SEDIMENTATION OF TWO SPHERES IN A SQUARE TUBE

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In this study, we simulated the sedimentation of two identical spheres having the same density in a square tube. Compared with the center-line and the diagonal planes (including the reverse diagonal plane), the sedimentation of spheres on other planes is more complicated. Results show that at relatively low and high Reynolds number, the spheres will deflect and eventually move to the diagonal plane of the square tube. At the medium Reynolds number, the spheres settle near the initial plane. The possible mechanisms underlying these behaviors are examined. Finally, it is shown that the distance between the spheres increases with an increase in the Reynolds number, which is applicable to all the initial settlement planes studied.

Key words: *sedimentation, Reynolds number, center-line plane*

Introduction

Multiple particles in a fluid generate hydrodynamic interactions, which will have a tremendous effect on the movement of particles and the overall properties of the suspension under the inertial force of the particles and the fluid, Because of its simplicity and convenience, 2-D flow has always been favored by researchers. The drifting-kissing-tumbling Fortes *et al.* [1] phenomenon has been revealed through the finite element method [2]. Other studies have examined the effect of particle size, Wang *et al.* [3], Reynolds number, Nie *et al.* [4], array mode, Yacoubi *et al.* [5] the distances between particles, and Nie *et al.* [6] on particle sedimentation.

Regarding the 3-D aspects of the settlement system, previous research has focused on the movement patterns of particles [7, 8] and the wall effect, Luo *et al.* [9] when they settle. It is worth noting that particles have different stable settlement planes under different Galileo numbers, Nie *et al.* [10], a topic that has inspired our research. The purpose of our study is to provide a comprehensive understanding of the interaction of two-particle sedimentation in a square tube.

Lattice Boltzmann method

In this work, 3-D lattice Boltzmann method (LBM) is employed. The discrete lattice Boltzmann equation of the single-relaxation time model is:

$$f_i(\boldsymbol{x}+e_i\Delta t, t+\Delta t) - f_i(\boldsymbol{x},t) = -\frac{1}{\tau}[f_i(\boldsymbol{x},t) - f_i^{(0)}(\boldsymbol{x},t)] \tag{1}$$

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where $f_i(\boldsymbol{x}, t)$ is the distribution function of the microscopic velocity, e_i, in the i^{th} direction, $f_i^{(0)}(\boldsymbol{x}, t)$ – the equilibrium distribution function, Δt – the time step of the simulation, and τ – the relaxation time and is related to the fluid viscosity, ν.

The density and velocity of fluid are computed:

$$\rho = \sum_i f_i \quad , \quad \rho \boldsymbol{u} = \sum_i f_i \boldsymbol{e_i} \tag{2}$$

For the 3-D particle settlement simulation, the 19-velocity (D3Q19) lattice model is used. The discrete velocity vector is:

$$\boldsymbol{e_i} = \begin{cases} (0,0,0)c & i = 0, \\ (\pm 1,0,0)c, (0,\pm 1,0), (0,0,\pm 1)c & i = 1\text{-}6 \\ (\pm 1,\pm 1,0)c, (\pm 1,0,\pm 1)c, (0,\pm 1,\pm 1)c & i = 7\text{-}18 \end{cases} \tag{3}$$

The equilibrium distribution function $f_i^{(0)}(x, t)$ is defined by:

$$f_i^{(0)}(\boldsymbol{x}, t) = w_i \rho \left[1 + \frac{3e_i \cdot u}{c^2} + \frac{9(e_i \cdot u)^2}{2c^4} - \frac{3\mathbf{u} \cdot \mathbf{u}^2}{2c^2} \right] \tag{4}$$

where $c = \Delta x/\Delta t$, in which Δx is the lattice separation distance. The speed of sound is related to $c_s^2 = c^2/3$, and the weights are set as $w_0 = 1/3$, $w_{1\text{-}6} = 1/18$, and $w_{7\text{-}18} = 1/36$.

For the convenience of calculation, the lattice grid, Δx, and time step, Δt, are both fixed at 1, settings adopted in most lattice Boltzmann simulations. In addition, an interpolation-based bounce-back scheme is employed to treat the curved boundaries of solid particles, Lallemand *et al.* [11]. To account for the lubrication effects between particles at small distances, a lubrication force model is also introduced, Nguyen *et al.* [12].

Problem description

In this work, we study the settlement motion of two interacting spheres in a square tube. The computational model is shown in fig. 1(a). Two spheres are released at rest in a square tube containing a fluid with a density, ρ, and a kinematic viscosity, ν. The diameter and density of these two spheres are expressed as d and ρ_s, respectively. The tube width, L, is $5d$ with a height, H, of $25d$. The center-line, diagonal, and reverse diagonal planes of the square tube are marked in fig. 1(b). We use a movement field to mimic an infinite tube. Nonslip boundary conditions are applied to all the walls of the tube. At the upper and bottom boundaries, the normal derivative of the velocity is 0.

