OPTIMAL SPACINGS FOR CHANNELS WITH HAGEN-POISEUILLE FLUID-FLOW AND MASS TRANSFER – The Role of the Bejan Number

by

Ventsislav D. ZIMPAROV**, Valentin M. PETKOV, and Hristo N. HRISTOV

Department of Mechanical Engineering, Gabrovo Technical University, Gabrovo, Bulgaria

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This work is a continuation of the recent studies [1, 2], revealing that the unique form of the Bejan number is robust (unchangeable) and appears independently in all Hagen-Poiseuille fluid-flows with heat or mass transfer by convection. The other dimensionless groups, derived from the First law of thermodynamics (related to the convection heat or mass transfer), and named Bejan numbers are combinations of the unique Bejan number with Prandtl or Schmidt numbers, respectively, and ratios of geometrical parameters of the system. In this paper we continue developing this idea through presenting new examples of problems in the field of convection mass transfer in pure laminar duct flows.

Key words: *Bejan number, optimal spacing, discriminated dimensional analysis, mass transfer*

Introduction

The Bejan number, derived from the fluid mechanics and First law of thermodynamics, has been defined for a first time in 1988 by Bhattacharjee and Grosshandler [3] performing scale analysis of the *x*-momentum equation in the wall region of a flow over high temperature wall. Bejan and Sciubba [4] and Bejan [5] published a study related to the selection of board-to-board optimal spacing, in order to maximize the heat transfer from a package of parallel plates that are cooled by forced convection. The obtained dimensionless group in the form:

$$\Pi = \frac{\Delta p L^2}{\mu \alpha}$$

has been named by Bejan [4, 5] *the pressure drop number.* Petrescu [6] defined this group as the Bejan number:

$$\mathrm{Be} = \frac{\Delta p L^2}{\mu \alpha}$$

If the thermal diffusivity in the complex $\Delta p L^2/\mu\alpha$ is replaced by the mass diffusivity, D, the new complex $\Delta p L^2/\mu D$ has been called by Awad [7] a *new definition of the Bejan number*. Later, Awad and Lage [8] defined a complex $\Delta p L^2/\rho\delta^2$ named a *general form of the Bejan number,* where δ is the corresponding diffusivity of the process in consideration: ν, α, or D. A

* Corresponding author, e-mails: ventsi.zimparov@gmail.com; vdzim@tugab.bg

historical view on the Bejan numbers is presented by Awad [9]. However, there are no any comments in these publications [6-9] related to the physical meaning of the defined dimensionless groups.

In the recently published paper [1], it was demonstrated by applying the classic fluid mechanics only, that the optimal board-to-board spacing can be derived, and in order to minimize the pressure drop across a package of parallel plates, it has to obey the scaling relationship

$$\frac{D_{\text{opt}}}{L} \sim \left(\frac{\Delta p L^2}{\rho \nu^2}\right) \tag{1}$$

The obtained dimensionless group has been named *unique* Bejan number since it appears as robust (unchangeable) and independent criterion of similarity in all Hagen-Poiseuille fluid-flows with heat or mass transfer convection. Its physical meaning is a ratio of geometrical parameters of the system under study (*slenderness ratio*). All other dimensionless groups [6-9], derived from the First law of thermodynamics (related to the convection heat or mass transfer), and named Bejan numbers are combinations of the *unique* Bejan number with Prandtl or Schmidt numbers, respectively, and ratios of geometrical parameters of the system.

When the pressure drop is imposed (fixed), the *unique* Bejan number is a criterion of similarity and governs the optimal spacings of the channels (*slenderness ratio*). If the geometrical parameters are fixed, the *unique* Bejan number convers into dimensionless variable related to Δp.

Many new additional evidences in order to extend the scope of implementation of the *unique* Bejan number and assess its role in Hagen-Poiseuille fluid-flow with convection heat transfer in pure laminar duct flows and laminar flows in channels filled with porous medium have been presented in [2].

This paper is a continuation of the recent studies [1, 2] developing this idea through presenting new examples of problems related to mass transfer and combination of mass and heat transfer convection in pure laminar duct flows. The discriminated dimensional analysis (DDA) [10] has been applied, as a tool revealing the role and physical meaning of the *unique* Bejan number.

