MASS TRANSFER AND CATTANEO-CHRISTOV HEAT FLUX FOR A CHEMICALLY REACTING NANOFLUID IN A POROUS MEDIUM BETWEEN TWO ROTARY DISKS

by

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This paper examines the transport of a chemically reacting nanofluid in a porous medium between two rotary disks with Cattaneo-Christov's heat flux. The non-linear ordinary differential system formed under Vonn Karman transformation of a non-linear partial differential system is solved via a shooting method with MATLAB bvp4c. The nanofluid thermodynamics profiles with variation in physical properties of thermal relaxation time, thermal radiation, porosity, and chemical reaction are observed. Axial, radial, and tangential velocities are found to be increasing functions of porous medium. A decrease in the fluid temperature is perceived as thermal radiation and thermal relaxation increase since more heat can be transported to neighboring surroundings. The concentration is enhanced with intensified Cattaneo-Christov's thermal relaxation but it oscillates with reacting chemicals. The rotary disks bound the oscillating nanofluid from downward to upward directions and vice versa. The axial velocity represents the change in force due to porosity and radial stretching of the disks.

Key words: *Cattaneo-Christov heat flux, transport equation, porous medium, chemical reaction, rotary disks, nanofluid*

Introduction

Rotary disks have plenty applications in biomedical technology, mechanical industries, and in thermal engineering. Nanofluid motion in a micro spacing between Tesla-tube co-spinning disks studied by Sengupta and Guha [1], uncovers tangential velocity in the computational domain that increases radial velocity profile. Transfer of heat from a flow between stretchable coaxial spinning disks in a thermally stratified medium was studied by Hayat *et al.* [2] with conclusions that radial and axial velocities decrease towards the lower disk by enhancing Reynolds number and that tangential velocity is a decreasing function of magnetic number. Diffusion-thermo and thermal diffussion effects on MHD flow of a viscous liquid between enlarging/contracting rotary disks with viscous dissipation were investigated by Srinivas *et al.* [3]. They deduced that the flow temperature increases while the concentration decreases with increment in Soret and Dufour parameters. A generalized modelling of an unbalanced mixed convection fluid adjourned with hybrid nanoparticles amid two spinning disks was recommended by Xu [4] as it matches with the simplified hybrid nanofluid design.

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Muhammad *et al.* [5] analysed the compression of a nanofluid flow and Cattaneo--Christov heat-mass fluxes with the conclusion that temperature and concentration profiles are decreased for solutal and thermally stratified parameters. Chemical reaction alters ionic and molecular structure of a fluid flow. The mixed radiative model with chemical reaction by Hayat *et al.* [6] is currently extended with new addition of porous medium into the momentum equation, Cattaneo-Christov heat flux into the energy equation and a Buongiorno's nanofluid transport equation. The new physical properties are thermal relaxation time, porous medium and the nanoparticle volume fraction for the hydromagnetic nanofluid flow between two rotary disks.

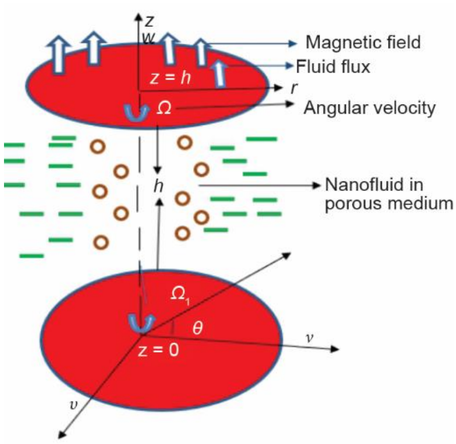

Figure 1. Geometry of the problem

Model formulation

Consider the fluid model as in fig. 1 where the disks are rotating at angular velocities velocities Ω_1 and Ω_2, respectively, while being radially stretched at the rates of a_1 and a_2, respectively. The analogous equations can be detailed out:

$$\frac{\partial u}{\partial r}+\frac{u}{r}+\frac{\partial w}{\partial z}=0 \tag{1}$$

$$u\frac{\partial u}{\partial r}-\frac{v^2}{r}+w\frac{\partial u}{\partial z}=-\frac{1}{\rho}\frac{\partial p}{\partial r}+\nu\left[\frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\frac{\partial u}{\partial r}+\frac{\partial^2 u}{\partial z^2}-\frac{u}{r^2}\right]+\mathrm{g}[\gamma_1(T-T_2)+\gamma_2(T-T_2)^2]+$$

$$+\mathrm{g}[\gamma_3(C-C_2)+\gamma_4(C-C_2)^2]-\left[\frac{\mu}{k_1\rho}+\frac{\sigma B_0{}^2}{\rho}\right]u \tag{2}$$

