TRANSPORTATION OF Al_2O_3-SiO_2-TiO_2 MODIFIED NANOFLUID OVER AN EXPONENTIALLY STRETCHING SURFACE WITH INCLINED MAGNETOHYDRODYNAMIC

by

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In the present study Al_2O_3-SiO_2-TiO_2/$C_2H_6O_2$ modified nanofluid flow over a stretching surface is considered with imposed inclined magnetic field. Three different suspended nanoparticles in a base fluid are considered in this next generation of hybrid nanofluid called as modified nanofluid. Ethanol glycol is taken as a base fluid with suspension of three nanoparticles of Al_2O_3, SiO_2, and TiO_2. The mathematical model of the flow is encountered by Runga-Kutta fourth order method using appropriate similarity transformations. As a key result it is observed that the capacity of heat transportation of modified nanofluid is higher as compared with nanofluids and hybrid nanofluids. Numerical solutions with graphical representation are presented. With increased inclined angle, parameter of magnetic field, and volume friction parameter a decrement in velocity field has been noticed for modified nanofluid.

Key words: *modified nanofluid, exponentially stretching sheet, inclined magnetic field*

Introduction

Nanofluids are the fluids with suspended nanometer sized particles with different shapes and size are disbursed in the base fluid. It is established by many researchers that heat transfer capacity of nanofluids are much higher than conventional fluid because the thermal conductivity of a fluid can be optimized by adding nanosized particles. Initially, Choi [1] investigated this phenomenon and named these fluids as nanofluids. Afterward, a number of studies have been presented by researchers that signify the role of nanofluids in heat transfer enhancement by their size, shape, and concentration, with various base fluids, nanoparticles on different geometries [2-4]. Nuclear plants, micro polymer films, heat exchangers, electronic devices, space technology, production of heat pipes are some applications of nanofluids where heat transfer plays significant role. Afterward combination of two different nanosized particles with a base liquid was introduced as hybrid nanofluids by the researchers. With appropriate blend or combination of nanometer-sized particles suspended into the base fluid, better results can be obtained in heat/mass transfer because advantage and disadvantage of nanoparticles can

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be managed in hybrid nanofluids. Chamkha *et al.* [5] studied the radiative MHD heat transportation of a hybrid nanofluid with Joule heating effect. Acharya *et al.* [6] analyzed the effects of inclined MHD of hybrid nanofluids flow through a slippery surface. Waini *et al.* [7] presented the transfer of heat for hybrid nanofluids past a porous stretching/shrinking surface. Dadheech *et al.* [8] presented comparative heat transfer analysis between $MoS_2/C_2H_6O_2$ nanofluid and SiO_2-$MoS_2/C_2H_6O_2$ hybrid nanofluid with natural convection and inclined magnetic field it is concluded by the authors that hybrid nanofluids are more efficient for heat transfer as compare to nanofluids because by using the different properties of nanoparticles in hybrid case we can achieve better thermal conductivity. In continuation to this hypotheses researchers have started to prepare a mixture three different nanoparticles suspended in a base fluid called *modified nanofluid* to optimize the heat transfer capacity. Nadeem *et al.* [9] studied the heat transportation phenomenon on modified nanofluid in porous medium with MHD, and established that modified nanofluids are more effective in heat transfer then hybrid and nanofluid cases. Furthermore Nadeem *et al.* [10] presented, modified nanofluids flow through a stretching Rega plate where viscosity is temperature dependent.

Although, unitary nanofluid are useful to cooling the nanomaterials or to increase the heat transport capacity in various conditions. However, mixer of different nanoparticles dispersed in a base fluid can be more effective to foster the heat transfer. Purpose of this analysis is indeed to find study Al_2O_3-SiO_2-$TiO_2/C_2H_6O_2$ modified nanofluids flow past an exponentially stretched surface, with imposed inclined MHD. A comparative study with previous findings of researchers is discussed. An excellent concurrence is achieved with the presentation of dependable conclusions.

