FREQUENCY MODELLING AND DYNAMIC IDENTIFICATION OF CROSS-FLOW WATER TURBINES

by

Daniel Catalin STROITA and Adriana Sida MANEA *

Faculty of Mechanical Engineering, Politehnica University of Timisoara, Timisoara, Romania

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The cross-flow turbines cover from the point of view hydraulic power the running domain of some well-known turbines such as Pelton, Francis or Kaplan. This type of turbine has a simple construction, long life and low execution cost, which makes it very suitable for on and off grid small to medium hydro power plants. It is quite difficult to establish an exact theoretical dynamic model for this type of turbines, due to the complex flow phenomenon (bi phase flow water and air). In order to obtain the exact dynamic behavior of the hydraulic machine, experimental dynamic identification will be done. In automation, the dynamic properties, represent the fundamental characteristic of the object which must be regulated. When the dynamic properties of the regulated object are obtained experimentally, we analyze the characteristics of the transient regime, which appears because of the application at the system inlet of some stochastic or deterministic signals (sine waves for our case). The hydraulic turbine is modeled as an informational quadrupole having the inlet parameters the movement of the wicket gate and the turbine head and outlet parameters the torque and the speed. In this paper it will be presented the frequency modelling of the cross-flow turbine and the validation of the mathematical model through experimental dynamic identification.

Key words: *cross-flow turbine, frequency modelling, fast Fourier transform, frequency place, Bode characteristic*

Introduction

The hydrographic potential of large rivers being largely exploited, the use of hydraulic energy of small and medium rivers is very important from an economic point of view, the demand for green energy being increasing in our days, due to high atmospheric pollution.

The cross-flow turbine has a simple construction, does not demand complex hydropower developments, has a facile design and low execution cost, making it very suitable for small to medium hydropower plants and it can be installed directly in river bed.

The energy produced by this type of turbines can be used on and off grid, this is why is very important to know the dynamic behavior, for the design of the speed regulator.

Compared with other types of turbines which demand complex constructions the use of cross-flow turbines does not affect too much the environment, making it quite eco friendly.

In cross-flow turbines the water gets out from the nozzle, passes through the blades going from periphery to center, then passes through the free zone from the interior of the run-

* Corresponding author, e-mails: adrianasida@yahoo.com; adriana.manea@upt.ro

ner and passes again through the blades, going from center to periphery this time, giving each time a part of energy to the runner. Like this the water stream energy is transferred to the runner in the process of double passing of the water through the blades. The water path and the velocity triangle for this type of hydraulic machines are presented in fig. 1.

The cross-flow turbines, due to their construction work at atmospheric pressure and the cavitation phenomenon does not appear disappearing the obligation of locating the runner under the water level, reducing quite a lot the costs.

The use of the general theory of system identification in order to solve the numerous problems found at their design phase as for the transient process analysis in the case of the exploiting signals is generally hard and extremely complex [1, 2].

These difficulties impose the creation of some engineering methods of approximate solving of the most known design problems and adjusting of the automation of processes. The most used method is the dynamic identification with periodic probe signals. The use of these probe signals bring the researched installation in forced oscillations regime, permitting the detection and extraction of the noises and different perturbations over the main signal from outlet.

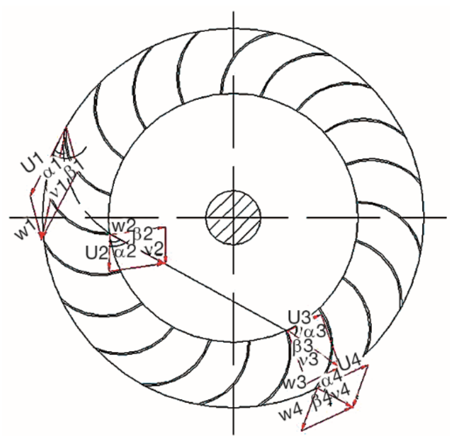

Figure 1. Water path and velocity triangles in a cross-flow runner

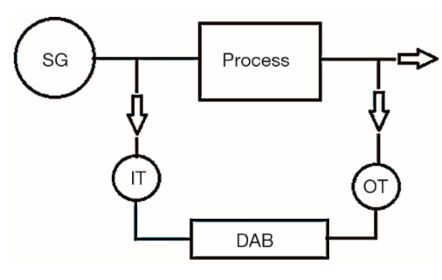

Figure 2. Block scheme for obtaining the frequency characteristics

This method obtains the frequency response of the researched process, which can be used directly in the design calculus of the system automation.

