

## ON HOMOGENEOUS-HETEROGENEOUS REACTIONS IN OBLIQUE STAGNATION-POINT FLOW OF JEFFREY FLUID INVOLVING CATTANEO-CHRISTOV HEAT FLUX

by

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*The existing investigation highlights oblique Jeffrey fluid with mixed convection on a stretched surface. Also chemical reactions with properties of homogeneity and heterogeneity are considered. The leading physical model is converted in a non-linear system of ODE by means of proper similarity alterations. Influence of all relative physical constraints on velocity, temperature as well as on concentration are expressed geometrically. Physical magnitudes of interest like friction measurements, thermal and concentration transport rates of chemical species at the surface are studied numerically.*

**Key words:** *Jeffrey fluid, homogeneous-heterogeneous reactions, mixed convection, Cattaneo-Christov heat flux model*

### Introduction

Many researchers have gained attractions towards stagnation point flows on a stretching surface. Non-Newtonian fluids with heat and mass transfer has attracted numerous researchers because of their practical engineering and industrial applications. One of simplest non-Newtonian liquid relation, which has gained significant consideration, is known as Jeffrey fluid, which incorporates the rheological features of viscoelastic liquid. The Jeffrey relation is one of the rate type materials that express some physiological and industrial fluids [1].

Mixed convection stagnation point flows are of vital significance in various engineering processes. Nadeem *et al.* [2] investigated numerically oblique magnetized flow of Walter's B liquid on convective sheet. Vajravelu and Hadjinicolaou [3] discussed heat transmission on a convective stretched plate. Ishak *et al.* [4] surveyed the incompressible mixed convective viscous flow to a tilted porous stretched surface. Nadeem *et al.* [5] discussed nanoparticle examination for non-tangential third grade fluid towards a stretched sheet, he also examined partial slip in non-aligned stagnation point nanoliquid flow over convective surface [6]. Some more such types of investigations are debated in [7-13].

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Fourier suggested heat conduction law [14] which is taken as base to estimate the heat transmission in various applications. Cattaneo [15] corrected this principal by including relaxation time for heat flux. Maximum chemical compositions contain homogeneous-heterogeneous properties. It can be used in artificial foodstuff process, metal type manufacturing, ceramic ware, polymer productions and heat circulation and dampness over agronomic grounds. Merkin [16] calculated the viscous flow on a plane wall along with characteristics of homogeneity heterogeneity replies.

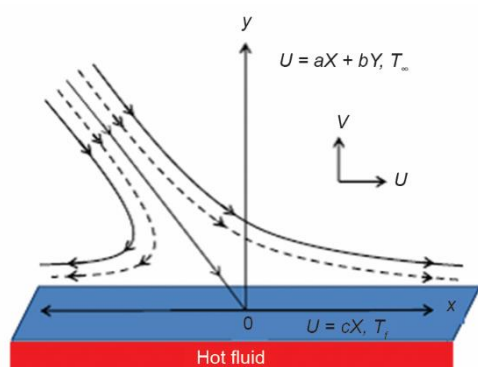


Figure 1. Geometrical description

Bachok *et al.* [17] analyzed same type of properties close to a stretched pane. Khan and Pop [18] examined homogeneous-heterogeneous effects of viscid elastic fluid over a stretched surface.

#### Mathematical scheme of governed problem

We are considering an oblique steady 2-D Jeffrey fluid on a mixed convective stretched sheet with homogeneous-heterogeneous effects. To keep surface stretched, origin is fixed by applying two equivalent and opposed forces along the  $x$ -axis as in fig. 1.

The equations accordingly to previously stated assumptions are [18]:

$$\frac{\partial u^*}{\partial x^*} + \frac{\partial v^*}{\partial y^*} = 0 \quad (1)$$

$$u^* \frac{\partial u^*}{\partial x^*} + v^* \frac{\partial u^*}{\partial y^*} + \frac{1}{\rho} \frac{\partial p^*}{\partial x^*} = \frac{1}{1 + \lambda_1} \left[ \nabla^{*2} u^* + 2\lambda_2 \frac{\partial}{\partial x^*} \left( u^* \frac{\partial^2 u^*}{\partial x^{*2}} + v^* \frac{\partial^2 u^*}{\partial x^* \partial y^*} \right) + \lambda_2 \frac{\partial}{\partial y^*} \left\{ u^* \left( \frac{\partial^2 u^*}{\partial x^* \partial y^*} + \frac{\partial^2 v^*}{\partial x^{*2}} \right) + v^* \left( \frac{\partial^2 v^*}{\partial x^* \partial y^*} + \frac{\partial^2 u^*}{\partial y^{*2}} \right) \right\} \right] + g_1 \beta_T (T^* - T_\infty) \quad (2)$$