Mathematical modelling

The 2-D, viscous incompressible, Al_2O_3-SiO_2-$TiO_2/C_2H_6O_2$ modified nanofluid boundary layer flows through an exponentially stretching surface. Magnetic field is imposed uniformly with inclination angle, α. Base fluid is ethanol glycol with suspension of three nanoparticles of Al_2O_3, SiO_2, and TiO_2 is chosen. We acknowledge that the fundamental fluids and the nanosize particles are in thermal equilibrium state. The laminar fluid flow is considered at $y \geq 0$ where x-axis is considered along the surface and y-axis is considered normal to the exponential stretching sheet. Ambient temperature, T_∞, and the surface temperature, T_w, are assumed different. Assuming the previous hypothesis, the governing equations of the flow are described [8]:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial v}{\partial y} = \frac{\mu_{\text{mnf}}}{\rho_{\text{mnf}}}\frac{\partial^2 u}{\partial y^2} - \frac{\sigma_{\text{mnf}} B_0^2 u \sin^2\alpha}{\rho_{\text{mnf}}} \tag{2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{\kappa_{\text{mnf}}}{(\rho C_p)_{\text{mnf}}}\frac{\partial^2 T}{\partial y^2} \tag{3}$$

Also the boundary conditions under previous presumptions are:

$$u = u_w, \quad v = 0, \quad T = T_w, \quad \text{at} \quad y = 0, \quad u \to 0, \quad T \to T_\infty, \quad \text{as} \quad y \to \infty \tag{4}$$

where velocity components along *x*- and *y*-axis are *u* and *v*, respectively, also C_s represents the surface heat capacity, μ_{mnf} – the viscosity modified nanofluid, ρ_{mnf} – the density, κ_{mnf} – the thermal conductivity, σ_{mnf} – the electrical conductivity, and $(\rho C_p)_{mnf}$ – the heat capacity. Here subscript mnf, hnf, nf, and f, defined for modified nanofluids, hybrid nanofluids, nanofluids, tab. 1, and base fluid, respectively, tab. 2. Magnetic field is B_0 and $U_w = U_0 e^{x/l}$, $T_w = T_\infty + T_0 e^{x/2l}$.

Table 1. Thermo-physical characteristics of hybrid nanofluids [8, 10]

$\mu_{mnf} = \frac{\mu_f}{(1-\phi_1)^{2.5}(1-\phi_2)^{2.5}(1-\phi_3)^{2.5}}$	(Effective dynamic viscosity)
$\rho_{mnf} = (1-\phi_3)(\{[(1-\phi_1)\rho_f + \phi_1\rho_{s1}](1-\phi_2)\} + \phi_2\rho_{s2}) + \phi_3\rho_{s3}$	(Effective density)
$\sigma_{mnf} = \left[\frac{2\sigma_{hnf} + \sigma_{s3} - 2(\sigma_{hnf} - \sigma_{s3})\phi_3}{2\sigma_{hnf} + \sigma_{s3} + (\sigma_{hnf} - \sigma_{s3})\phi_3}\right]\sigma_{hnf}$ where $\sigma_{hnf} = \left[\frac{2\sigma_{nf} + \sigma_{s2} - 2(\sigma_{nf} - \sigma_{s2})\phi_2}{2\sigma_{nf} + \sigma_{s2} + (\sigma_{nf} - \sigma_{s2})\phi_2}\right]\sigma_{nf}$ and $\sigma_{nf} = \left[\frac{\sigma_{s1} + 2\sigma_f - 2\phi_1(\sigma_f - \sigma_{s1})}{\sigma_{s1} + 2\sigma_f + \phi_1(\sigma_f - \sigma_{s1})}\right]\sigma_f$	(Electrical conductivity)
$\kappa_{mnf} = \left[\frac{\kappa_{s3} + (n-1)\kappa_{hnf} - (n-1)(\kappa_{hnf} - \kappa_{s3})\phi_3}{\kappa_{s3} + (n-1)\kappa_{hnf} + (\kappa_{hnf} - \kappa_{s3})\phi_3}\right]\kappa_{hnf}$ where $\kappa_{hnf} = \left[\frac{\kappa_{s2} + (n-1)\kappa_{nf} - (n-1)(\kappa_{nf} - \kappa_{s2})\phi_2}{\kappa_{s2} + (n-1)\kappa_{nf} + (\kappa_{nf} - \kappa_{s2})\phi_2}\right]\kappa_{nf}$ $\kappa_{nf} = \left[\frac{\kappa_{s1} + (n-1)\kappa_f - (n-1)(\kappa_f - \kappa_s)\phi_1}{\kappa_{s1} + (n-1)\kappa_f + (\kappa_f - \kappa_s)\phi_1}\right]\kappa_f$	(Thermal conductivity)
$(\rho C_p)_{mnf} = (1-\phi_3)(\{(1-\phi_2)[(1-\phi_1)(\rho C_p)_f + (\rho C_p)_{s1}\phi_1]\} + (\rho C_p)_{s2}\phi_2) + \phi_3(\rho C_p)_{s3}$	(Heat capacitance)