In the block scheme from fig. 2, is shown the principle of system identification with sine wave probe signals [1, 2], where SG is the signal generator, IT represents the process inlet transducers and OT are the ones from the outlet, the signals being recorded in real time through the data acquisition board (DAB). In the case of a linear system the oscillations from inlet differ from the ones from outlet through amplitude and phase. In order to determine the frequency characteristics the two signals are compared for different pulsations. The inlet and outlet signals for different pulsations are recorded on the same graphic and based on this are obtained the amplitude-phase and pulsation-phase characteristics also called the BODE diagrams.

Further we present the two parts of this paper the proposed mathematical model and the experimental dynamic identification for the researched cross-flow turbine [1].

Frequency modelling of cross-flow turbine dynamics

Frequency modeling of cross-flow turbine dynamics, which is an impulse type turbine is based on the movement equation [1, 3-10]:

$$T_{\mathrm{a}}\frac{\mathrm{d}\varphi}{\mathrm{d}t} = C_{\mathrm{t}}\Delta\mu - \theta_{\mathrm{a}} + C_f\zeta \tag{1}$$

where $T_a = J\omega^2/P$ is the accelerating time of the turbine at startup, $C_t = 1/(1 - \mu_r)$ – a constant which depends on turbine type, μ_r – the relative wicket gate opening in permanent regime, θ_a – the global auto regulation coefficient of the turbine and generator, $C_f = (\mu_c - \mu_r)/(1 - \mu_r) + 0.5\beta_m$ – a constant which characterize the running regime of the aggregate, μ_c – the relative open of the guide vane in the new established regime, β_m – the turbine auto regulation coefficient, φ – the relative deviation of the angular speed of the aggregate, and ζ – the relative changing of turbine's head.

The relative deviation of the servomotor rod is noted with $\Delta\mu$, different by μ, which corresponds to completely closed position of the guide vane. Considering the maximum elasticity of water and of pipe wall, in order to obtain ζ, we have the equation:

$$\zeta = -T_{\mathrm{wa}}\left(\frac{\mathrm{d}\Delta\mu}{\mathrm{d}t} + 0.5\mu_c \frac{\mathrm{d}\zeta}{\mathrm{d}t}\right) \tag{2}$$

where $T_{\mathrm{wa}} = (L_B v_{\max})/(gH_0)$ is the water mass accelerating time in the penstock.

Considering the water elasticity and the penstock walls, the changing of the head is obtained through the solving of partial derivative equation:

$$\frac{\partial v}{\partial t} = g\frac{\partial H}{\partial L}, \quad \frac{\partial H}{\partial t} = \frac{a^2}{g}\frac{\partial v}{\partial L} \tag{3}$$

where L is the distance from turbine to a reference distance of the penstock, H – the turbine head, v – the water velocity, and a – the celerity.

A limit condition for getting the water hammer in the considered equations is to maintain constant the head in the penstock: $L = L_B$ and $H = H_0$.

Doing the math and taking the ratio between the outlet parameter φ and inlet parameter $\Delta\mu$, we obtain the transfer function for the turbine:

$$W_a(s) = W_a'(s)\left\lfloor 1 + \frac{C_f}{C_t}W_{lb}(s)\right\rfloor, \quad W_a'(s) = \frac{C_t}{T_a s + \theta_a} \tag{4}$$

In the second term we have the transfer function of the water hammer considered as a separate part included in the regulation system.

Without considering the elasticity we have $W'_{\mathrm{WH}}(s)$ when it is taken in consideration the elasticity we have the following transfer function $W''_{\mathrm{WH}}(s)$:

$$W_{lb}(s) = W'_{\mathrm{WH}}(s) = -\frac{T_{\mathrm{wa}}s}{0.5\mu_c T_{\mathrm{wa}}s + 1}, \quad W_{lb}(s) = W''_{\mathrm{WH}}(s) = -\frac{1 - e^{-\tau s}}{b_1 + b_2 e^{-\tau s}} \tag{5}$$

where $\tau = 2L/c$ is reflection time of waves.

