MARANGONI CONVECTION OF NANOFLUID IN A WAVY TRAPEZOIDAL ENCLOSURE

by

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The purpose of this study is to investigate the behaviour of natural convection in a wavy trapezoidal enclosure that is filled with nanofluid. The left wavy wall has wavelength, λ, and amplitude, A. The top and bottom walls are adiabatic while the side walls are set to constant temperature, and shear stress occurs at the top of the enclosure. The numerical approach used in this study in order to discretize the governing equations with its boundary conditions is the finite element method where the Galerkin technique is adopted. The solutions obtained are for various values of the Marangoni number, Rayleigh number, and solid particle volume fraction. The graphs of the streamlines, isotherms, local Nusselt, and average Nusselt numbers are then presented and discussed.

Key words: *Marangoni convection, heat transfer, wavy trapezoidal enclosure*

Introduction

The Marangoni effect (also known as the Marangoni convection), which was named after the Italian physicist, Carlo Giuseppe Matteo Marangoni [1], is an effect where heat and mass move to an area with a higher surface tension within a liquid. Because of this, many studies have been conducted, as the phenomenon has significant effect in low-gravity hydrodynamics and small-scale systems [2] which improves heat transfer performance. Many investigations have been conducted on the Marangoni convection and heat transfer performance in a cavity. Zebib *et al.* [3] observed that boundary-layers formed in the cold stagnation area of a square cavity as Ra $\rightarrow \infty$. This is further studied by Saleh and Hashim [2] where they examined a square cavity with shear stress taking place at the free surface on top of the enclosure. They discovered that when nanoparticles are added to the fluid, the flow rate at where the shear stress occurs is greater than the secondary cell. The heat transfer rate also improved with the presence of solid nanoparticles in the fluid. Singh and Bhargava [4] conducted a numerical study of Marangoni convection in a wavy enclosure and concluded that the heat transfer rate increases significantly with Rayleigh number and if the Prandtl number is less than 1. Chen *et al.* [5] conducted an investigation on the onset of double-diffusive Marangoni convection in a rectangle cavity and came to the conclusion that the Rayleigh number destabilises before stabilising fluid circulation. Various other geometries were also studied, including trapezoidal enclosures by Nasrin and Parvin [6] and Zaharuddin *et al.* [7]. Narsin and

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Parvin [6] concluded that the heat transfer rate improves significantly when the Prandtl number is high and the length of the top width is short. Zaharuddin *et al.* [7], on the other hand, observed the formation of a second cell flow that circulates in an anti-clockwise direction inside the cavity at high Marangoni numbers. Furthermore, for high Rayleigh number, there exists a critical Marangoni number that causes the average Nusselt number to reach a stationary state. Hence, in this study, a wavy trapezoidal enclosure filled with nanofluid is examined with varying Marangoni number, Rayleigh number, and volume fraction, ϕ, where the wavy side has wavelength 1, 2, and 5.

Physical and mathematical formulation

The left wavy wall of the trapezoidal enclosure is described by:

$$x = A\left[1-\cos\left(\frac{2\pi\lambda y}{H}\right)\right] \tag{1}$$

The wavy side is set to a constant temperature of T_h, and the right side of the wall, T_c, where the temperature of the former is higher than that of the latter. The height, H, and width, W, are assumed to be the same, $H = W$. The top and bottom walls are insulated and the fluid in the enclosure is water-based nanofluid. All physical properties of the fluid and nanoparticles are assumed to be constant. In this study, the flow is assumed to be steady, laminar, and incompressible, which implies that there is no viscous dissipation. The gravitational force acts in the vertical downward direction and the no-slip boundary condition is applied to all walls aside from the top one where the slip boundary condition is imposed. The fluid used in this study is water-based nanofluid containing Al_2O_3 nanoparticles and their thermophysical properties are referenced from Zaharuddin *et al.* [7]. For a 2-D flow inside a wavy trapezoidal cavity, the dimensionless governing equations are written:

$$\frac{\partial U}{\partial X}+\frac{\partial V}{\partial Y}=0 \tag{2}$$

$$U\frac{\partial U}{\partial X}+V\frac{\partial U}{\partial Y}=-\frac{\partial P}{\partial X}+\mathrm{Pr}\frac{\dfrac{\mu_{\mathrm{nf}}}{\mu_{\mathrm{f}}}}{\dfrac{\rho_{\mathrm{nf}}}{\rho_{\mathrm{f}}}}\left(\frac{\partial^2 U}{\partial X^2}+\frac{\partial^2 U}{\partial Y^2}\right) \tag{3}$$

