

## STUDY ON THE INTERACTIONS BETWEEN TWO LIGHT PARTICLES RISING IN A VERTICAL CHANNEL

by

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*In this study, a 2-D lattice Boltzmann method was used to numerically study the interaction between two light particles rising freely in a channel. The influence of the Reynolds number and the density difference between the particles as they rose was studied from the aspects of particle velocity, motion trajectory and motion pattern. The results show that a change of Reynolds number changed the relative position and distance between the particles, and a change in density changed the inertial force of the particles, which affected the interaction between them. Two movement patterns have been revealed: relatively static and a periodic movement pattern. The influence of differing density on the movement period of the particles was also studied.*

Key words: *interaction, movement pattern, lattice Boltzmann method*

### Introduction

Two-phase flow is widely present in nature and engineering equipment, such as a dusty atmosphere and clouds, sandy water flow, blood vessel flow, and air bubbles in the oceans [1]. In two-phase flow, the hydrodynamic interactions between particles are very important for studying the flow characteristics and particle movement patterns.

For particle-fluid interactions of a single particle that settles or rises in a fluid, the effect of particle size and density on particle sedimentation has been revealed, and different motion patterns of particles during sedimentation have also been discovered [2, 3]. The free rise of spherical particles with a density less than that of the fluid is considered to obey the law of free sedimentation, because the same force is applied to the particles but in the opposite direction. However experiments and simulations have found that compared with falling spheres, rising spheres under certain conditions exhibit different behaviors [3-5].

For particle-particle interactions in the particle-fluid system of two or more particles, the drafting, kissing, and tumbling [6] particle pattern mode has been revealed through experiments by Lomholt *et al.* [7] and 2-D, by Feng and Michaelides [8], and 3-D simulations, Glowinski *et al.* [9]. Based on the different responses of light and heavy particles to turbulent fluctuations, it is reliable to use density and size as indices to quantify the degree of aggregation and separation [10].

From the mentioned literature survey, it is clear that particle-fluid and particle-particle interactions are of great significance to the study of two-phase flow. Most of the literature

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only focuses on the interactions of two heavy particles when they settle, however, it is worth noting that few studies discuss the interaction of two rising particles in different configurations. Therefore, the aim of our work is to demonstrate a deeper understanding of the interactions between two unequal particles using the lattice Boltzmann method (LBM).

### Lattice Boltzmann method

This work used a single-relaxation-time LBM [11] to solve the motion of the fluid. The discrete lattice Boltzmann equation is expressed:

$$f_i(\mathbf{x} + e_i \Delta t, t + \Delta t) - f_i(\mathbf{x}, t) = -\frac{1}{\tau} [f_i(\mathbf{x}, t) - f_i^{(0)}(\mathbf{x}, t)] \quad (1)$$

where  $f_i(\mathbf{x}, t)$  is the distribution function for the microscopic velocity,  $e_i$ , in the  $i^{\text{th}}$  direction,  $f_i^{(0)}(\mathbf{x}, t)$  – the equilibrium distribution function,  $\Delta t$  – the time step of the simulation, and  $\tau$  – the relaxation time related to the fluid viscosity,  $\nu$ .

The density and velocity of fluid are computed through the follow formulations,

$$\rho = \sum_i f_i, \quad \rho \mathbf{u} = \sum_i f_i e_i \quad (2)$$

The D2Q9 lattice model was adopted in this work, for which the discrete velocity vectors are given by,

$$e_i = \begin{cases} (0, 0), & \text{for } i = 0, \\ (\pm 1, 0)c, (0, \pm 1)c, & \text{for } i = 1-4 \\ (\pm 1, \pm 1)c, & \text{for } i = 5-8 \end{cases} \quad (3)$$

The equilibrium distribution function  $f_i^{(0)}(\mathbf{x}, t)$  is chosen:

$$f_i^{(0)}(\mathbf{x}, t) = w_i \rho \left[ 1 + \frac{3e_i \cdot \mathbf{u}}{c^2} + \frac{9(e_i \cdot \mathbf{u})^2}{2c^4} - \frac{3\mathbf{u} \cdot \mathbf{u}^2}{2c^2} \right] \quad (4)$$

where  $c = \Delta x / \Delta t$ . Note that  $\Delta x$  is the lattice spacing and the weights are:  $w_0 = 4/9$ ,  $w_{1-4} = 1/9$ , and  $w_{5-8} = 1/36$ .

