HEAT AND MASS TRANSFER EFFECTS OF PERISTALTIC MOTION OF A JEFFERY FLUID IN A TUBE

by

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The impact of heat and mass transfer were analyzed in the present investigation by considering the peristaltic transport of a Jeffery fluid with nanoparticles in a uniform tube. Lubrication theory hypotheses have been considered and expressions have been defined for axial velocity, pressure drop, frictional force, heat and mass transfer effects. The patterns of the stream line and trapped bolus were graphically depicted at the end. It is found that with regard to the modification of various parameters, pressure drop and frictional force exhibit identical behavior. By increasing/decreasing the local temperature Grashof number, Brownian motion parameter and local nanoparticle Grashof number, pressure drop and axial velocity can be regulated.

Key words**:** *Jeffery fluid, peristaltic transport, temperature profile, nanoparticle phenomena*

Introduction

Peristalsis is a common transmission process of fluids from the area of lower pressure to the region of higher pressure. It occurs in the body for the transport of physiological fluids from one part of the body to another part of the body. Many researchers contributed to this field. To name few of them are, Shapiro *et al.* [1], Chu and Fang [2], Maruthi and Radhakrishnamacharya [3], and Prasad *et al.* [4].

Now a days nanotechnology has considerable attention by the researchers because of its applications to biomedical and industrial fields. Choi and Eastman [5] was the forerunner to do research in the nanotechnology. Eastman *et al.* [6] experimentally proved that thermal conductivity of the base fluids can be enhanced by 60% by adding the nanoparticles to the base fluids. Many more researchers also done their research in this field. Prasad *et al.* [4], Ellahi [7], Subadra *et al.* [8] contributed significant to this field of research.

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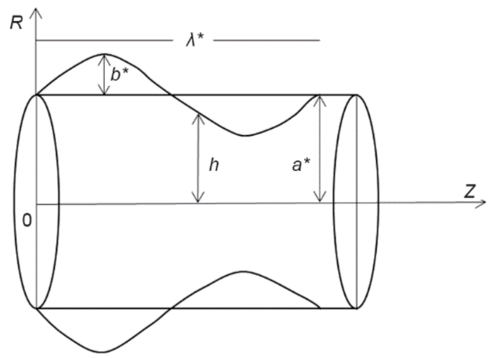

Figure 1. Peristaltic motion of a Jeffery fluid

The peristaltic motion of a nanoparticles immersed in Jeffery fluid in a conduit is considered in this paper, and the effects of heat and mass transfer have been studied. Analytical expressions have been calculated and illustrated by graphs for axial velocity, pressure drop, frictional force, heat and mass transfer effects. Stream line patterns and trapped bolus have been depicted.

Mathematical formulation

ln a tube, an incompressible Jeffery fluid with nanoparticles was taken into account. The geometry of the problem is explained by fig. 1. To examine the flow, cylindrical polar co-ordinate system (R, θ, Z) is used.

The equation given below represents wall deformation:

$$R = H(Z,t) = a^* + b^* \sin \frac{2\pi}{\lambda^*}(Z - c^* t) \tag{1}$$

where a^* is the radius of the tube, b^* – the amplitude, c^* – the wave speed, and λ^* – the wave length.

The following transformations are used to transfer the equations from stationary frame to wave frame of reference:

$$z = Z - c^* t, \quad r = R, \quad \theta = \theta, \quad w = W - c^*, \quad u = U \tag{2}$$

By applying non-dimensionalization; lubrication theory approximations; the equations of the Jeffery fluid flow, Nadeem *et al.* [10], with nanoparticles are:

$$\frac{\partial p}{\partial r} = 0 \tag{3}$$

$$\frac{1}{1+\lambda_1}\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial w}{\partial r}\right) = \frac{\partial p}{\partial z} + \mathrm{Gr}\theta_1 + \mathrm{Br}\sigma_1 \tag{4}$$

$$0 = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \theta_1}{\partial r}\right) + N_b\frac{\partial \sigma_1}{\partial r}\frac{\partial \theta_1}{\partial r} + N_t\left(\frac{\partial \theta_1}{\partial r}\right)^2 \tag{5}$$

$$0 = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \sigma_1}{\partial r}\right) + \left(\frac{N_t}{N_b}\right)\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \theta_1}{\partial r}\right) \tag{6}$$

where p is the pressure, w – the axial velocity, θ_1 – the temperature profile, σ_1 – the nanoparticle phenomena, N_b – the Brownian motion parameter, N_t – the thermophoresis parameter, Gr – the local temperature Grashof number, and Br – the local nanoparticle Grashof number.