$$u\frac{\partial v}{\partial r}+\frac{uv}{r}+w\frac{\partial v}{\partial z}=-\frac{1}{\rho}\frac{\partial p}{\partial z}+\nu\left[\frac{\partial^2 v}{\partial r^2}+\frac{1}{r}\frac{\partial v}{\partial r}+\frac{\partial^2 u}{\partial z^2}-\frac{v}{r^2}\right]-\left(\frac{\mu}{k_1\rho}+\frac{\sigma B_0^2}{\rho}\right)v \tag{3}$$

$$u\frac{\partial w}{\partial r}+w\frac{\partial v}{\partial z}=-\frac{1}{\rho}\frac{\partial p}{\partial z}+\nu\left[\frac{\partial^2 w}{\partial r^2}+\frac{1}{r}\frac{\partial w}{\partial r}+\frac{\partial^2 w}{\partial z^2}\right]-\frac{\mu}{k_1}w \tag{4}$$

$$u\frac{\partial T}{\partial r}+w\frac{\partial T}{\partial z}=\frac{k}{\rho c_p}\left(\frac{1}{r}\frac{\partial T}{\partial r}+\frac{\partial^2 T}{\partial r^2}+\frac{\partial^2 T}{\partial z^2}\right)+\frac{1}{\rho c_p}\frac{16\sigma^*}{3k^*}\frac{\partial}{\partial z}\left(T^3\frac{\partial T}{\partial z}\right)+\frac{Q_0}{\rho c_p}(T-T_2)+$$

$$-\frac{\sigma}{\rho c_p}B_0{}^2(u^2+v^2)-\gamma\left[u^2\frac{\partial^2 T}{\partial z^2}+2uw\frac{\partial^2 T}{\partial z\partial r}+\left(u\frac{\partial u}{\partial r}+w\frac{\partial u}{\partial z}\right)\frac{\partial T}{\partial r}+\left(u\frac{\partial w}{\partial r}+w\frac{\partial w}{\partial z}\right)\frac{\partial T}{\partial z}\right]+$$

$$+\frac{(\rho c_p)_s}{(\rho c_p)}\left\{D_\mathrm{B}\left(\frac{\partial T}{\partial r}\frac{\partial C}{\partial r}+\frac{\partial T}{\partial z}\frac{\partial C}{\partial z}\right)+\frac{D_T}{T\infty}\left[\left(\frac{\partial T}{\partial r}\right)^2+\left(\frac{\partial T}{\partial z}\right)^2\right]\right\} \tag{5}$$

$$u\frac{\partial C}{\partial z}+w\frac{\partial C}{\partial z}=D_{\mathrm{B}}\left[\frac{\partial^2 C}{\partial r^2}+\frac{1}{r}\frac{\partial C}{\partial r}+\frac{\partial^2 C}{\partial z^2}\right]+\frac{D_T}{T\infty}\left[\frac{\partial^2 T}{\partial r^2}+\frac{1}{r}\frac{\partial T}{\partial r}+\frac{\partial^2 T}{\partial z^2}\right]-k_2(C-C_2) \tag{6}$$

with boundary conditions:

$$u=ra_1,\quad v=r\Omega_1,\quad w=0,\quad k\frac{\partial T}{\partial z}=-h_1(T_1-T),\quad D\frac{\partial C}{\partial z}=-h_2(C_1-C)\quad \text{at } z=0$$

$$u=ra_2,\quad v=r\Omega_2,\quad w=0,\quad k\frac{\partial T}{\partial z}=-h_3(T-T_2),\quad D\frac{\partial C}{\partial z}=-h_4(C-C_2)\quad \text{at } z=h \tag{7}$$

We now use von Karman [7] transformations:

$$u=r\Omega_1 f'(\eta),\quad v=r\Omega_1 g(\eta),\quad w=-2h\Omega_1 f(\eta),\quad \theta(\eta)=\frac{T-T_2}{T_1-T_2,},\quad \phi(\eta)=\frac{C-C_2}{C_1-C2},$$

$$p=\rho\nu\Omega_1\left[p(\eta)+\frac{r^2}{2h^2}\in\right],\quad \eta=\frac{z}{h} \tag{8}$$

on eqs. (1) to (7) with eq. (1) is identically satisfied and we obtain:

$$f'''+\mathrm{Re}\left[2ff''-f'^2+\mathrm{g}^2-f'\frac{1}{\beta}-\mathrm{M}f'+\beta_1\theta(1+\beta_T\theta)+\beta_1 N\phi(1+\beta_T\phi)\right]-\in=0 \tag{9}$$