Spacings for channels with mass transfer

Hagen-Poiseuille fluid flow in duct with mass transfer and fixed Δp

Figure 1. Stack of parallel boards with forced convection mass transfer and constant species at the wall

Consider (fig. 1) a stack of parallel boards (CV = $L \times H \times 1$), discussed also in [1, 2], where Hagen-Poiseuille fluid-flow with heat transfer takes place. Now, for the same geometrical configuration we address a Hagen-Poiseuille flow where mass transfer takes place. The mass transfer is based on the difference between species concentration of the exposed side the wall, C_w, and the bulk concentration of the stream, C_b. The flow is driven by the imposed Δp through parallel-plates channels of length, L, and width 1, fig. 1. The objective is to define the optimal distance between the parallel-plates, $D_{i,\text{opt}}$ = ?.

Using the DDA of Huntley [10], the list of relevant quantities is:

$$D_{\mathrm{i,opt}} = f(\dot{m}, \Delta p, L, \rho, \nu, D) \tag{2}$$

where $\dot{m}$ is the mass fluid-flow, D – the mass diffusivity, and assuming that $D_{\mathrm{i,opt}}$ depends on $\dot{m}$. Following Huntley [10], eq. (2) can be presented in the form:

$$D_{\mathrm{i,opt}} \sim \dot{m}^a \Delta p^b L^c \rho^d \nu^e D^f \tag{3}$$

Applying the DDA in the same way as in [2], the number of fundamental physical dimensions have been enlarged through dividing the basic unit of mass, M, into two new basic units, M_i and M_μ, which present its internal diversity, as a matter of inertia M_i or substance M_μ [10]. For this reason, the fundamental physical dimensions become L_x, L_y, L_z, M_i, M_μ, and T. Accordingly, the dimensional equations are:

$$[D_{\mathrm{i,opt}}] = L_y,\ [\dot{m}] = M_\mu T^{-1},\ [\Delta p] = M_i L_x L_y^{-1} L_z^{-1} T^{-2},\ [L] = L_x,\ [\rho] = M_\mu L_x^{-1} L_y^{-1} L_z^{-1}$$
$$[\nu] = L_y^2 M_\mathrm{i} M_\mu^{-1} T^{-1},\ [D] = L_y^2 T^{-1}$$

The value of the exponents in eq. (3) can be obtained from the solution of the next set of equations:

$$0 = b + c - d$$
$$1 = -b - d + 2e + 2f$$
$$0 = -b - d$$
$$0 = b + e$$
$$0 = a + d - e$$
$$0 = -a - 2b - e - f$$

The values of the exponents are: $a = 0$, $b = -1/4$, $c = 1/2$, $d = 1/4$, $e = 1/4$, and $f = 1/4$. The value $a = 0$ reveals that D_{opt} does not depend on the prescribed mass flow and has to be omitted from the list of selected variables. As a result, eq. (3) becomes:

$$D_{\mathrm{i,opt}} \sim \Delta p^{-1/4} L^{1/2} \rho^{1/4} \nu^{1/4} D^{1/4} \tag{4}$$

and can be arranged in the form:

$$\frac{D_{\mathrm{i,opt}}}{L} \sim \left(\frac{\Delta p L^2}{\rho \nu^2}\right)^{-1/4} \left(\frac{\nu}{D}\right)^{-1/4} \tag{4a}$$

or

$$\frac{D_{\mathrm{i,opt}}}{L} \sim \mathrm{Be}^{-1/4} \mathrm{Sc}^{-1/4} \tag{4b}$$

where Sc is the Schmidt number.

The next important conclusions can be derived from this result, namely:

– Dividing the basic unit of mass M into two new basic units: M_{i} and M_{μ}, to enlarge the number of fundamental physical dimensions as L_x, L_y, L_z, M_{i}, M_{μ}, and T, the unknown value of the exponent n in eq. (38) [1] has been defined as $n=-1/4$. Equation (4a) can be also transformed in the form:

$$\frac{D_{\mathrm{i,opt}}}{L} \sim \left(\frac{\Delta p L^2}{\mu D}\right)^{-1/4} \tag{5}$$

where the new complex $\Delta p L^2/\mu D$ has been called by Awad [6] a *new definition of the Bejan number*.

– If we consider the same CV and Hagen-Poiseuille fluid flow with heat transfer, and fulfill the same procedure, outlined foregoing, eq. (4a) will take the form:

$$\frac{D_{\mathrm{i,opt}}}{L} \sim \left(\frac{\Delta p L^2}{\rho \nu^2}\right)^{-1/4} \left(\frac{\nu}{\alpha}\right)^{-1/4} \tag{6}$$

where the mass diffusivity, D, is replaced with the thermal diffusivity, α. This result has already been discussed in [2].