Dimensionless boundary conditions are:

$$\frac{\partial w}{\partial r}=0,\quad \frac{\partial \theta_1}{\partial r}=0 \quad \text{and} \quad \frac{\partial \sigma_1}{\partial r}=0 \quad \text{at} \quad r=0 \tag{7}$$

$$w=-1 \quad \text{at} \quad r=h(z) \tag{8}$$

Solution of the problem

The homotopy perturbation technique of He [10] is employed to find the approximate solutions for θ_1 and σ_1 from eqs. (5) and (6).

By applying boundary conditions and by assuming the initial approximations for θ_1 and σ_1 as $\theta_{10}(r, z) = -[(r^2 - h^2)/4]$ and $\sigma_{10}(r, z) = -[(r^2 - h^2)/4]$, the expressions for θ_1 and σ_1 are:

$$\theta_1=(N_b-N_t)(N_b-2N_t)\left(\frac{r^6-h^6}{1152}\right)-(N_b-2N_t)\left(\frac{r^4-h^4}{64}\right) \tag{9}$$

$$\sigma_1=-\left(\frac{N_t}{N_b}\right)(N_b-N_t)\left(\frac{r^4-h^4}{64}\right) \tag{10}$$

Using eqs. (9) and (10) for θ_1 and σ_1 and applying boundary conditions eqs. (7) and (8) in the eq. (4), the expression for w is:

$$w=-1+(1+\lambda_1)\frac{\mathrm{d}p}{\mathrm{d}z}\left(\frac{r^2-h^2}{2}\right)+\frac{\mathrm{Gr}}{1152}(1+\lambda_1)(N_b-N_t)(N_b-2N_t)\left(\frac{r^8}{64}-\frac{r^2h^6}{4}-\frac{15h^8}{64}\right)-$$
$$-\frac{\mathrm{Gr}}{64}(1+\lambda_1)(N_b-2N_t)\left(\frac{r^6}{36}-\frac{r^2h^4}{4}+\frac{2h^6}{9}\right)-\frac{\mathrm{Br}}{64}(1+\lambda_1)\left(N_t-\frac{N_t^2}{N_b}\right)\left(\frac{r^6}{36}-\frac{r^2h^4}{4}+\frac{2h^6}{9}\right) \tag{11}$$

Dimensionless flux q in the moving frame is given by:

$$q=\int_0^h 2rw\,\mathrm{d}r \tag{12}$$

Pressure drop over a wave length Δp_λ is given by:

$$\Delta p_\lambda=-\int_0^1 \frac{\mathrm{d}p}{\mathrm{d}z}\mathrm{d}z \tag{13}$$

The expression for Δp_λ is given by:

$$\Delta p_\lambda=qL_1+L_2 \tag{14}$$

where $L_2=\dfrac{4}{(1+\lambda_1)}\displaystyle\int_0^1 \frac{1}{h^2}\mathrm{d}z$

$$L_2 = \frac{4}{(1+\lambda_1)}\int_0^1 \frac{1}{h^2}\mathrm{d}z + \frac{57}{40320}\mathrm{Gr}(N_b - N_t)(N_b - 2N_t)\int_0^1 h^6 \mathrm{d}z + \frac{5\mathrm{Gr}}{768}(N_b - 2N_t)$$

$$\int_0^1 h^4 \mathrm{d}z + \frac{5\mathrm{Br}}{768}N_t\left(1 - \frac{N_t}{N_b}\right)\int_0^1 h^4 \mathrm{d}z$$

Utilizing the same technique of Shapiro *et al*. [1] the average time flux was supplied by the laboratory frame over one period is:

$$\bar{Q} = 1 + \frac{\varepsilon^2}{2} + q \tag{15}$$

At the wall the dimensional less frictional force $\bar{F}$ is:

$$\bar{F} = \int_0^1 h^2 \left(-\frac{\mathrm{d}p}{\mathrm{d}z}\right)\mathrm{d}z \tag{16}$$

At the wall, heat transfer coefficient and mass transfer coefficient are:

$$Z_\theta(r,z) = \left(\frac{\partial h}{\partial z}\right)\left(\frac{\partial \theta_1}{\partial r}\right) \quad \text{and} \quad Z_\sigma(r,z) = \left(\frac{\partial h}{\partial z}\right)\left(\frac{\partial \theta_1}{\partial r}\right) \tag{17}$$

Results and discussions of the problem

The MATHEMATICA 11.0 software has been used to draw the graphs of axial velocity, pressure drop, frictional force, heat transfer coefficient, and mass transfer coefficient.