$$\mathrm{g}''+\mathrm{Re}(2f\mathrm{g}'+2f'\mathrm{g}-\frac{\mathrm{g}}{\beta}-\mathrm{Mg})=0 \tag{10}$$

$$p'+4\mathrm{Re}\,ff'+2f''+\frac{2f}{\beta}=0 \tag{11}$$

$$\theta''+\frac{4}{3}Rd[\theta(\theta_w-1)+1]^2[\theta(\theta_w-1)+1]\theta''-4\lambda\,\mathrm{Pr}\,\mathrm{Re}\,f^2\theta''+Nb\,\mathrm{Pr}\,\theta'\phi'+\mathrm{Pr}\,Nt\theta'^2+\mathrm{Pr}\,\alpha\theta+$$

$$+4Rd[\theta(\theta_w-1)+1]^2(\theta_w-1)\theta'^2+2\,\mathrm{Pr}\,\mathrm{Re}\,f\theta'+\mathrm{Pr}\,\mathrm{Re}\,M\,\mathrm{Ec}(f'^2+\mathrm{g}^2)-4\lambda\,\mathrm{Pr}\,\mathrm{Re}\,f'\theta'=0 \tag{12}$$

$$\phi''+2\,\mathrm{Pr}\,\mathrm{Le}\,\mathrm{Re}\,f\phi'-\mathrm{Sc}\,Cr+\frac{Nt}{Nb}\theta''=0 \tag{13}$$

with boundary conditions:

$$f(0)=0,\quad f'^{(0)}=A_1,\quad \mathrm{g}(0)=1,\quad \theta'(0)=-\lambda_1[1-\theta(0)],\quad \phi'(0)=-\lambda_2[1-\phi(0)]$$

$$p(0)=0,\quad f(1)=0,\quad f'(0)=A_2,\quad \mathrm{g}(1)=\tau,\quad \theta'(1)=-\lambda_3\theta'(1),\quad \phi'(1)=\lambda_4\phi(1) \tag{14}$$

where

$$\mathrm{Re}=\frac{h^2\Omega_1}{\nu},\quad \mathrm{Pr}=\frac{\rho c_p\nu}{k},\quad \mathrm{M}=\frac{\sigma B_0^2}{\rho\Omega_1,},\quad \mathrm{Le}=\frac{k}{\rho c_p D_{\mathrm{B}}},\quad \mathrm{Ec}=\frac{\rho r^2\Omega_1^2}{(\rho c_p)_f(T_1-T_2)}$$

$$A_1=\frac{a_1}{\Omega_1,},\quad A_2=\frac{a_2}{\Omega_2},\quad \tau=\frac{\Omega_1}{\Omega_2},\quad Cr=\frac{k_2h^2}{D},\quad \alpha=\frac{Q_{0h^2}}{k},\quad \theta_w=\frac{T_1\nu-T_2\Omega_1 r^2}{\nu T_1}$$

$$\lambda = \gamma\Omega_1, \quad \lambda_1 = \frac{\beta_1 h}{k}, \quad \lambda_2 = \frac{\beta_2 h}{D}, \quad \lambda_3 = \frac{\beta_3 h}{k}, \quad \lambda_4 = \frac{\beta_4 h}{D}$$

$$N = \frac{\lambda_{3(C_1 - C_2)}}{\lambda_1 (T_1 - T_2)}, \quad Nb = \frac{(\rho c_p)_s (C_1 - C_2)}{(\rho c_p)_\mathrm{f} \nu}, \quad Nt = \frac{(\rho c_p)_s (T_1 - T_2)}{(\rho c_p)_\mathrm{f} \nu T\infty}$$

$$\beta = \frac{k_1 \Omega_1}{\nu}, \quad \beta_1 = \frac{\mathrm{g}\lambda_1 (T_1 - T_2)}{r\rho^2}, \quad \beta_T = \frac{\lambda_2}{\lambda_1}(T_1 - T_2), \quad \beta_C = \frac{\lambda_4}{\lambda_3}(C_1 - C_2) \tag{15}$$

Equation (9) can be simplified by differentiating it with respect to η and we obtain:

$$f'''' + \mathrm{Re}\left[2ff''' + 2\mathrm{gg}' - \frac{f''}{\beta} - \mathrm{M}f'' + \beta_1 \theta'(1 + \beta_T \theta) + \beta_1 N \phi'(1 + 2\beta_C \phi)\right] = 0 \tag{16}$$

Hence, the eqs. (10)-(14) and (16) can then solved by using a shooting technique with MATLAB *bvp4c* algorithm.