For simplicity, the lattice spacing,  $\Delta x$ , and the time step,  $\Delta t$ , were both fixed at 1, which is common for lattice Boltzmann simulations. In addition, an interpolation-based bounce-back scheme [12] was employed to treat the curved boundaries of the solid particles. To account for the lubrication effects between particles at small distances, a lubrication force model [13] was introduced.

### Problem description

The purpose of this work is to study the interaction of two light particles in the fluid during their freely rising period, as shown in fig. 1, where two round particles with the same diameter,  $d$ , but different densities ( $\rho_1$  and  $\rho_2$ ) are released freely, and the 2-D channel is filled with a fluid with density,  $\rho$ , and kinematic,  $\nu$ . In the simulation, unless otherwise specified, the parameters are fixed as follows:  $d = 30$ ,  $L = 5d$ ,  $H = 25d$  (corresponding to a  $300 \times 1500$  lattice unit calculation grid), the initial distance between particles is  $L_1 = 2d$ , and  $H_1 = 10d$ , and  $H_2 = 15d$ . Since  $\rho > \rho_2 > \rho_1$ , the particles will move upward (in the opposite direction of gravity)

due to having a lower density than that of the fluid. It is worth noting that the mentioned parameters are all based on lattice units that are common in lattice Boltzmann simulations. To simulate an infinite channel, a moving computational domain was used in this work.

Owing to the unpredictability of the final velocity of the particle, the velocity scale is set:

$$U_0 = \sqrt{\left(\frac{\rho_1}{\rho} - 1\right)gd} \quad (6)$$

where  $g$  is gravitational acceleration. The time scale is fixed as  $T_0 = d/U_0$ . The length scale is the diameter of the particle,  $d$ . Some dimensionless parameters are configured as: density difference of the particles  $\lambda = (\rho_2 - \rho_1)/\rho$ , Reynolds number  $Re = U_0d/\nu$ , position of the light particle  $X_1' = X_1/d$  and  $Y_1' = Y_1/d$ , position of the heavy particle  $X_2' = X_2/d$  and  $Y_2' = Y_2/d$ , average rising velocity of the heavy particle  $V_{y2}' = V_{y2}/U_0$ .