The relations in order to obtain the turbine amplitude-phase coordinates from the presented transfer functions are obtained for $s = i\omega$ using the rules of complex numbers. Representing $W_a(i\omega)$ as a complex number $W_a = x_a - iy_a$, we obtain the coordinates of turbine's hodograph:

$$x_a = \left\lfloor x\left(\frac{C_t}{C_f} - x_{\mathrm{WH}}\right) - yy_{\mathrm{WH}}\right\rfloor\frac{C_f}{C_t}, \quad y_a = \left\lfloor y\left(\frac{C_t}{C_f} - x_{\mathrm{WH}}\right) - xy_{\mathrm{WH}}\right\rfloor\frac{C_f}{C_t} \tag{6}$$

where x, x_{WH} are corresponds to real part of the amplitude-phase without and with water hammer and y, y_{WH} – are the imaginary parts of the amplitude-phase characteristic.

Developing eq. (4) we have:

$$x = \frac{C_{\mathrm{t}}\theta_{\mathrm{a}}}{\theta_{\mathrm{a}}^2 + T_{\mathrm{a}}^2\omega^2}, \quad y = \frac{C_{\mathrm{t}}T_{\mathrm{a}}\omega}{\theta_{\mathrm{a}}^2 + T_{\mathrm{a}}^2\omega^2} \tag{7}$$

The expressions for x_{WH} and y_{WH} depend on the water hammer took into consideration:

$$x_{\mathrm{WH}} = x'_{\mathrm{WH}} = \frac{0.5\mu_{\mathrm{c}}T_{\mathrm{wa}}^2\omega^2}{1 + 0.25\mu_{\mathrm{c}}^2T_{\mathrm{wa}}^2\omega^2}, \quad y_{\mathrm{WH}} = y'_{\mathrm{WH}} = \frac{T_{\mathrm{wa}}\omega}{1 + 0.25\mu_{\mathrm{c}}^2T_{\mathrm{wa}}^2\omega^2} \tag{8}$$

If we consider the elasticity of water and of the penstock walls we obtain:

$$x_{\mathrm{WH}} = x''_{\mathrm{WH}} = \frac{(b_1 - b_2)[1 - \cos(\omega\tau)]}{(b_1 + b_2)^2 - 2b_1b_2[1 - \cos(\omega\tau)]}$$

$$y_{\mathrm{WH}} = y''_{\mathrm{WH}} = \frac{(b_1 + b_2)\sin(\omega\tau)}{(b_1 + b_2)^2 - 2b_1b_2[1 - \cos(\omega\tau)]} \tag{9}$$

As input parameters for the modelled turbine we have [1, 11]

– Turbine power P = 100 W
– Nominal speed n0 = 460 rpm
– Turbine head H0 = 2.5 m
– Penstock length LB = 3.9 m
– Medium velocity through penstock at maximum load vmax = 1 m/s
– Autoregulation turbine coefficient βm = 1
– Autoregulation coefficient at generator coupling γ = 0
– Inertia momentum of moving parts $J = 7.688 \cdot 10^{-4}$ kgm^2

With the previous input data we can calculate:

– Turbine acceleration time at startup: $T_{\mathrm{a}} = (J\omega^2)/P_0 = 0.018$ seconds
– Penstock time: $T_{\mathrm{wa}} = (L_{\mathrm{B}}v_{\mathrm{max}})/(\mathrm{g}H_0) = 0.191$ seconds
– Wave reflection time: $\tau = (2L_{\mathrm{B}})/c = 0.0065$ seconds
– Turbine constants at maximum load $C_{\mathrm{t}} = 1.253$; $C_f = 1.375$

In fig. 3 is presented the frequency function $F(\varphi/\Delta\mu)$ with and without the water and penstock elasticity for $\omega = 0...25$ rad per seconds pulsations at a relative wicket gate variation between 0.2 and 0.9, for the analyzed cross-flow turbine.