where the subscript 1, 2, and 3 stands for first, second and third nanoparticles. The ϕ_1, ϕ_2, and ϕ_3 are the volume fraction and for spherical nanoparticles, also we have considered $n = 3$. Here $\phi_1 = \phi_2 = 0.05$ vol are fixed.

Table 2. Thermophysical values [3, 4, 8]

	ρ [kgm^{-3}]	C_p [Jkg^{-1}K^{-1}]	k [Wm^{-1}K^{-1}]	σ [Sm^{-1}]
$C_2H_6O_2$	1116.6	2382	0.249	0.01485
Al_2O_3	3970	765	40	35×10^6
SiO_2	2650	730	1.5	1×10^{-18}
TiO_2	4175	692	8.4	6.27×10^{-5}

Similarity transformation

With the help of the similarity transformations difined [10]:

$$u = U_0 e^{x/l} f'(\eta), \quad v = -\sqrt{\frac{\nu_f U_0}{2l}} e^{x/2l} [f(\eta) + \eta f'(\eta)] \quad \text{and} \quad \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty} \tag{5}$$

where

$$\eta = y\sqrt{\frac{U_0}{2l\nu_f}} e^{x/2l}$$

the non-linear equations of the flow (momentum and energy equations) are now converted in the following ordinary differential equations using aforementioned transformations:

$$Af''' + B(ff'' - 2f'^2) - \frac{\sigma_{mnf}}{\sigma_f} M \sin^2 \alpha f' = 0 \tag{6}$$

$$\frac{1}{\Pr} \frac{\kappa_{mnf}}{\kappa_f} C\theta'' + f\theta' - f'\theta = 0 \tag{7}$$

Here

$$A = \frac{1}{(1-\phi_1)^{2.5}(1-\phi_2)^{2.5}(1-\phi_3)^{2.5}}$$

$$B = (1-\phi_3)\left(\left\{(1-\phi_2)\left[(1-\phi_1) + \phi_1 \frac{\rho_{s1}}{\rho_f}\right]\right\} + \phi_2 \frac{\rho_{s2}}{\rho_f}\right) + \phi_3 \frac{\rho_{s3}}{\rho_f}$$

$$\text{and } C = \left[(1-\phi_3)\left(\left\{(1-\phi_2)\left[(1-\phi_1) + \phi_1 \frac{(\rho C_p)_{s1}}{(\rho C_p)_f}\right]\right\} + \phi_2 \frac{(\rho C_p)_{s2}}{(\rho C_p)_f}\right) + \phi_3 \frac{(\rho C_p)_{s3}}{(\rho C_p)_f}\right]^{-1}$$