$$U\frac{\partial V}{\partial X}+V\frac{\partial V}{\partial Y}=-\frac{\partial P}{\partial Y}+\mathrm{Pr}\frac{\dfrac{\mu_{\mathrm{nf}}}{\mu_{\mathrm{f}}}}{\dfrac{\rho_{\mathrm{nf}}}{\rho_{\mathrm{f}}}}\left(\frac{\partial^2 V}{\partial X^2}+\frac{\partial^2 V}{\partial Y^2}\right)+\frac{\beta_{\mathrm{nf}}}{\beta_{\mathrm{f}}}\mathrm{Ra}\,\mathrm{Pr}\,\theta \tag{4}$$

$$U\frac{\partial \theta}{\partial X}+V\frac{\partial \theta}{\partial Y}=\alpha_{\mathrm{nf}}\left(\frac{\partial^2 \theta}{\partial X^2}+\frac{\partial^2 \theta}{\partial Y^2}\right) \tag{5}$$

With boundary conditions:

$$U=V=0 \quad \text{and} \quad \theta=0 \quad \text{at} \quad Y=-2X+2, \quad \frac{1}{2}\le X\le 1 \tag{6}$$

$$U=V=0 \quad \text{and} \quad \theta=1 \quad \text{at} \quad X=A[1-\cos(2\pi\lambda Y)], \quad 0\le Y\le 1 \tag{7}$$

$$U = V = 0 \quad \text{and} \quad \frac{\partial \theta}{\partial Y} = 0 \quad \text{at} \quad Y = 0 \tag{8}$$

$$U = V = 0, \quad \frac{\partial \theta}{\partial Y} = 0 \quad \text{and} \quad \frac{\partial U}{\partial Y} = \mathrm{Ma}\frac{\partial \theta}{\partial X} \quad \text{at} \quad Y = 1 \tag{9}$$

with the dimensionless stream function:

$$\frac{\partial V}{\partial X} - \frac{\partial U}{\partial Y} = -\nabla^2 \Psi \tag{10}$$

where the following substitutions are used:

$$X = \frac{x}{W}, \quad Y = \frac{y}{W}, \quad U = \frac{uW}{\alpha_\mathrm{f}}, \quad V = \frac{vW}{\alpha_\mathrm{f}}, \quad \theta = \frac{T - T_c}{T_h - T_c}, \quad A = \frac{a}{H}, \quad P = \frac{pW^2}{\rho_\mathrm{f}\alpha_\mathrm{f}^2}$$

$$\mathrm{Pr} = \frac{\mu_\mathrm{f}}{\rho_\mathrm{f}\alpha_\mathrm{f}}, \quad \mathrm{Ra} = \frac{\mathrm{g}\beta_\mathrm{f} W^3 (T_h - T_c)}{\upsilon_\mathrm{f}\alpha_\mathrm{f}}, \quad \mathrm{Ma} = -W\frac{\partial \sigma}{\partial T}\frac{(T_h - T_c)}{\mu_\mathrm{f}\alpha_\mathrm{f}}, \quad \Psi = \frac{\psi}{\alpha_\mathrm{f}}$$

As nanofluid is being used in this study, the density, dynamic viscosity, thermal diffusivity, thermal expansion coefficient, heat capacitance, and thermal conductivity are:

$$\rho_\mathrm{nf} = (1-\phi)\rho_\mathrm{f} + \phi\rho_s, \quad \mu_\mathrm{nf} = \frac{\mu_\mathrm{f}}{(1-\phi)^{2.5}}, \quad \alpha_\mathrm{nf} = \frac{k_\mathrm{nf}}{(\rho C_p)_\mathrm{nf}}, \quad \beta_\mathrm{nf} = (1-\phi)\beta_\mathrm{f} + \phi\beta_s$$

$$(\rho C_p)_\mathrm{nf} = (1-\phi)(\rho C_p)_\mathrm{f} + \phi(\rho C_p)_s, \quad k_\mathrm{nf} = k_\mathrm{f}\frac{k_s + 2k_\mathrm{f} - 2\phi(k_\mathrm{f} - k_s)}{k_s + 2k_\mathrm{f} - \phi(k_\mathrm{f} - k_s)}$$

The heat transfer rate for the hot wall is computed by local and average Nusselt number. The local Nusselt number at the heated wall is given by:

$$\mathrm{Nu}_\mathrm{loc} = -\frac{k_\mathrm{nf}}{k_{bf}}\sqrt{\left(\frac{\partial \theta}{\partial X}\right)^2 + \left(\frac{\partial \theta}{\partial Y}\right)^2} \tag{11}$$

from Nasrin and Parvin [4] and hence, the average Nusselt number is:

$$\mathrm{Nu}_\mathrm{avg} = \int_0^1 \mathrm{Nu}_\mathrm{loc}\frac{\partial \theta}{\partial N}\mathrm{d}N \tag{12}$$

Numerical technique

The system of eqs. (2)-(9), are solved using the Galerkin technique of the finite element method in COMSOL 5.3, which is described in detail by Taylor and Hood [8], Nasrin and Parvin [6], and Dechaumphai [9].