Figure 3 shows that in the case of short penstock the model can be used without considering the elasticity of water and penstock, both curves being almost superposed. If same hydro aggregate will have a 1000 m penstock and a head of 50 m after using the frequency model we can see a complete different frequency function shown in fig. 4.

Experimental study of cross-flow turbine dynamics

The experimental tests were done in the Hydraulic Machinery Laboratory of Mechanical Faculty of Politehnica University of Timisoara, at a constant turbine head $H = 2.5$ m, varying the volumetric flow rate from $Q_{\mathrm{min}} = 2.85$ Lps to $Q_{\mathrm{max}} = 7.9$ Lps. The speed range achieved was from $n = 0$ to 960 rpm, for wicket gate openings $\alpha = 5$, 7.5, 10, 12.5, 15, 17.5, 20°. The cross-flow turbine sketch together with the measurements apparatuses and testing rig are presented in figs. 5 and 6.

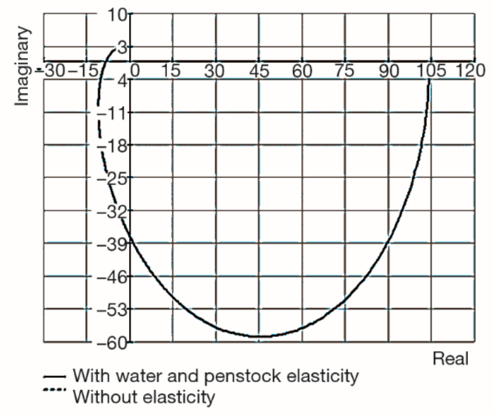

Figure 3. The frequency function $F(\varphi/\Delta\mu)$ for the tested cross-flow turbine (as installed in laboratory)

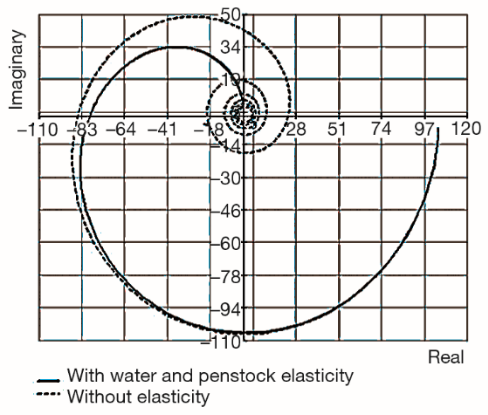

Figure 4. The frequency function $F(\varphi/\Delta\mu)$ for the tested cross-flow turbine with an imaginary 1000 meters penstock and 50 m head

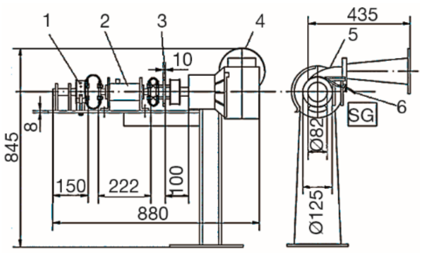

Figure 5. Testing rig scheme;
1 – band brake, 2 – torque transducer, 3 – speed transducer, 4 – flow transducer, 5 – pressure transduce, 6 – position transducer

Figure 6. Testing rig

In laboratory conditions in stationary regime have been obtained efficiencies between 10% and 50%, which are presented in the hill chart of the cross-flow turbine in fig. 7.

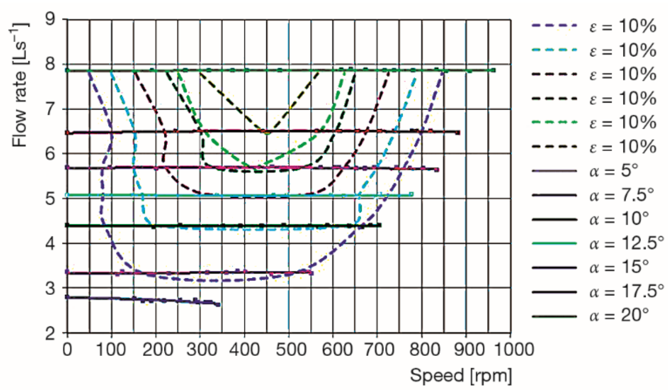

Figure 7. Cross-flow turbine hill chart

The method used for experimental dynamic identification of cross-flow turbine is the one with periodical sine probe signals [1, 2]. The sinusoidal variation of the movement of the wicket gate is ensured with a system with a continuous current electric motor and cylindrical cams, which can be seen in fig. 6. The cams will impose a sinusoidal variation at the inlet of the system with an amplitude equal with the cam eccentricity and the frequency will be controlled changing the voltage (speed) of the continuous current motor.