Using previous transformation, boundary condition (5) is also transformed:

$$\theta(0) = 1, \quad f(0) = 0, \quad f'(0) = 1, \quad \theta(\infty) = 0, \quad f'(\infty) = 0 \tag{8}$$

where $M = (2lB_0^2\sigma_f)/(U_0\rho_f e^{x/l})$ is the magnetic parameter and $\Pr = [(\rho C_p)_f \nu_f]/\kappa_f$ – the Prandtl number. Another physical quantity is local Nusselt number that is given by $\mathrm{Nu}_x = -[(\kappa_{hnf}/\kappa_f) + 4R/3]\psi_1 x\theta'(0)$.

Numerical solution

After applying suitable similarity solutions, the fundamental eqs. (1)-(3) have converted to the ODE along with boundary condition. To tackle this further we have exercised, Runga-Kutta method of order four with the shooting procedure. Furthermore, we have changed eqs. (6) and (7), alongside boundary conditions (8) into first-order initial value problems:

$$f = h_1,\quad f' = h_2,\quad f'' = h_3,\ \theta = h_4,\ \text{and}\ \theta' = h_5$$

$$h_3' = \frac{B}{A}(2h_2^2 - h_1 h_3) + \frac{M}{A}\frac{\sigma_{\text{hnf}}}{\sigma_{\text{f}}} h_2 \sin^2\alpha\ ,\quad h_5' = \frac{\Pr}{C}\frac{\kappa_{\text{f}}}{\kappa_{\text{mnf}}}(h_2 h_4 - h_1 h_5)$$

With boundary condition, $h_1(0)=0,\ h_2(0)=1,\ h_4(0)=1,$ and some initial guesses best approximated results will be obtained for numerical solutions, by the shooting method. Prandtl number for $C_2H_6O_2$ is 204 and step size $\Delta\eta = 0.01$ considered Number of iterations has been performed in the mentioned technique to get the proper arrangements up to the exactness 10^{-7}.

Result and discussions

The analysis of modified nanofluids flow through an exponentially stretched sheet is considered. Inclined magnetic field is applied together. Numerical outcomes for temperature and velocity profiles are presented graphically for different flow pertinent parameters.

In fig. 1, the effect of M on $f'(\eta)$ for Al_2O_3-SiO_2-TiO_2/$C_2H_6O_2$ modified nanofluid is depicted. Results show that with increased M, we get a decreased in the velocity profile. This concurs with the way that the electromagnetic force makes the opposite force called as Lorentz force, to the fluid with expanded M. The effects of the α on $f'(\eta)$, are depicted in fig. 2. By enhancing the α parameter, $f'(\eta)$ profile decreases. This is because magnetic field gets stronger