– Note that the dimension of the kinematic viscosity ν $(L_y^2 M_{\mathrm{i}} M_{\mu}^{-1} T^{-1})$ is different compared to those of thermal diffusivity α and mass diffusivity $(L_y^2 T^{-1})$. This is another way to reveal the difference between the momentum transfer with this one of heat or mass transfer.

Hagen-Poiseuille fluid-flow in duct with mass transfer and unknown Δp

Consider now, the case with the control volume $(D_{\mathrm{i}} \times L \times 1)$, fig. 1, where $\dot{m}, H, L$, and D_{i} are imposed, but the pressure drop Δp is unknown. In this case, the Bejan number has completely different physical meaning since it converts into dimensionless variable with respect to Δp, namely:

$$\frac{\Delta p L^2}{\rho \nu^2} \sim \mathrm{Sc}^{-1} \left(\frac{D_{\mathrm{i}}}{L}\right)^{-4} \tag{7}$$

Hagen-Poiseuille flow in duct with simultaneous heat and mass transfer and fixed Δp

Consider again the stack of parallel boards $(\mathrm{CV} = L \times H \times 1)$, fig. 1. In this example, Hagen-Poiseuille fluid-flow with simultaneous heat and mass transfer takes place. The mass transfer is based on the difference between species concentration of the wall, C_{w}, and the bulk concentration of the stream, C_{b}, whereas the heat transfer is based on the difference between the wall temperature, T_{w}, and the bulk temperature of the stream, T_{b}. The flow is driven by the imposed Δp through parallel-plates channels of length L and width 1, fig. 1. The objective is to define the optimal distance between the parallel-plates, $D_{\mathrm{i,opt}}$ = ?.

In this case, the list of relevant quantities is:

$$D_{\mathrm{i,opt}} = f(\Delta p, L, \rho, \nu, D, \alpha) \tag{8}$$

where D and α are the mass and thermal diffusivities. The mass fluid-flow, $\dot{m}$, has been excluded from the list of variables. Following Huntley [10], eq. (8) can be presented in the form:

$$D_{\mathrm{i,opt}} \sim \Delta p^a L^b \rho^c \nu^d D^e \alpha^f \tag{9}$$

Applying again the DDA in the same way as foregoing, and using the fundamental physical dimensions as L_x, L_y, L_z, M_i, M_μ, and T, the dimensional equations are:

$$[D_{\mathrm{i,opt}}] = L_y,\ \ [\Delta p] = M_\mathrm{i} L_x L_y^{-1} L_z^{-1} T^{-2},\ \ [\rho] = M_\mu L_x^{-1} L_y^{-1} L_z^{-1},\ \ [L] = L_x,\ \ [\nu] = L_y^2 M_\mathrm{i} M_\mu^{-1} T^{-1}$$
$$[D] = L_y^2 T^{-1},\ \ [\alpha] = L_y^2 T^{-1}$$

The value of the exponent of each variable in eq. (9) can be obtained from the solution of the next set of equations:

$$0 = a + b - c$$
$$1 = -a - c + 2(d + e + f)$$
$$0 = -a - c$$
$$0 = a + d$$
$$0 = c - d$$
$$0 = -2a - d - e - f$$

The values of the exponents are: a = –1/4, b = 1/2, c = 1/4, d = 1/4, e = e, f = 1/4 – e. One of the exponents has not been obtained since in the set of equations there are only five independent equations. As a result, eq. (9) becomes:

$$D_{\mathrm{i,opt}} \sim \Delta p^{-1/4} L^{1/2} \rho^{1/4} \nu^{1/4} D^e \alpha^{1/4-e} \tag{10}$$

which can be arranged in the form:

$$\frac{D_{\mathrm{i,opt}}}{L} \sim \left(\frac{\Delta p L^2}{\rho \nu^2}\right)^{-1/4} \left(\frac{\nu}{\alpha}\right)^{-1/4} \left(\frac{\alpha}{D}\right)^{-e}, \text{ or} \tag{11a}$$

$$\frac{D_{\mathrm{i,opt}}}{L} \sim \mathrm{Be}^{-1/4} \mathrm{Pr}^{-1/4} \mathrm{Le}^{-e} \tag{11b}$$

where $\mathrm{Le} = \alpha/D$ is the Lewis number. The Lewis number can also be expressed in terms of the Prandtl and Schmidt numbers as $\mathrm{Le} = \mathrm{Sc}/\mathrm{Pr}$. In this case, eq. (11b) yields:

$$\frac{D_{\mathrm{i,opt}}}{L} \sim \mathrm{Be}^{-1/4}\, \mathrm{Pr}^{-1/4+e}\, \mathrm{Sc}^{-e} \tag{11c}$$