Because of the unpredictability of the final velocities of particles, the velocity scale is placed at:

$$U_0 = \sqrt{\left(\frac{\rho_s}{\rho} - 1 \right) \mathrm{g} d} \tag{5}$$

where g is the gravitational acceleration, which is fixed at $\mathrm{g} = 9.8 \times 10^{-4}$. The density of the fluid and the two spheres are fixed at $\rho = 1$, $\rho_s = 1.5$. The diameters of the two spheres are fixed at $d = 20$. The time scale is fixed at $T_0 = d/U_0$. In addition, all parameters are lattice parameters. Spheres 0 and 1 are used to distinguish these two identical spheres. Before releasing the spheres under gravity, they are placed as shown in fig. 1(b). In fig. 1(b), $D_0 = 2d$, where D_0 is the distance between the centers of the two spheres, Note that θ_0 is the angle between the initial and center-line planes, which has four values of 5°, 15°, 25°, and 35°. In the simulation, the initial positions of Spheres 0 and 1 are determined by the angle θ_0.

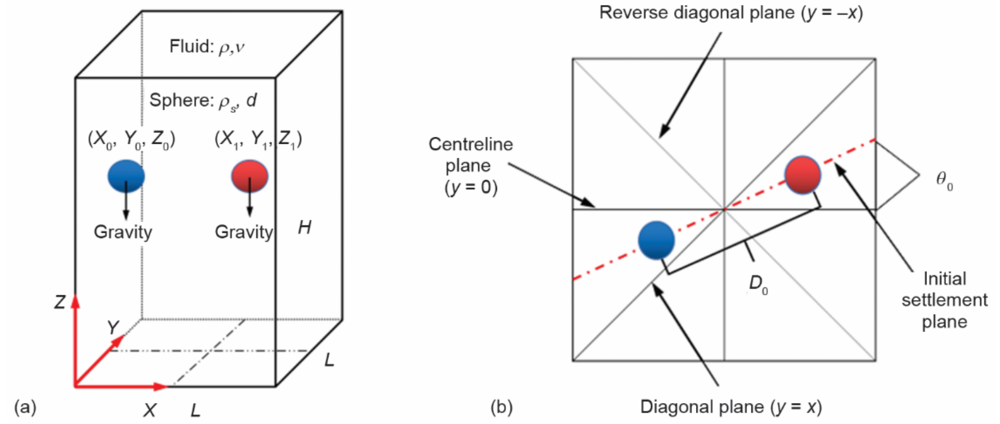

Figure 1. Diagrammatic description of two spheres of sedimentation in a square tube

Numerical results

In the simulation, when the particle sedimentation was fully developed, the particles eventually reached steady-state sedimentation on a single plane. To illustrate more effectively the situation of particle sedimentation in the square tube, we used different Reynolds numbers (Re = 20, 60, 120, and 140) at θ_0 = 5° to present the particle trajectories after steady-state sedimentation, as shown in fig. 2.

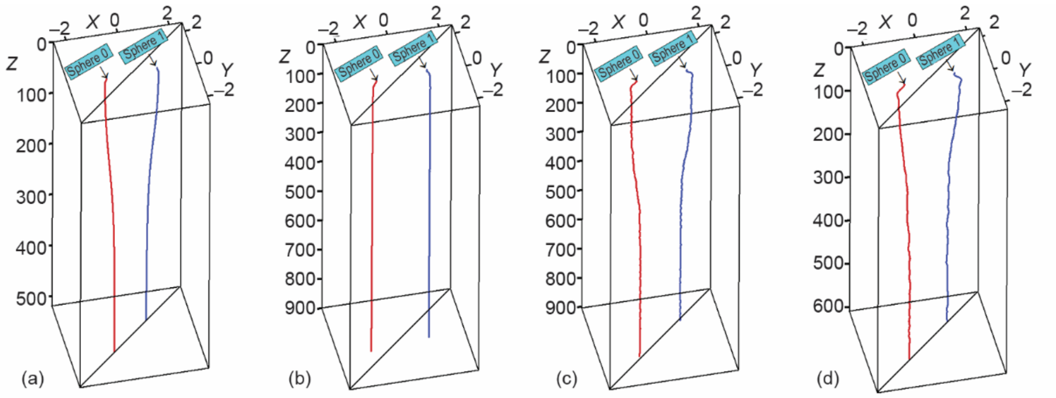

Figure 2. Trajectories of the spheres for θ_0 = 5°; (a) Re = 20, (b) Re = 60, (c) Re = 120, and (d) Re = 140