In order to apply the dynamic identification with sinusoidal probe signals there are necessary two up to four oscillations. In the 1024 analyzed samples we are having always more than 4.

In order to obtain the two most important parameters, dominant amplitude and fundamental frequency, necessary for the construction of the frequency place of the transfer function, we used the mathematical model from the fast Fourier transform (FFT).

The turbine response to the wicket gate movement was recorded for nine different frequencies, monitoring the interest parameters: wicket gate movement, inlet pressure (corresponding to turbine head), torque and speed (corresponding to output power).

The measurements done for the two cams with 5 mm and 21 mm eccentricity at three different frequencies, figs. 8-10, 13-15 together with the frequency place and the BODE characteristics for the researched cross-flow water turbine are presented in figs. 11, 12, 16, and 17.

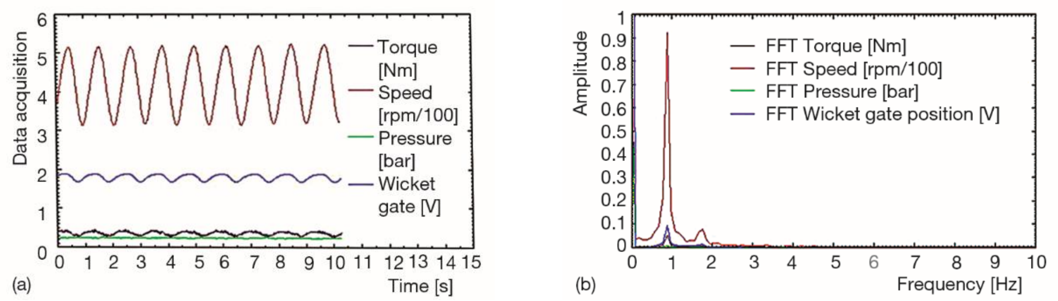

Figure 8. Measurements with 5 mm eccentricity cam, 0.9 Hz frequency and fast fourier transform of data

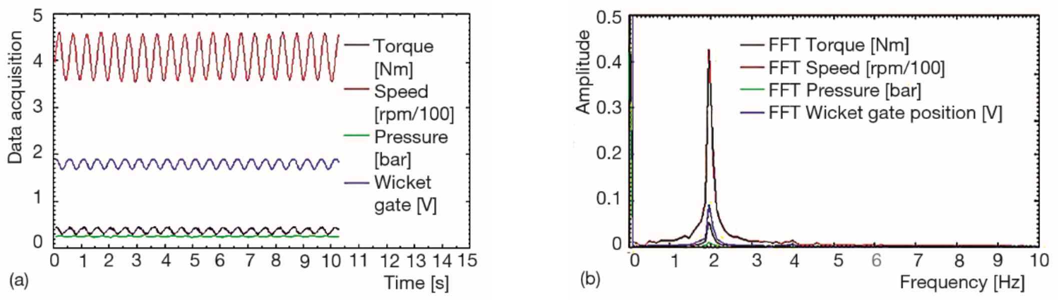

Figure 9. Measurements with 5 mm eccentricity cam, 1.95 Hz frequency and fast fourier transform of data