From figs. 2 and 3, it is observed that, pressure drop, Δp_λ, and frictional force, $\bar{F}$, enhances with the rise of N_t, Gr, and diminishes with the rise of N_b, Br. Moreover, with the increase of λ_1, Δp_λ, and $\bar{F}$ increases in the region (0, 0.05) and decreases in the region (0.05, ∞).

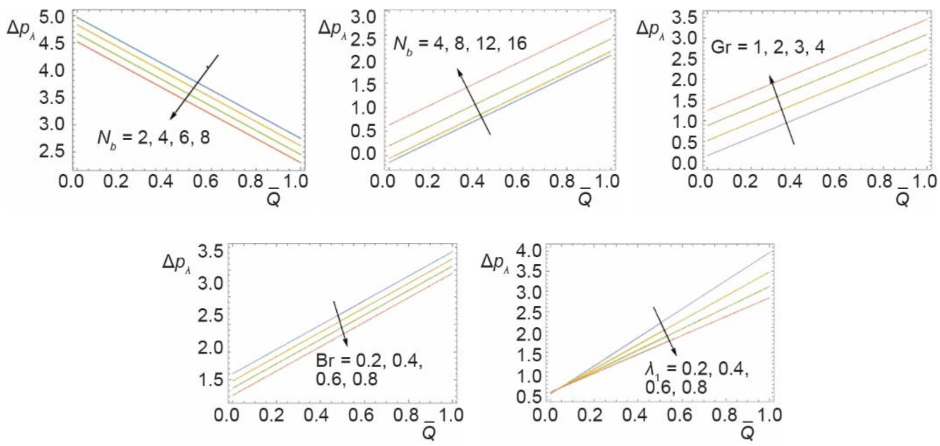

Figure 2. Graphs for pressure drop Δp_λ showing the effects of changing N_b, N_t, Gr, Br, and λ_1

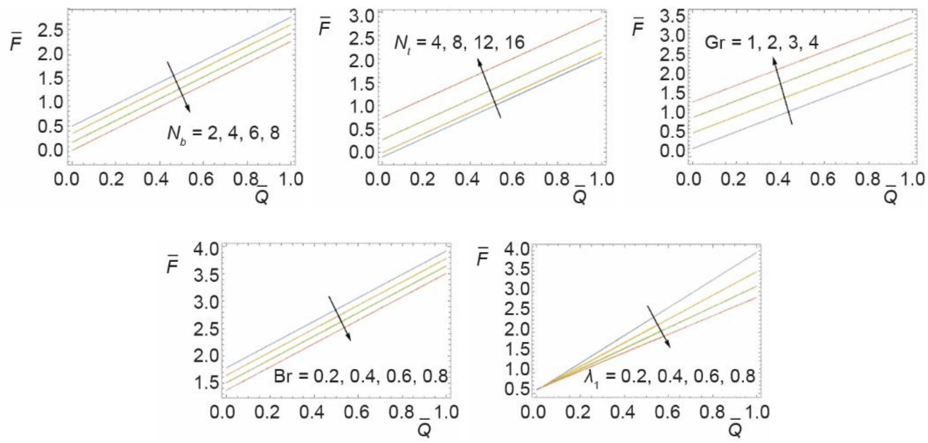

Figure 3. Graphs for $\bar{F}$ depicting the effects of varying N_b, N_t, Gr, Br, and λ_1

From fig. 4, it is noticed that the value of w reduces with the increase of Gr, Br and, λ_1.

From figs. 5, it is noted that, θ_1 amplifies with the rise of N_b and diminishes with the rise of N_t and finally converges to 1 in both the cases and σ_1 rises with the increase of N_b and N_t and finally converges to 1.