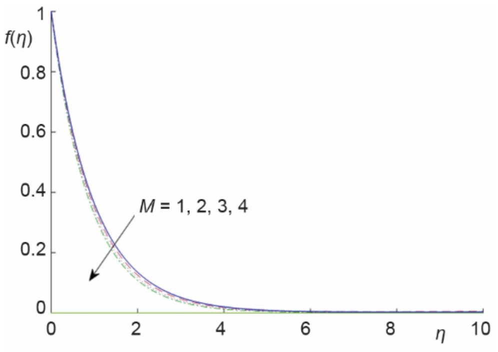

Figure 1. Profile of velocity for M; α = π/3, Pr = 204, ϕ_3 = 0.005

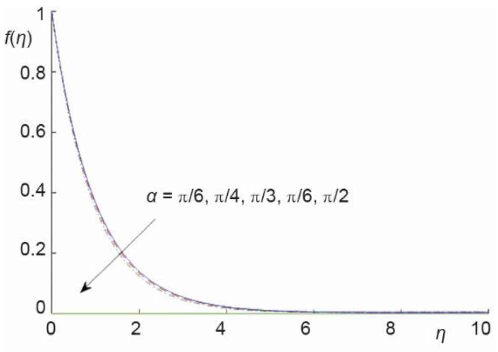

Figure 2. Profile of velocity for α; M = 0.2, Pr = 204, ϕ_3 = 0.005

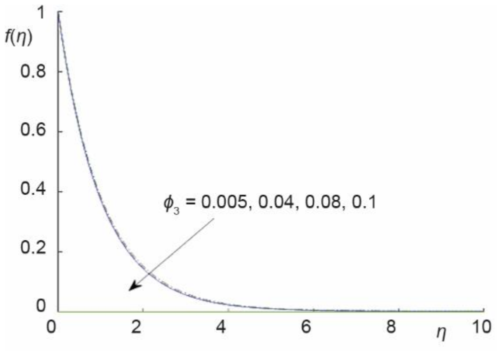

Figure 3. Profile of velocity for ϕ_3; α = π/3, Pr = 204, M = 0.2

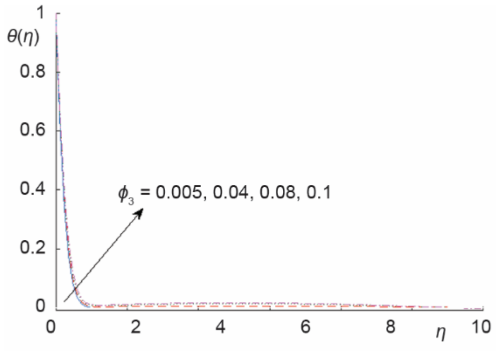

Figure 4. Profile of temperature for ϕ_3; α = π/3, Pr = 204, M = 0.2

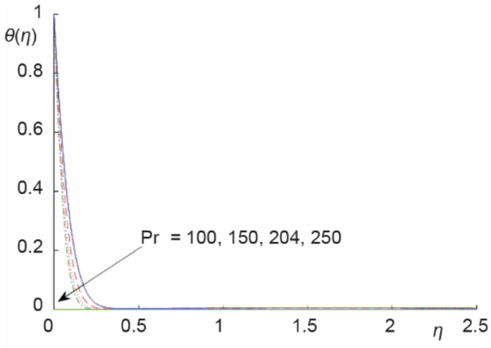

Figure 5. Profile of temperature for Pr; α = π/3, ϕ_3 = 0.005, *M* = 0.2

with increased angle of inclination so by Lorentz force velocity field gets cut down. Figure 3 depicts the effect of volume friction ϕ_3 on $f'(\eta)$. It is noticed that with increased ϕ_3 we get decreased velocity profile. It is also observed for ϕ_3 that, the speed of the fluid is lesser in the neighborhood of the sheet and away from it the the speed is increased with increasing ϕ_3. Figure 4, represents the impact of ϕ_3 on $\theta(\eta)$. It is noticed that with increased ϕ_3 we get an increased temperature profile for modified nanofluids. With increased nanoparticles more energy can be extracted, so it may results to increased temperature field. Figure 5, represents the effect of Prandtl number on $\theta(\eta)$. From the figures we can conclude that with increased Prandtl number along with $\theta(\eta)$ decreased temperature profile is observed.

Conclusions

Heat transfer effect, of modified nanofluids, flow past an exponential stretching surf along with imposed inclined magnetic field has been investigated. Ethylene glycol based modified nanofluid with suspension of Al_2O_3-SiO_2-TiO_2 nanoparticles has presented graphically. The governing model of the flow is solved by Runga-Kutta fourth order method using appropriate similarity transformations.

The following important outcomes are obtained.

- With increased inclined angle α and parameter of magnetic field a decrement in velocity field has been noticed for modified nanofluid.
- The velocity curve for ϕ_3, near the wall, the fluid velocity is decreases in the neighborhood of the sheet, and away from the sheet, it is increases.
- For an increment in the values of ϕ_3 for velocity field a decrement is noticed and reverse effect is noticed with the temperature profile.

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