Results and discussion

In this section, the results obtained by numerically solving the systems of eqs. (2)-(5) along with its boundary conditions (6)-(9) are presented in the form of streamlines, isotherms, Nu_loc and Nu_avg at the heated wall for various parameter values where $0 \le \mathrm{Ma} \le 10^3$,

$1 \le \lambda \le 5$, Pr = 0.052, $10^3 \le \text{Ra} \le 10^4$, and $0\% \le \phi \le 3\%$. The temperature of the wavy wall is set to be higher than that of the right wall, and the top and bottom walls are adiabatic with shear stress taking place on the top wall. Figures 1 and 2 show three different trapezium enclosures where the left side of the enclosure is designed to be wavy, with amplitude, *A*, and wavelength, λ. The evolution of the fluid-flow and temperature distribution rely on the Marangoni number and hence, three different values were considered. From fig. 1, when shear stress is absent, the fluid in the enclosure moves in a clockwise direction, moving up of the left heated wall and then to the right of the cold wall, flowing down.

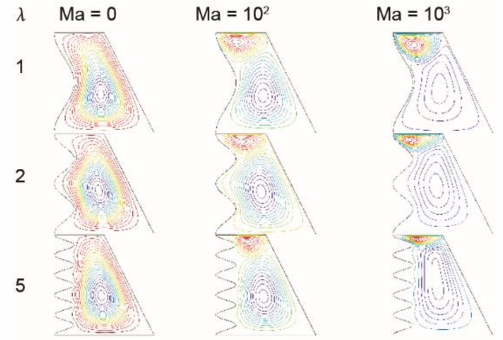

Figure 1. Streamlines for λ = 1, 2, 5, and Ra = 10^3

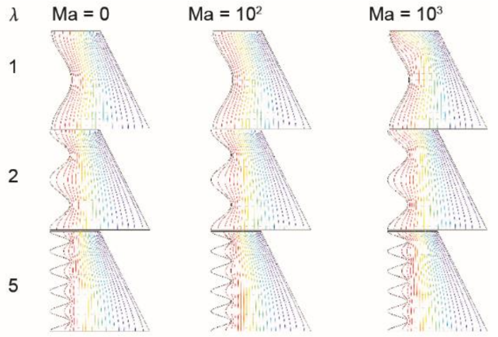

Figure 2. Isotherms for λ = 1, 2, 5, and Ra = 10^3

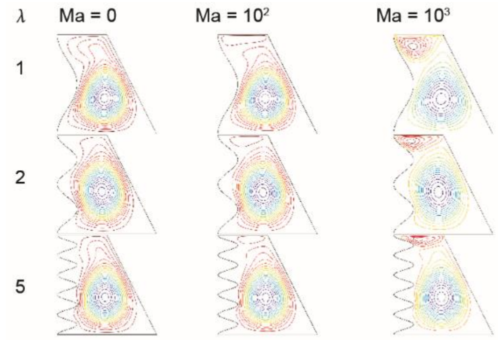

Figure 3. Streamlines for λ = 1, 2, 5, and Ra = 10^4

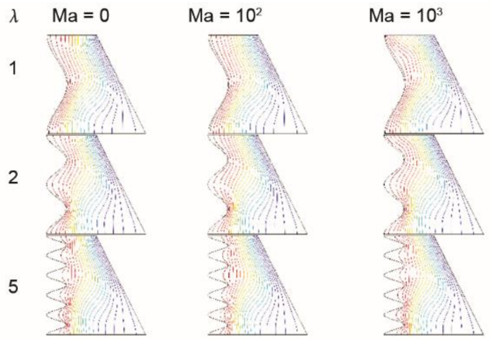

Figure 4. Isotherms for λ = 1, 2, 5, and Ra = 10^4

At high Marangoni numbers for all wavelengths, two distinctive cells form in the cavity where the top cell circulates in a counter-clockwise direction while the bottom one moves clockwise, based on the positive and negative signs. Note that at Ma = 10^3, the concentration of the top cell is higher for λ = 1 compared to λ = 2 and λ = 5, though the strength of the vorticity of the bottom cell is more prominent for λ = 2. This is supported by the isotherms and it is evident that the deformations are much more noticeable for enclosures with wavelength 1 and 2 for Ma = 10^3, as seen in fig. 2, where they form into a boomerang-like shape. The reason as to the absence of significant contortion for λ = 5 is due to the wavy lines themselves, warping the distribution of temperature even when there is no shear stress. If the Rayleigh number is increased from 10^3 to 10^4, the fluid inside the cavity is expected to have a much stronger flow and this can be seen in fig. 3. Even if the shear stress in