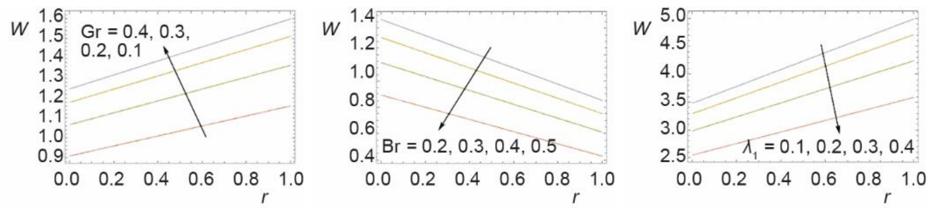

Figure 4. Graphs for axial velocity w showing the effects of changing Gr, Br, and λ_1

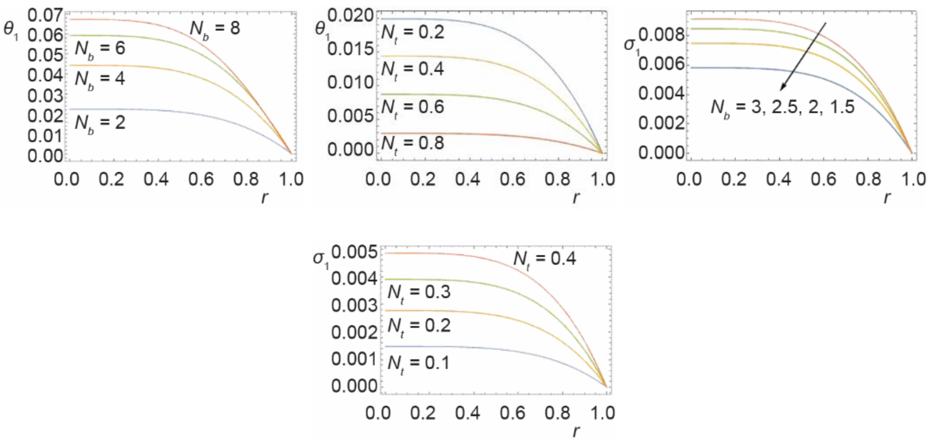

Figure 5. Graphs for θ_1 and σ_1 showing the effects of changing N_b and N_t,

From fig. 6, it is known that heat transfer coefficient Z_θ is invariable in (0,0.2) and decreases in (0.2, ∞) with the rise of N_b. It displays reverse behavior with the increase of N_t and mass transfer coefficient Z_σ is invariable in the region (0,0.2) and decreases in (0.2, ∞) with the rise of N_b and N_t.

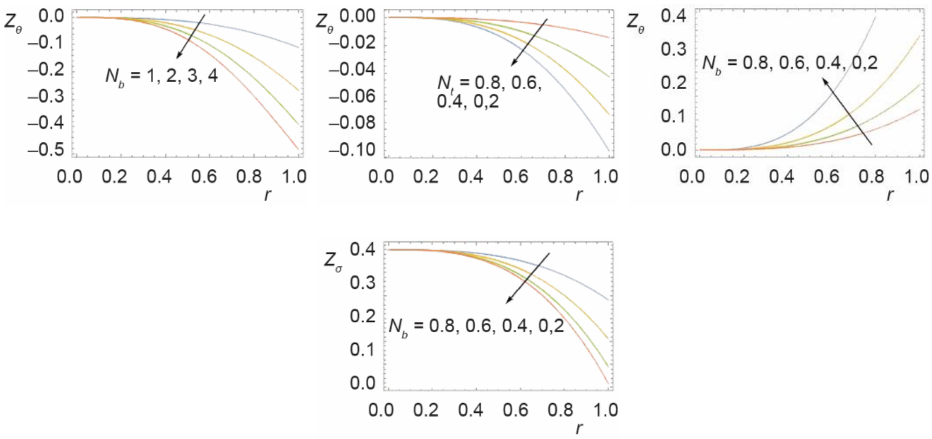

Figure 6. Graphs for Z_θ and Z_σ depicting the effects of varying N_b and N_t

Figures 7 and 8 depicts that the volume of the trapped bolus rises with the increase of N_b, N_t, Gr, and λ_1 and diminishes with the increase of Br.