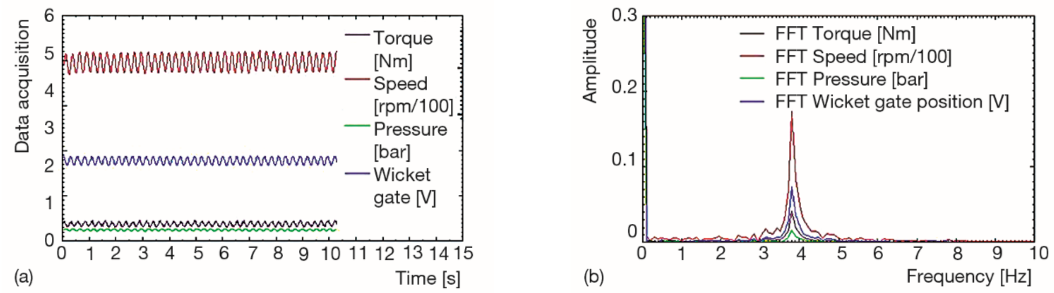

Figure 10. Measurements with 5 mm eccentricity cam, 3.8 Hz frequency and fast fourier transform of data

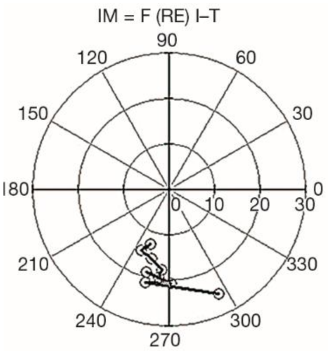

Figure 11. Frequency place corresponding to the transfer function between the wicket gate movement and speed for the 5 mm amplitude cam

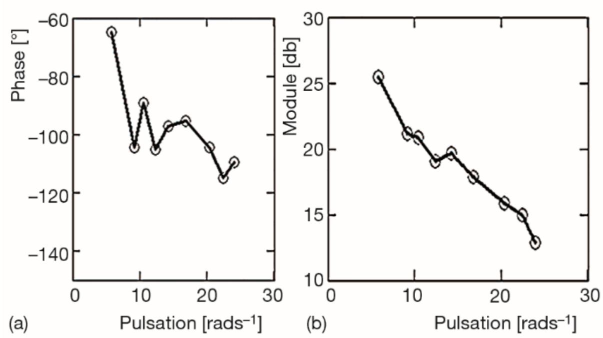

Figure 12. Bode characteristic corresponding to the transfer function between wicket gate movement and speed for the 5 mm amplitude; (a) phase-pulsation characteristic corresponding to transfer function between wicket gate movement and speed and (b) amplitude-pulsation characteristic corresponding to transfer function between wicket gate movement and speed

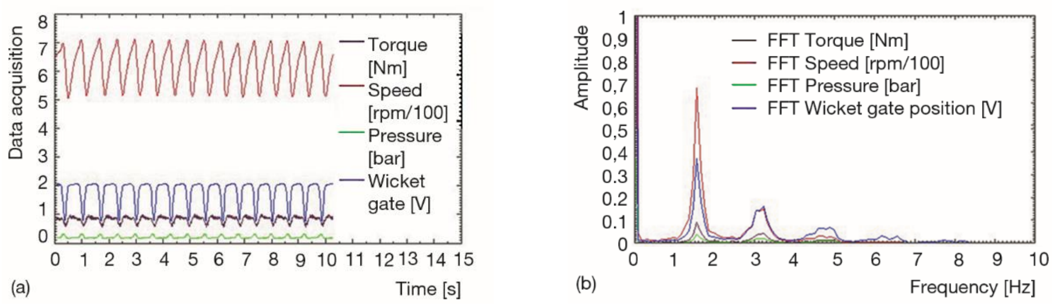

Figure 13. Measurements with 21 mm eccentricity cam, 1.55 Hz frequency and fast fourier transform of data

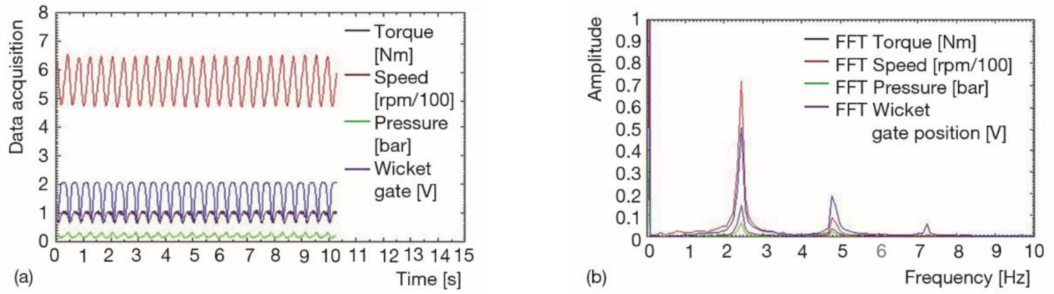

Figure 14. Measurements with 21 mm eccentricity cam, 2.5 Hz frequency and fast fourier transform of data