non-existent, the walls of the single cells lose its uniform flow, seen in fig. 1 for all three cases and depict a slight turbulence. However, similar to the results in fig. 1, at high Marangoni values, the single cell splits into two and there is a noticeable increase in the intensity of the flow compared to the aforementioned. In addition, comparing the isotherms in fig. 4 to the ones in fig. 2, there is barely a difference in the distortion despite an increase in the Rayleigh number. Due to the wavy geometry, the plots along the hot side of the wall where the local Nusselt numbers are calculated are also wavy in nature. Figure 5 represents the local Nusselt for Ra = 10^3 and Ra = 10^4, and various Marangoni numbers. For the smaller value of Rayleigh number, it indicates that the heat transfer rates increase significantly as λ increases, with λ = 5 having the highest maximum $\mathrm{Nu_{loc}}$ for all three cases of Marangoni number. However, as the Marangoni number gets bigger, the profiles for λ = 2 and 2 seem to lose its periodic behaviour as $Y \to 1$ for Ma < 10^3. Furthermore, the local Nusselt value profiles seem to be decreasing with increasing Y for Ma < 1000 in all cases. This is likely due to the laminar flow shifting to turbulent flow, which affects the shear stress at the top part of the cavity by reducing its influence on the heat transfer rate. Figure 6 shows $\mathrm{Nu_{avg}}$ with respect to the Marangoni number from 0 to 10^3 for λ = 1, 2, 5. The Rayleigh number is set to 10^3 and 10^4, respectively, and from the figure, at high Marangoni numbers, specifically around the 750 mark, λ = 1 and 2 share the same values before the former's $\mathrm{Nu_{avg}}$ surpasses that of the latter's, though λ = 5 still has the highest $\mathrm{Nu_{avg}}$ overall. In the case when the Rayleigh number is increased to 10^4, λ = 1 has the lowest $\mathrm{Nu_{avg}}$ with λ = 2 surpassing λ = 5 as $\mathrm{Ma} \to 1000$, suggesting that for $\mathrm{Ma} \geq 1000$, λ = 2 is more efficient at transferring heat. Observing fig. 7, $\mathrm{Nu_{avg}}$ increases linearly with increasing ϕ. It can also be noted that $\lambda = 1$ has higher $\mathrm{Nu_{avg}}$ compared to the rest when Ma = 10^3. However, when the Rayleigh number is set to 10^4, λ = 2 has the highest $\mathrm{Nu_{avg}}$.

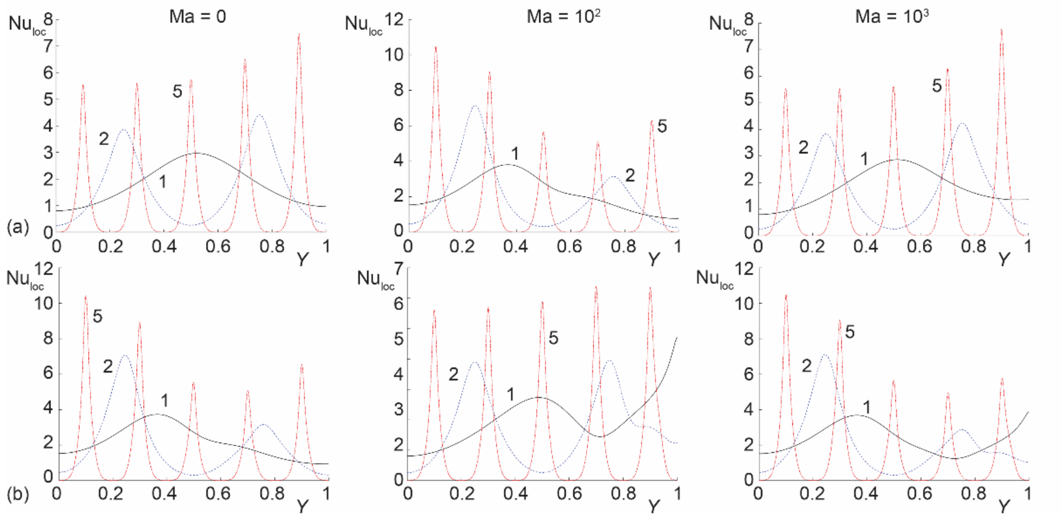

Figure 5. Local Nusselt values for (a) Ra = 10^3 and (b) Ra = 10^4 with λ = 1, 2, and 5