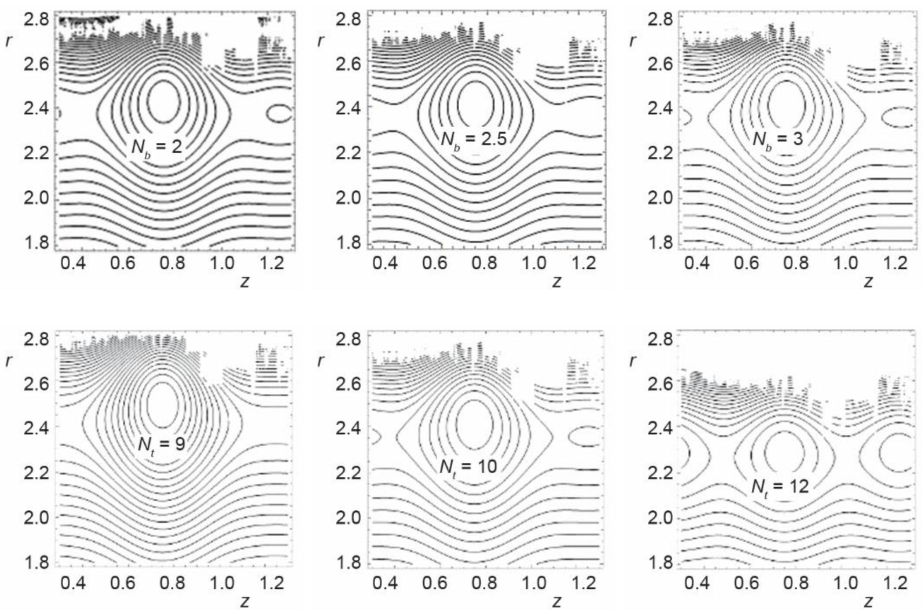

Figure 7. Graphs for stream line patterns and trapped bolus showing the effects of changing N_b and N_t

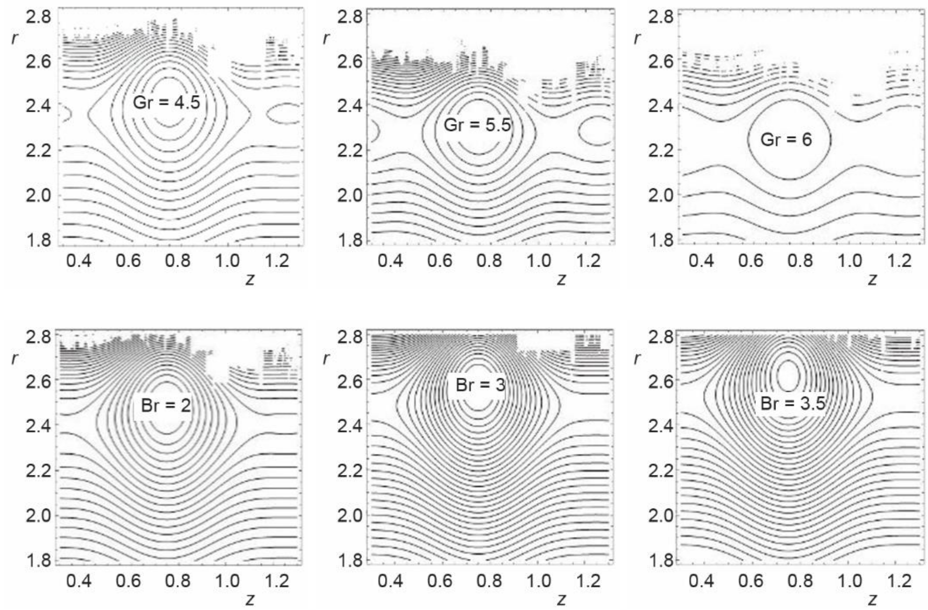

Figure 8. Graphs for stream line patterns and trapped bolus showing the effects of changing Gr and Br

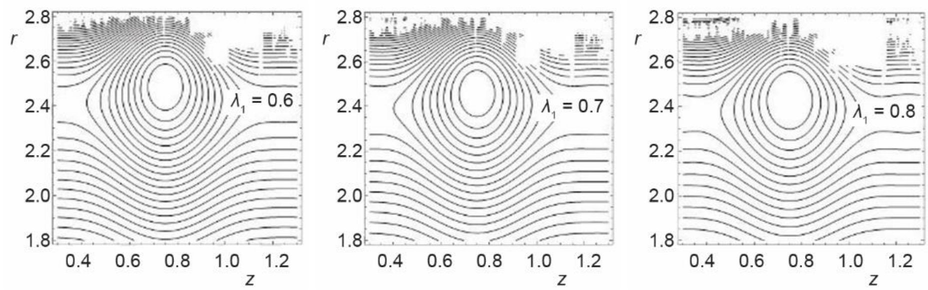

Figure 9. Graphs for stream line patterns and trapped bolus showing the effects of changing λ_1

Conclusions

The major conclusions of this paper are as follow.

- Axial velocity can be controlled by increasing/decreasing of Gr, Br, and λ_1.
- Size of the trapped bolus rises with the increase of N_b, N_t, Gr, λ_1 and diminishes with the increase of Br.

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