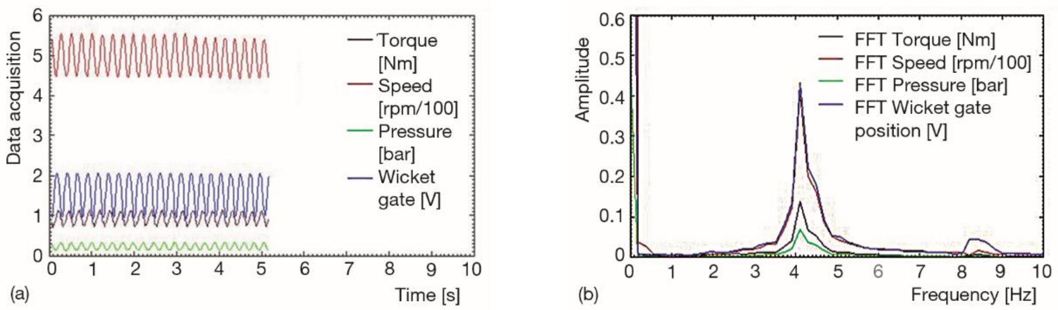

Figure 15. Measurements with 21 mm eccentricity cam, 4.1 Hz frequency and fast fourier transform of data

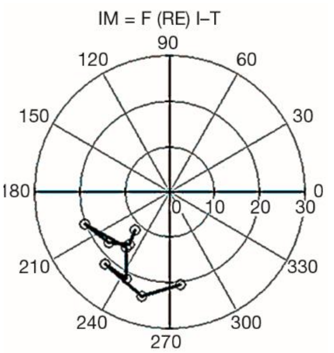

Figure 16. Frequency place corresponding to the transfer function between the wicket gate movement and speed for the 21 mm amplitude cam

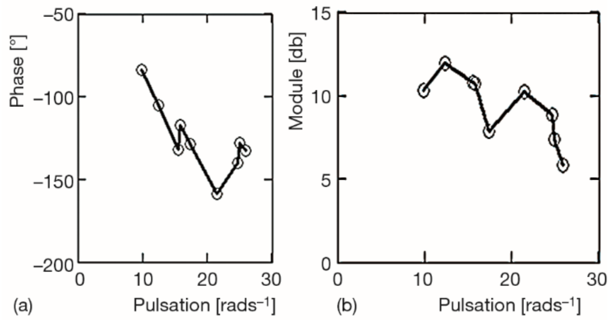

Figure 17. Bode characteristic corresponding to the transfer function between wicket gate movement and speed for the 21 mm amplitude; (a) phase-pulsation characteristic corresponding to transfer function between wicket gate movement and speed and (b) amplitude-pulsation characteristic corresponding to transfer function between wicket gate movement and speed

Figures 18 and 19 show the comparison between the proposed mathematical model and the results obtained through experimental dynamic identification.