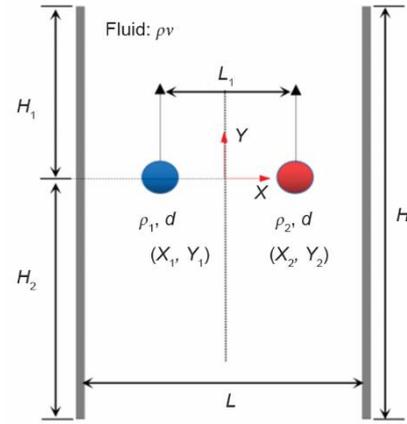


Figure 1. Schematic diagram of the particle rise mode

### Numerical results

A numerical study of the interaction of two unequal particles in a 2-D channel during the rising process was carried out. The Reynolds number range was 5-50 ( $5 \leq Re \leq 50$ ) and the density difference between the particles ranged from 0.01-0.07 ( $0.01 \leq \lambda \leq 0.07$ ). When  $\lambda$  changed, the situation was different, fig. 2 shows the average rising velocity ( $V_{y2}'$ ) of the relatively heavy particle (referred to as heavy particle) at different  $\lambda$  for a specific Reynolds number. Note that  $V_{y2}'$  suddenly increased between  $\lambda = 0.03$  and  $\lambda = 0.04$  at  $Re = 5$ , which is different from the other results. The reason for this mutation is that the pattern of the particle's movement had changed. When  $\lambda \leq 0.03$ , the heavy particle made circular arc-shaped periodic motions to the right of the light particle, fig. 3(a). The light particle was affected by the heavy particle, and exhibited horizontal oscillating motion (note that simultaneously, both particles were rising). The periodic motion of the heavy particle can be represented by its displacement in the horizontal direction, fig. 3(b). However, when  $\lambda \geq 0.03$ , this periodic motion disappeared and was replaced by a stable pattern, which was characterized by two particles in the center of the channel, and a rising pattern where the heavy particle is on the bottom and the light particle is on the top, as shown in fig. 3(c). This is precisely because the center of the channel receives the least wall effect, which explains the sudden rise in  $V_{y1}'$  at  $Re = 5$ .

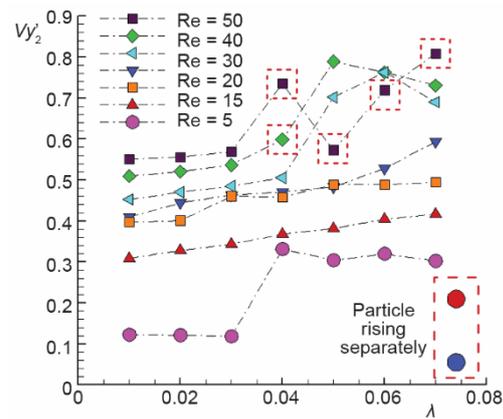
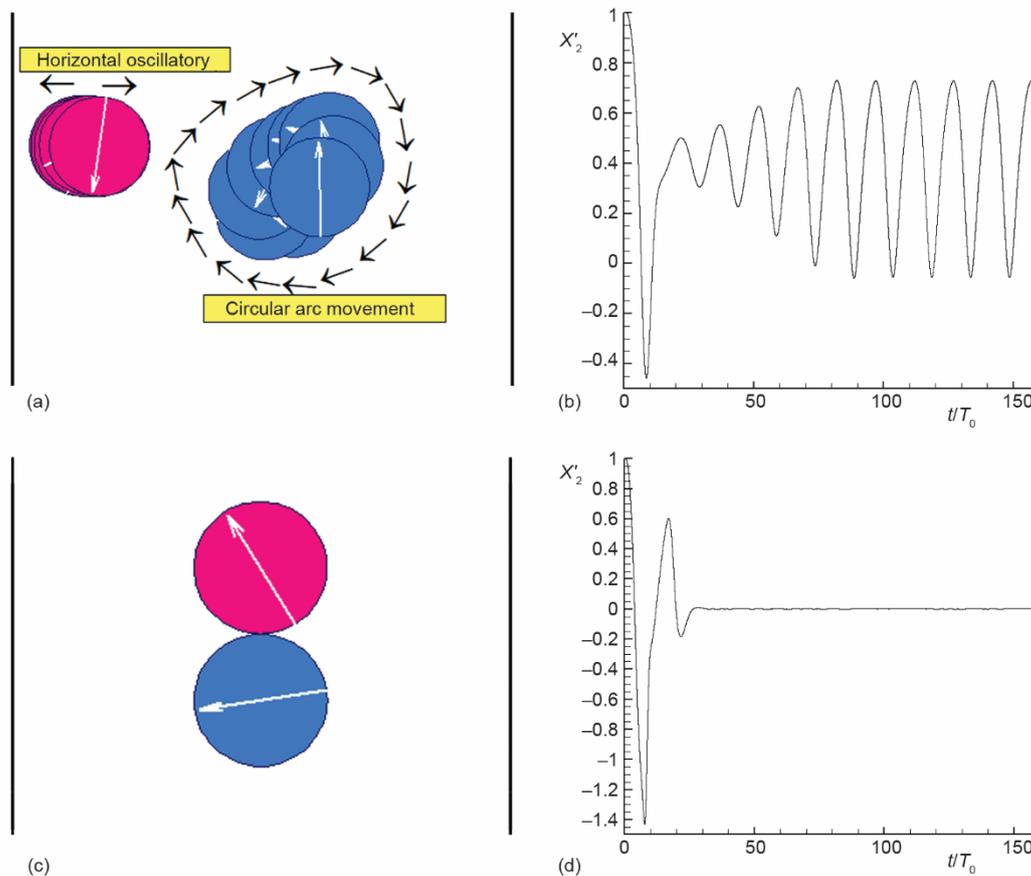
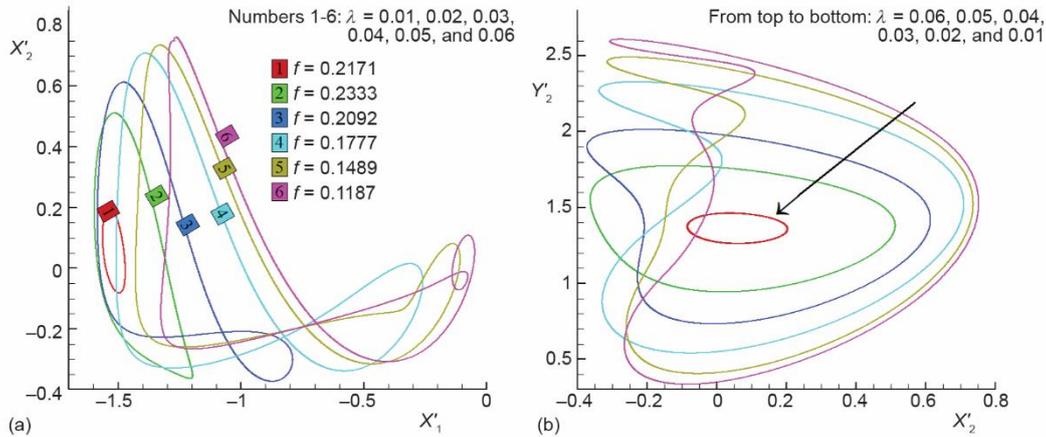