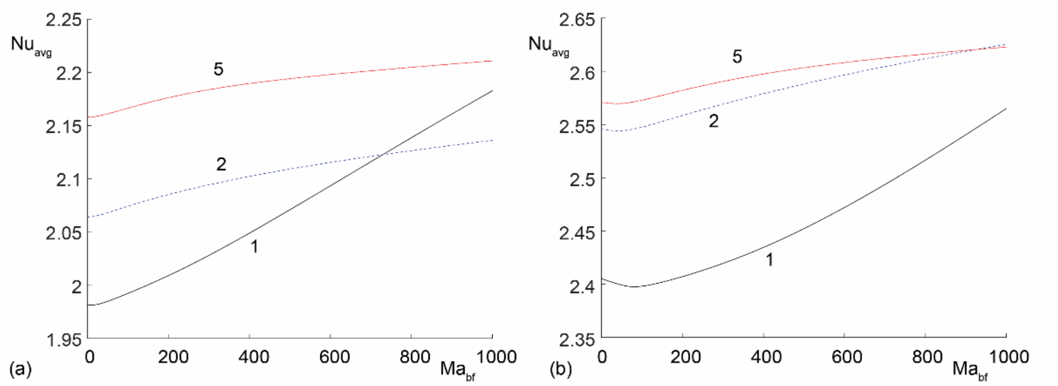

Figure 6. Average Nusselt values for (a) Ra = 10^3 and (b) Ra = 10^4 for varyng Marangoni number with λ = 1, 2, and 5

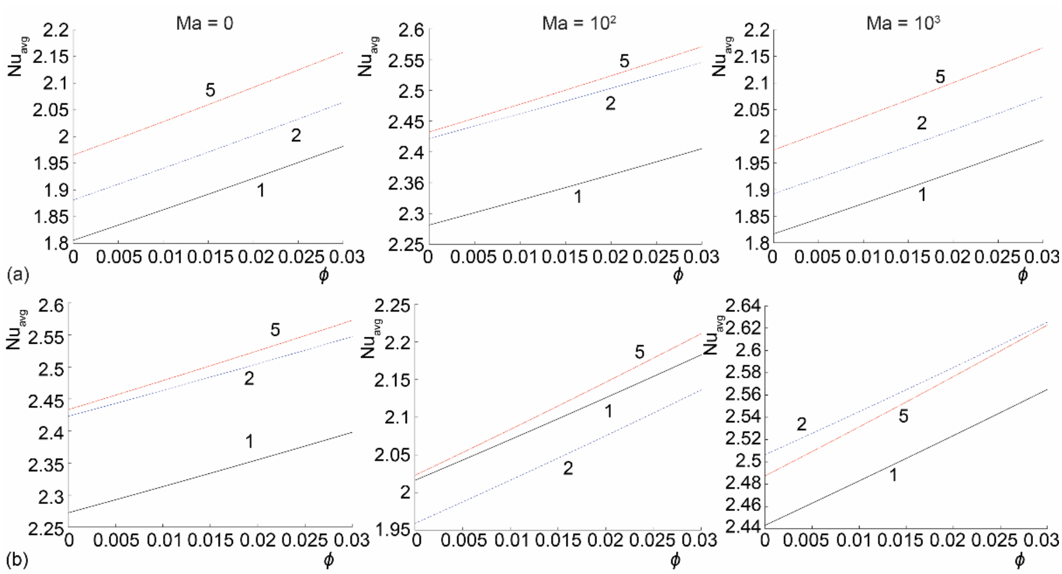

Figure 7. Average Nusselt values for (a) Ra = 10^3 and (b) Ra = 10^4 for varyng ϕ with λ = 1, 2, and 5

Conclusion

The problem of free convection and fluid-flow within a wavy trapezoidal enclosure with the left side of the wall heated has been studied. The dimensionless governing equations were solved using the finite element method. The results consisting of the streamlines, isotherms, $\mathrm{Nu_{loc}}$ and $\mathrm{Nu_{avg}}$ were presented and discussed. The observations made from this study are that there exists a secondary flow cell in the enclosure for all values of Rayleigh number when Marangoni number is big with the smaller cell at the top leaning into the hot wall. Furthermore, $\mathrm{Nu_{avg}}$ increases linearly with ϕ for all values of Marangoni number and Rayleigh number and finally, the heat transfer rate improves significantly with increasing wavelength.

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