Figure 2(a) presents the trajectories of the two spheres for θ_0 = 5° at Re = 20, where it can be observed that the two spheres first repelled each other, and then began to leave the initial plane. A period then passed before they moved to the diagonal plane. Nevertheless, the spheres may remain at the initial settlement plane at a relatively medium Reynolds number value, and fig. 2(b) shows the trajectories of the two spheres for θ_0 = 5° at Re = 60. Different behaviors were observed, namely, the spheres did not seem to leave the initial plane after repelling each other. When the Reynolds number reached relatively high values, such as

Re = 120 and 140, as shown in figs. 2(c) and 2(d), respectively, the same movement as that of a lower Reynolds number reappeared.

Figure 3 shows the 3-D structure of the flow field around the particles after stable settlement for θ_0 = 5° at Re = 120. The vortical structure presented in fig. 3 was proposed by Jeong and Hussain [13], who considered the tensor $\mathbf{S}^2 + \mathbf{\Omega}^2$ to determine whether there was a local pressure minimum derived from vortical motion, where $\mathbf{S}$ and $\mathbf{\Omega}$ are the symmetrical and anti-symmetrical parts of the velocity gradient tensor, respectively. Because of the symmetry of $\mathbf{S}^2 + \mathbf{\Omega}^2$, only real eigenvalues exist for $\mathbf{S}^2 + \mathbf{\Omega}^2$. In addition the vortical structures are identified by $\lambda_2 < 0$ within the vortex core, where λ_2 is the second largest eigenvalue of $\mathbf{S}^2 + \mathbf{\Omega}^2$. Figure 3 shows that, in our study, the vortex motion was concentrated in the thin torus-area surrounding the spheres.

Here, the parameter δ was introduced to describe the effect of θ_0 on particle settlement, where δ is determined by $\delta = (\theta_1-\theta_0)/(45°-\theta_0)$ to describe the deflection rate of particles, and θ_1 represents the angle between the steady settlement and center planes. The result is shown in fig. 4, where the effects of θ_0 on different Reynolds number values were different. It was observed that the value of θ_0 had little effect on the δ of particles when Reynolds number was relatively high or low, as shown in fig. 4. However, the δ of particles was more dependent on the value of θ_0 at a relatively moderate Reynolds number. For example, large fluctuations could not be observed when Re = 20, 40, and 120 but appeared at Re = 60, 80, and 100.

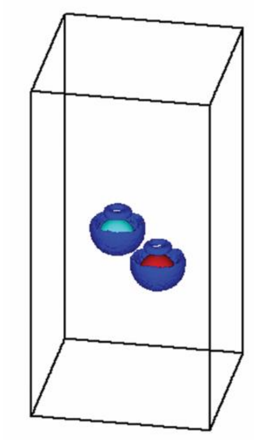

Figure 3. Reference results for θ_0 = 5° (Re = 120), the surface where λ_2 = –0.0004

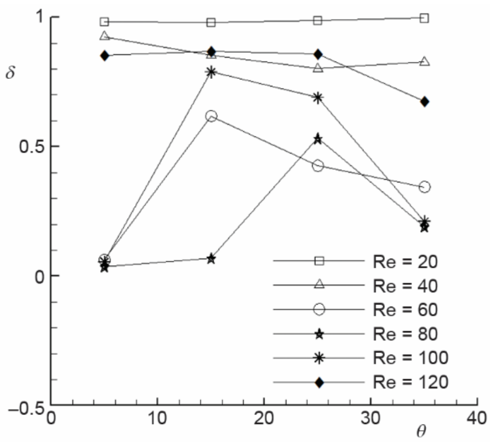

Figure 4. Deflection rate, δ, of particles with θ_0 = 5°,15°, 25°, and 35° at different Reynolds number

To examine further settlement behavior of the particles in the tube, we observed angle θ_1, which was affected by θ_0 and Reynolds number. Figure 5(a) shows the corresponding results for θ_0 = 5°, 15°, 25°, and 35° at different Reynolds number values. As fig. 5(a) indicates, for all θ_0, θ_1 showed a trend of first declining and then increasing. However we observed that an abnormal increase occurred in the curve shown in fig. 5(a) at Re = 70-90 for θ_0 = 25° and also at Re = 80-100 for θ_0 = 35°, which were different from the results of θ_0 at lower values. This was because the larger the θ_0, the closer were the spheres to the diagonal plane of the tube, and lower was the wall effect caused by the tube. As fig. 5(b) shows, D_1 is defined as $D_1 = D_0/d$, where D_1 represents the spacing between the two spheres at the time of stable settlement. The effect of Reynolds number on D_1 was significant, and D_1 increased as Reynolds number increased. In addition, a difference in D_1 could still be observed between the larger and smaller θ_0. As fig. 5(b) indicates, D_1 abruptly increased when Re = 70-90 for θ_0 = 25° and also abruptly increased when Re = 80-100 for θ_0 = 35°, which was consistent with θ_1.