Figure 2. Average rising velocity of the heavy particle,  $V_{y2}'$ , under different parameters; the velocity marked by the dotted box is the velocity before the particles rose separately



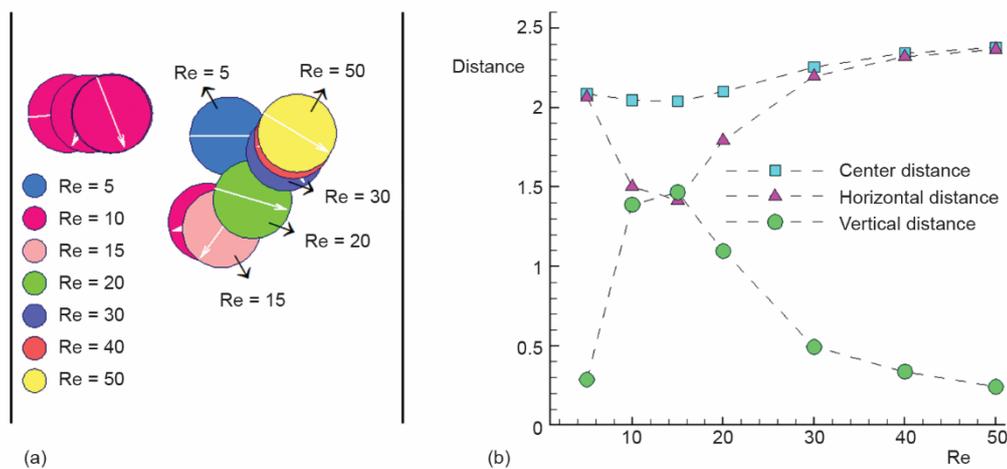
**Figure 3. Movement pattern of the particles and the displacement of the heavy particle in the X-direction; (a)  $\lambda = 0.01$ ,  $Re = 5$  and (b)  $\lambda = 0.04$ ,  $Re = 5$**

