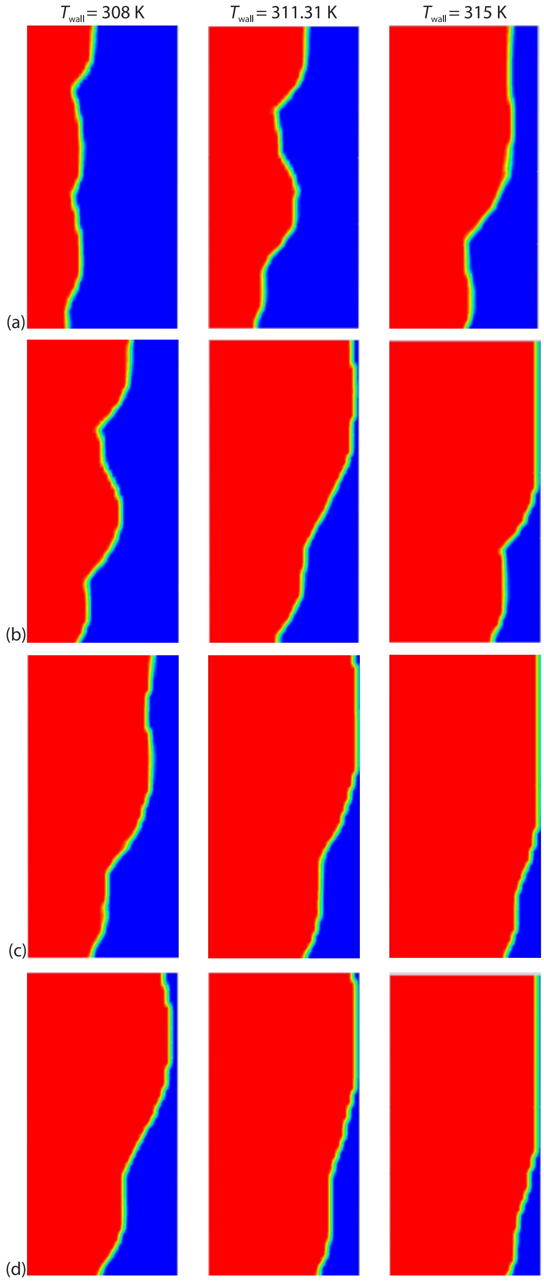

Figure 10. Melting front location for model *Ar* = 2 at different wall temperature and same time steps; (a) time 2 minutes, (b) time 5 minutes, (c) time 8 minutes, and (d) time 12 minutes

The time internals that are presented are 2, 5, 8, and 12 minutes. In reference to fig. 9 showing 0.5 *Ar* results, it is logical that increasing the wall temperature leads to move the melting front more towards the cold wall. Also, the buoyancy force reflect is very clear, which assures that the natural-convection is the main leading aspect in heat transfer of this case. The profiles show that, as long as times passes, the more homogenous is the temperature inside the studied field.

While for 0.5 *Ar*, the below profiles represented in fig. 9 show that there is a serious lag in melting from hot side towards the cold wall. In this ratio, the temperature distribution is less homogenous as compared to $Ar = 1$. However, as time moves on and as long the hot wall is of higher temperature, then, the more hot and homogeneous temperature distribution appears.

The better homogeneous temperature distribution is observed at $Ar = 2$, see below profiles at fig. 10. This is due to the slim configuration in combination with buoyancy force. At a short time of about five minutes, the temperature distribution seems to be fair and well distributed.

Conclusions

The experimental, analytical, and numerical approaches showed good agreements as indicated in figs. 5 and 6. They showed that the melting process is originated by many parameters based on which stage the melting process is passing through.

It was also concluded that convection is the main parameter, and it affects greatly the melting process and melting shape in all melting stages except first pure conduction stage. Convection is affected greatly by melting time, melting shape and other properties as temperature distribution, and conductivity.

Convection share can be estimated by replacing normal PCM material conductivity, K, with effective conductivity, K_{eff}, which takes into consideration convection factors based on melting stages:

– The $K_{\text{eff}} = K$ for Pure conduction stage.
– The $K_{\text{eff}} = K\,(\text{Nu} + 1)$ for mixed conduction and convection stage and pure convection stages.

Also, melting equations are proved that they can be easily estimated with try and error process though MATLAB once validation information is availed through previous experimental or through validated CFD analysis through ANSYS as the subject case of gallium heated from one side where the following equations are generated to simulate the behaviour of melting during different melting stages:

– The $\text{Nu} = 0.1\text{Ra}^{0.6}$ for mixed conduction and convection stage
– The $\text{Nu} = 0.1\text{Ra}^{0.4}$ for pure convection stage.

Detailed steps are provided for clear calculation and estimation of melting process inside rectangular cavities heated from one side and validated against literature experimental and analytical solution by MATLAB. All melting parameters can be estimated in no time by MATLAB and can be used instead since calculated time for melting process also is proved that it can be reduced greatly as it takes 10 seconds to have results similar to CFD model by ANSYS which is very important onday life, by using enthalpy method [19] and conduction equitation in 2-D.

In addition, different geometrical aspects were investigated and presented to show the reflect of different temperatures and aspects on melting as indicated in figs. 8-10. It was deduced that the higher the aspect ratio (more slim configuration), the more homogeneous the distribution.

Nomenclature

Ar – aspect ratio
C – Nusselt equation 1st constant
C_p – specific heat, [JKg^{-1}K^{-1}]
Err – error in $X_{\rm melt}$ for each time step
g – gravitational acceleration, [ms^{-2}]
$h_{\rm melt}$ – latent melting heat, [Jkg^{-1}]
H – model height, [cm]
K – conductivity, [Wm^{-1}K^{-1}]
$K_{\rm eff}$ – effective conductivity, [Wm^{-1}K^{-1}]
L – model width, [cm]
m – Nusselt equation 2nd constant
$M.\ Err$ – error in $X_{\rm melt}$ for each try
N (x or z) – MATLAB model mesh density
Nu – Nusselt number
Ra – Rayleigh number based on melting front position
RaH – Rayleigh number based on model height
$T_{\rm cold}$ – right model wall temperature, [K]
$T_{\rm m}$ – melting temperature, [K]
$T_{\rm wall}$ – left model wall temperature, [K]
t – time, [s]
X – model width axis, [cm]
$X_{\rm melt}$ – melting front position is X-direction, [m]
$X_{\rm melt\ av}$ – melting front average position is X-direction, [m]
Y – model height axis [cm]

Greek symbols

α – thermal diffusion coefficient, [m^{2}s^{-1}]
β – thermal expansion coefficient, [K^{-1}]
μ – dynamic viscosity, [kgm^{-1}s^{-1}]
ρ – density, [kgm^{-3}]
τ – dimensionless time
$\tau_{\rm conv}$ – dimensionless time at convention start effect
ϑ – kinematic viscosity, [m^{2}s^{-1}]

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Paper submitted: October 25, 2021
Paper revised: March 13, 2022
Paper accepted: March 21, 2022