Analytical results

Present results are validated with previous solutions in tab. 1 when $A_1 = A_2 = \phi = 0$ and Re = 1. Figure 2(a) shows the axial velocity as an increasing function of porous medium because additional stretching force is being exerted onto the disk surface. Figure 2(b) shows radial velocity as an increasing function of porosity at nearly $\eta = 0.3$ and $\eta = 0.7$. Figure 2(c) depicts tangential velocity as also an increasing function of porosity β at some different values of η. Figure 3(a) shows that as thermal relaxation λ increases $\phi(\eta)$, increases while fig. 3(b) shows that $\theta(\eta)$, decreases with increasing values of λ. When time is relaxed to transfer heat to neighboring substances, this causes temperature to fall in the long run. Moreover, fig. 4(a) shows that temperature is a reducing function of *Rd*. As thermal radiation increases, more heat energy being discharged from the fluid body to the neighboring space, hence temperature decreases significantly. Figure 4(b) depicts that, as values of *Cr* increases, the corresponding nanoparticle volume fraction profile

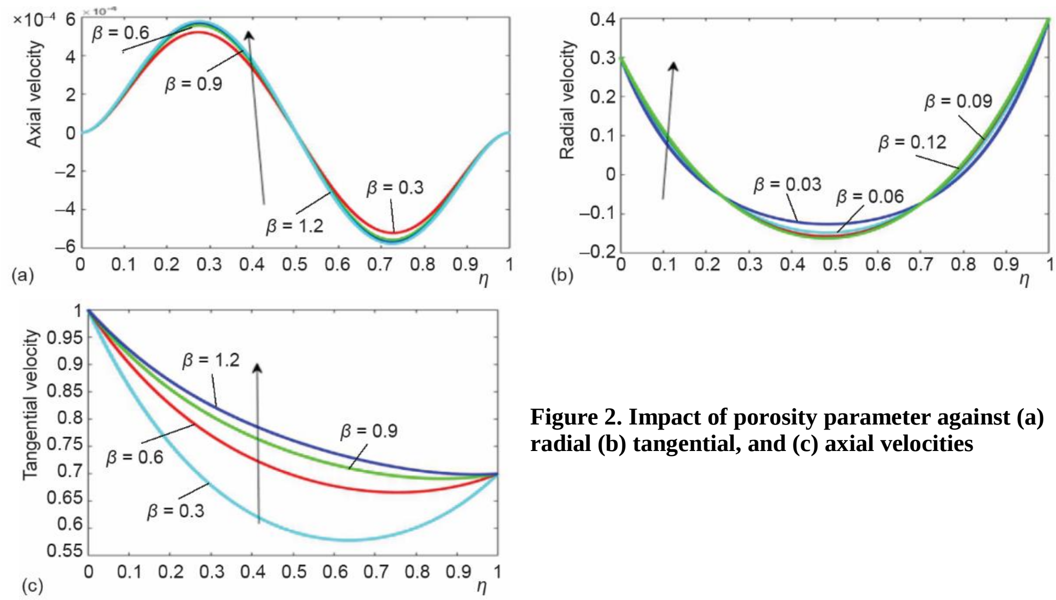

Figure 2. Impact of porosity parameter against (a) radial (b) tangential, and (c) axial velocities

decreases, hence concentration is a decreasing function of *Cr*. Chemical reaction prevents constant settlement of the nanoparticles thus the concentration field is scattered.