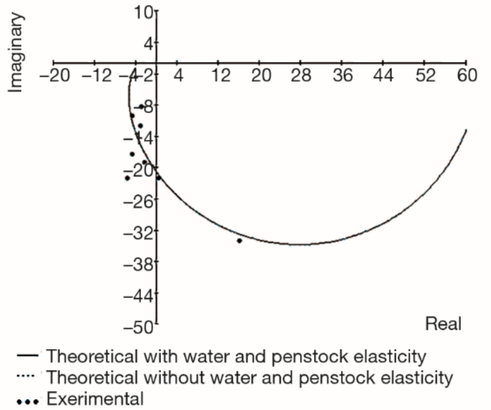

Figure 18. Transfer function for 5 mm amplitude cam

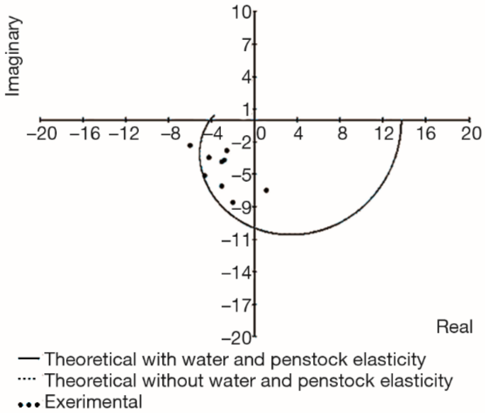

Figure 19. Transfer function for 21 mm amplitude cam

Conclusions

- Although the generation of sinusoidal signals at the inlet of some systems is quite difficult, the use of this method is motivated through quite easy data processing in order to obtain the transfer function, being necessary just the comparison of the inlet signal with the outlet one.
- The selection of the complexity of the mathematic model for a hydraulic turbine is done as a function of the studied physical process and of the phenomenon which appear. For the studied cross-flow turbine case, the frequency modelling highlights the influence of the penstock length and of the water elasticity.
- At small oscillation frequencies of the inlet signal, it has been observed a periodical variation of the recorded data, but different from a pure sinusoidal variation. Once with the increase of the frequency the periodic variation becomes quasi sinusoidal, showing the non-linear characteristic of the hydraulic machine.
- Comparing the transfer functions obtained experimental with the theoretical ones shows that the proposed mathematic model offers a good first approximation for the cross-flow turbines dynamic behavior.
- Analyzing the dynamic measurements after FFT, it has been observed that with the increase of the inlet signal frequency, the amplitude of the inlet pressure and torque increases and the speed amplitude decreases.
- The signal characteristics in order to draw the frequency place corresponding to the studied transfer functions were obtained through FFT, which is a helpful mathematic tool that can be used also in system identification.

Nomenclature

a	– celerity, [ms^{-1}]
b_1	– constants $0.5 = [(T_r/T_W + \mu_C)]$, [–]
b_2	– constants $0.5 = [(T_r/T_W + \mu_C)]$, [–]
C_f	– is a constant which characterize the running regime of the aggregate, [–]
C_t	– represents a constant which depends on turbine type, [–]
H	– the turbine head, [m]
H_0	– studied turbine head, [m]
J	– inertia momentum of moving parts, [kgm^2]

L – the distance from turbine to a reference distance of the penstock, [m]
L_B – studied turbine penstock length, [m]
n_0 – studied turbine nominal speed, [rpm]
P – studied turbine power, [W]
T_a – studied accelerating time of the turbine at startup, [s]
T_{wa} – studied turbine penstock time, [s]
t – time, [s]
v – water velocity, [ms^{-1}]
v_{max} – medium velocity through penstock at maximum load, [ms^{-1}]
$W_a(s)$ – turbine transfer function considering the watter hammer
$W'_a(s)$ – water hammer part of transfer function
$W'_{WH}(s)$ – turbine transfer function considering the water hammer without water and penstock walls elasticity
$W''_{WH}(s)$ – turbine transfer function considering the water hammer and water and penstock walls elasticity
x_a, y_a – real and imaginary part of W_a transfer function
x, x_{WH} – real parts of the amplitude-phase characteristics without and with water hammer
y, yx_{WH} – imaginary parts of the amplitude-phase characteristic without and with water hammer
x'_{WH}, x''_{WH} – real parts of amplitude-phase characteristic without and with taking into consideration the water and penstock elasticity
y'_{WH}, y''_{WH} – imaginary parts of amplitude-phase characteristic without and with taking into consideration the water and penstock elasticity

Greek symbols

β_m – turbine auto regulation coefficient, [–]
γ – autoregulation coefficient at generator coupling, [–]
ζ – relative changing of turbine's head, [–]
θ_a – global auto regulation coefficient of the turbine and generator, [–]
μ – completely closed position of the guide vane
μ_c – relative open of the guide vane in the new established regime, [–]
μ_r – relative wicket gate opening in permanent regime, [–]
$\Delta\mu$ – relative deviation of the servomotor rod, [–]
φ – relative deviation of the angular speed of the aggregate, [–]
τ – reflection time of waves, [s]
ω – pulsation, [rads^{-1}]

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