When Reynolds number is relatively medium ( $Re = 10, 15, 20$ ), as presented in fig. 2, there is an increase correlation between  $V_{y2}'$  and Reynolds number, especially when  $Re = 10$ . The results show that the motion pattern here is similar to that in fig. 3(a), both of which are the periodic arc motion of the heavy particle relative to the light particle. To better understand the impact of  $\lambda$  and Reynolds number on this pattern, the phase diagrams constructed with  $X_1'$  and  $X_2'$  are presented in fig. 4(a). For a fixed value of Reynolds number, as the value of  $\lambda$  increases, the size of the limit cycles increases significantly, fig. 4(a), which also affects the period of particle motion. Note that a single cycle is observed in the limit cycles of  $\lambda = 0.01$  and  $0.02$ , and double cycles are observed in the limit cycles of other  $\lambda$  ( $\lambda = 0.03-0.06$ ). To explain this, the limit cycles constructed by  $X_2'$  and  $Y_2'$  are shown in fig. 4(b). The difference is that there is an oscillating path on the left of the limit cycles, and the wake effect is the primary cause of the difference. When the heavy particle is in the wake region of the light particle, the velocity of the heavy particles increases suddenly owing to the smaller resistance, which explains the sudden increase in  $f$  between  $\lambda = 0.01$  and  $\lambda = 0.02$ . When  $\lambda \geq 0.02$ ,  $f$  decreases monotonously with an increase in  $\lambda$  because the particles take a longer time to complete the larger limit cycle.



**Figure 4.** Limit cycles as a function of  $\lambda$  for  $Re = 10$  constructed by (a)  $X_1'$  and  $X_2'$  and (b)  $X_2'$  and  $Y_2'$ ; the figure also presents the frequency of particles oscillations ( $f = T_0/T'$ , where  $T'$  is the period of particles oscillations)

It is worth noting that, when  $\lambda$  is specified (specifically  $\lambda = 0.01, 0.02,$  and  $0.03$ ), an abnormal phenomenon appears with the change in Reynolds number. When Reynolds number increases ( $Re \geq 20$ ), the heavy particle no longer continues in a circular arc periodic motion, fig. 3(a), but is relatively stable at the lower right of the light particle. Figure 5 shows the position and distance information of the particles when  $\lambda = 0.01$ . As presented in fig. 5(a), with the change in Reynolds number, the relative position between particles also constantly changes (note that the position of  $Re = 5-15$  is the critical position before the particles start to move periodically). To better understand this phenomenon, the distance information between particles (including the center, horizontal and vertical distances) is given in fig. 5(b). According to fig. 5(b), the distance between the centers of the two particles increases with  $Re$ . The horizontal distance decreases first, then increases with the increase in  $Re$  and the vertical distance increases



**Figure 5.** Position information of the particles at different Reynolds number for  $\lambda = 0.01$ ; (a) schematic diagram of particle location and (b) distance between particles

first, then decreases with the increase in Reynolds number. This explains why there is no periodic movement of heavy particles as Reynolds number increases, when Reynolds number is greater than 15, the horizontal distance increase because of the enhancement of the flow field, resulting in a decrease in the interaction between the two particles, and a relatively stable motion pattern appears.

### Conclusion

Here, a numerical study of rising particles in a 2-D channel was carried out. The results indicated that the change in Reynolds number and  $\lambda$  will affect the particles' rising velocity, distance and movement pattern, and the influence is significant. Based on this influence, two different particle rising patterns were discovered: a relatively static and a periodic movement pattern. In addition, according to different limit cycles, a periodic pattern was identified as periodic motion without wake and periodic motion with wake.

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