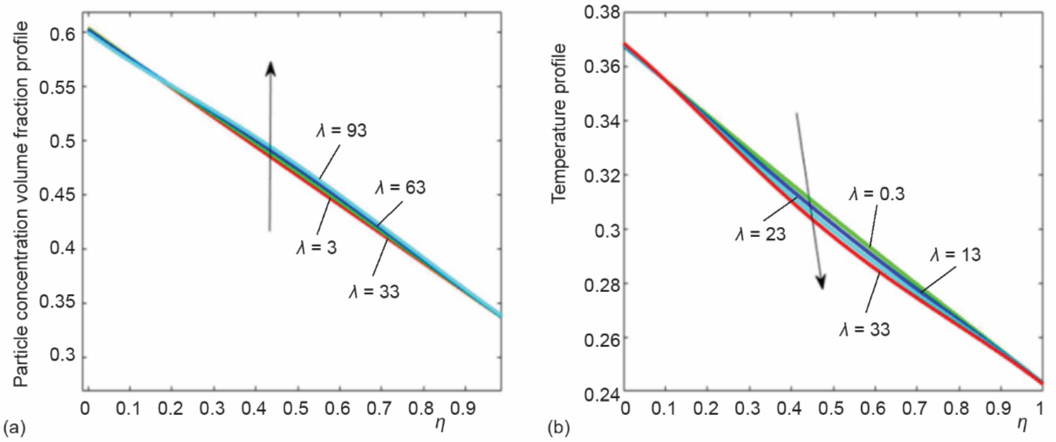

Figure 3. Impact of thermal relaxation on (a) concentration and (b) temperature

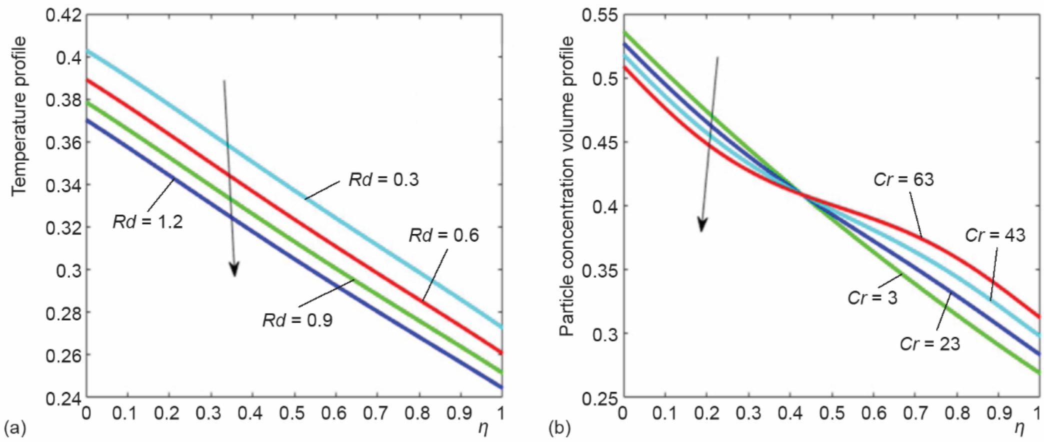

Figure 4. Effects of (a) thermal radiation (temperature) and (b) chemical reaction (concentration)

Table 1. Validation of *f″*(0) and –g′(0) values

t	*f*′(0) [8]	–g′(0) [8]	*f*″(0) [9]	–g′(0) [9]	*f*″(0) [10]	–g′(0) [10]	*f*″(0) present	–g′(0) present
– 1.0	0.06666	2.00095	0.06666	2.00095	0.06666	2.00095	0.06666312	2.00094754
– 0.8	0.08394	1.80259	0.08394	1.80259	0.08399	1.80259	0.08394203	1.80258252
– 0.3	0.10395	1.30442	0.10395	1.30442	0.10395	1.30443	0.10395083	1.30441653
0.0	0.09997	1.00428	0.09997	1.00428	0.09997	1.00428	0.09997217	1.00427150
0.5	0.06663	0.50261	0.06663	0.50261	0.0667	0.50261	0.06663000	0.50261174

Conclusion

The chemical reacting nanofluid between two stretchable rotary disks is investigated under the effects of porous medium, mass transfer and Cattaneo-Christov heat flux. Axial, radial and tangential velocities are found to be increasing functions of porous medium. Thermal radiation and thermal relaxation allow more heat to be transported to neighboring space. Moreover, the concentration enhances with intensified Cattaneo-Christov's thermal relaxation but oscillates with reacting chemicals.

Nomenclature

B_0 – uniform magnetic field, [T]
C – concentration field, [molm^{-3}]
c_p – specific heat capacity, [Jkg^{-1}K^{-1}]
Cr – chemical reaction parameter
D_B – Brownian diffusion coefficient
Ec – Eckert number
g – gravitational acceleration, [ms^{-2}]
h_1-h_4 – heat transfer coefficients
k – thermal conductivity, [Wm^{-1}K^{-1}]
Le – Lewis number
M – Hartmann number
Nb – Brownian parameter
Nt – thermophoresis parameters
Pr – Prandtl number
Rd – thermal radiation parameter
Sc – Schmidt number
T, T_1, T_2 – temperature, [K]
u, v, w – components of velocities, [ms^{-1}]

Greek symbols

α – heat generation parameter
$\beta_1, \beta_C, \beta_T$ – convection parameters
γ_1, γ_2 – thermal expansion coefficients
γ_3, γ_4 – concentration expansion coefficients
θ – dimensionless temperature
λ – thermal relaxation parameter
λ_1–λ_4 – Biot numbers
ν – kinematic viscosity, [ms^{-2}]
ρ – density, [kgm^{-3}]
σ – electrical conductivity [Sm^{-1}]
ϕ – dimensionless concentration

Subscripts

f – fluid
s – nanoparticle
w – disk surface

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