The conclusions from this result can be outlined:

– The optimal distance between the parallel-plates (*slenderness ratio*) $D_{\mathrm{i,opt}}/L$ depends on three criteria of similarity: *unique* Bejan number, Prandtl number, and Lewis numbers.
– Using the DDA, the values of powers of Bejan and Prandtl numbers have been obtained.
– If there is no mass transfer in the channels, $e = 0$ and eq. (11c) reduces to the form of eq. (6):

$$\frac{D_{\mathrm{i,opt}}}{L} \sim \mathrm{Be}^{-1/4}\,\mathrm{Pr}^{-1/4}$$

– If there is no heat transfer in the channels, $-1/4 + e = 0$ and eq. (11c) reduces to the form of eq. (4b):

$$\frac{D_{\mathrm{i,opt}}}{L} \sim \mathrm{Be}^{-1/4}\mathrm{Sc}^{-1/4}$$

Hagen-Poiseuille flow in duct with simultaneous heat and mass transfer and unknown Δp

In this case, $\dot{m}, H, L,$ and D_i are imposed, but the pressure drop Δp is unknown, and the *unique* Bejan number converts into dimensionless variable with respect to Δp, namely:

$$\frac{\Delta p L^2}{\rho \nu^2} \sim \mathrm{Pr}^{-1}\,\mathrm{Le}^{-e}\left(\frac{D_i}{L}\right)^{-4} \tag{12}$$

Conclusions

This paper presents additional evidences in order to extend the scope of implementation of the *unique* Bejan number and assess its role in Hagen-Poiseuille fluid-flow with convection mass transfer in pure laminar duct flows and laminar flows in channels filled with porous medium.

The *unique* Bejan number is robust (unchangeable) and present independently in Hagen-Poiseuille fluid-flow with heat or mass transfer convection. Its physical meaning is a ratio of the geometrical parameters of the system under study (*slenderness ratio*). Other dimensionless groups, derived from the First law of thermodynamics (related to the convection heat or mass transfer), and named Bejan numbers are combinations of the *unique* Bejan number with Prandtl or Schmidt numbers, respectively, and ratios of geometrical parameters of the system.

When the pressure drop is imposed (fixed), the *unique* Bejan number is a criterion of similarity and governs the optimal distance of the channels (*slenderness ratio*). If the geometrical parameters are fixed, the *unique* Bejan number converts into dimensionless variable related to Δp.

The power of the DDA has been successfully applied, as a tool to reveal the role and physical meaning of the *unique* Bejan number.

Declaration of competing interest

The authors declared that there is no conflict of interest.

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References

[1] Zimparov, V., *et al.*, New Insight into the Definitions of the Bejan Number, *Int. Comm. Heat Mass Transfer, 116* (2020), July, ID 104637

[2] Zimparov, V., *et al*., Critical Review of the Definitions of the Bejan Number – First Law of Thermodynamic, *Int. Comm. Heat Mass Transfer*, *124* (2021), May, ID 105113
[3] Bhattacharjee, S., Grosshandler, W. L., The Formation of a Wall Jet Near a High Temperature Wall Under Microgravity Environment, *Proceedings ASME 1988*, National Heat Transfer Conference, Houston, Tex., USA, 1988, Vol. 1, pp. 711-716
[4] Bejan, A., Sciubba, E., The Optimal Spacing of Parallel Plates Cooled by Forced Convection, *Int. J. Heat Mass Transfer*, *35* (1992), 12, pp. 3259-3264
[5] Bejan, A., *Heat Transfer*, John Wiley & Sons, Inc., New York, USA, 1993
[6] Petrescu, S., Comments on the Optimal Spacing of Parallel Plates Cooled by Forced Convection, *Int. J. Heat Mass Transfer, 37* (1994), 8, ID 1283
[7] Awad, M. M., A New Definition of Bejan Number, *Thermal Science, 16* (2012), 4, pp. 1251-1253
[8] Awad, M. M., Lage, J. L., Extending the Bejan Number to a General Form, *Thermal Sciences*, *17* (2013), 2, pp. 629-631
[9] Awad, M. M., The Science and the History of the Two Bejan Numbers, *Int. J. Heat Mass Transfer, 94* (2016), Mar., pp. 101-103
[10] Huntley, H. E., *Dimensional Analysis*, Dover Publications Inc., New York, USA, 1967

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