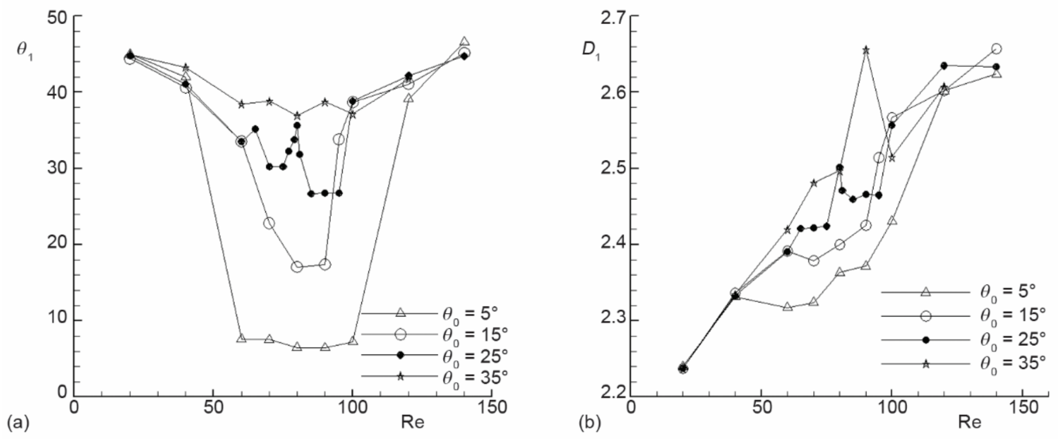

Figure 5. Angle, θ_1, and distance of the two spheres for different Re = 20-140

The instantaneous pressure distributions in the top view plane are presented to examine the motion pattern of the spheres for θ_0 = 25° at Re = 70, 80, and 90, when they were stable. As fig. 6 shows, a high pressure area could be observed at the opposite corner of the square tube, which created a strong repulsive force and caused the spheres to approach the diagonal plane. As both spheres–spheres or spheres-walls had a higher pressure at Re = 80, a higher θ_1 existed when Re = 80, which caused the pressure between the spheres and walls to increase. In addition, because the pressure between the spheres was greater, the distance between the spheres was greater, which corresponded to the result shown in fig. 5. Because of the hydrodynamic interaction, the spheres were forced to leave the initial settlement plane. Therefore the initial settlement plane was not a stable settlement plane for the spheres in our study.

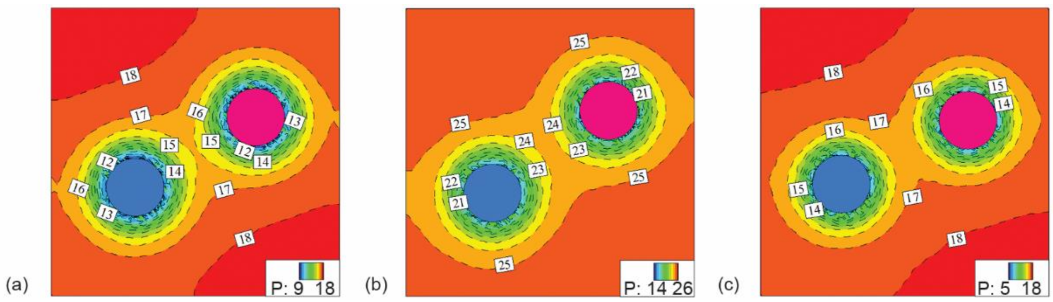

Figure 6. Instantaneous pressure distributions (top view) when the settlement is stable at θ_0 = 25°; (a) Re = 70, (b) Re = 80, and (c) Re = 90

Conclusion

In our work, a 3-D LBM was adopted to simulate the sedimentation of two identical spheres in a square tube. The two resting spheres were released in the initial plane. The effects of the angle, θ_0, of the initial settlement plane and different Reynolds numbers, on the stable sedimentation plane were studied. The results of a lower θ_0 ($\theta_0 = 5°$ and $\theta_0 = 15°$) showed a strong regularity. However, a more complex sedimentation regularity occurred at a higher θ_0 ($\theta_0 = 25°$ and $\theta_0 = 35°$) because of the lower effect of the wall. The results also indicated that at lower or higher Reynolds number (Re $\leq$ 40 or Re $\geq$ 120) the two spheres always reached stable sedimentation at the diagonal plane, irrespective of their initial settlement θ_0. However at a medium Reynolds number, the spheres settled near the initial plane.

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