$$u^* \frac{\partial v^*}{\partial x^*} + v^* \frac{\partial v^*}{\partial y^*} + \frac{1}{\rho} \frac{\partial p^*}{\partial y^*} = \frac{1}{1 + \lambda_1} \left\{ \nabla^{*2} v^* + 2\lambda_2 \frac{\partial}{\partial y^*} \left( v^* \frac{\partial^2 v^*}{\partial x^{*2}} + u^* \frac{\partial^2 v^*}{\partial x^* \partial y^*} \right) + \lambda_2 \frac{\partial}{\partial x^*} \left[ u^* \left( \frac{\partial^2 u^*}{\partial x^* \partial y^*} + \frac{\partial^2 v^*}{\partial x^{*2}} \right) + v^* \left( \frac{\partial^2 v^*}{\partial x^* \partial y^*} + \frac{\partial^2 u^*}{\partial y^{*2}} \right) \right] \right\} \quad (3)$$

$$\rho C_p \left( u^* \frac{\partial T^*}{\partial x^*} + v^* \frac{\partial T^*}{\partial y^*} \right) = -\nabla^{*2} q^* \quad (4)$$

$$u^* \frac{\partial c_1^*}{\partial x^*} + v^* \frac{\partial c_1^*}{\partial y^*} = D_A \frac{\partial^2 c_1^*}{\partial y^{*2}} - k_c c_1^* c_2^{*2} \quad (5)$$

$$u^* \frac{\partial c_2^*}{\partial x^*} + v^* \frac{\partial c_2^*}{\partial y^*} = D_B \frac{\partial^2 c_2^*}{\partial y^{*2}} + k_c c_1^* c_2^{*2} \quad (6)$$

Here we make the assumption that  $u^*$  is  $x^*$ , and  $v^*$  is  $y^*$  constituents of velocity,  $\rho$  is known as density,  $\nu$  – the effective kinematic viscosity,  $\mu$  – the dynamic viscosity,  $p^*$  – the pressure,  $T^*$  is taken to be temperature,  $T_\infty$  is considered as ambient fluid temperature,  $\lambda_1$  – the ratio of relaxation to retardation times,  $\beta_T$ , is known as thermal expansion coefficient,  $C_p$  – the specific heat at persistent temperature and  $q^*$  – the heat flux and after simplification eq. (4) becomes:

$$\begin{aligned} u^* \frac{\partial T^*}{\partial x^*} + v^* \frac{\partial T^*}{\partial y^*} = & \frac{k}{\rho C_p} \frac{\partial^2 T^*}{\partial y^{*2}} - \lambda_2 \left( u^{*2} \frac{\partial^2 T^*}{\partial x^{*2}} + v^{*2} \frac{\partial^2 T^*}{\partial y^{*2}} \right) + 2u^* v^* \frac{\partial^2 T^*}{\partial x^* \partial y^*} + \\ & + \left( u^* \frac{\partial u^*}{\partial x^*} + v^* \frac{\partial u^*}{\partial y^*} \right) \frac{\partial T^*}{\partial x^*} + \left( u^* \frac{\partial v^*}{\partial x^*} + v^* \frac{\partial v^*}{\partial y^*} \right) \frac{\partial T^*}{\partial y^*} \end{aligned} \quad (7)$$

The concerned boundary conditions takes the form [17, 18]:

$$u^* = cx^*, \quad v^* = 0, \quad \frac{\partial T^*}{\partial y^*} = -\frac{h}{k}(T_f - T^*), \quad D_A \frac{\partial c_1^*}{\partial y^*} = k_s c_1^*, \quad D_B \frac{\partial c_2^*}{\partial y^*} = -k_s c_1^* \text{ at } y^* = 0 \quad (8)$$

$$u^* = ax^* + by^*, \quad T^* = T_\infty, \quad c_1^* \rightarrow c_0, \quad c_2^* \rightarrow 0, \quad \text{as } y^* \rightarrow \infty \quad (9)$$

Here  $c_1^*$  and  $c_2^*$  are absorptions of the organic classes A and B,  $k_c$ , and  $k_s$  are known to be rate factors, by assuming that two reaction progressions are the same. The  $D_A$  and  $D_B$  are the dispersion quantities. Using scaling transformation in the set of eqs. (1)-(9), we have:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (10)$$

$$\begin{aligned} u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + \frac{\partial p}{\partial x} = & \frac{\nu}{1 + \lambda_1} \left\{ \nabla^2 u + 2\omega \frac{\partial}{\partial x} \left( u \frac{\partial^2 u}{\partial x^2} + v \frac{\partial^2 u}{\partial x \partial y} \right) + \right. \\ & \left. + \omega \frac{\partial}{\partial y} \left[ u \left( \frac{\partial^2 u}{\partial x \partial y} + \frac{\partial^2 v}{\partial x^2} \right) + v \left( \frac{\partial^2 v}{\partial x \partial y} + \frac{\partial^2 u}{\partial y^2} \right) \right] \right\} + \lambda T \end{aligned} \quad (11)$$

$$u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + \frac{\partial p}{\partial y} = \frac{\nu}{1 + \lambda_1} \left\{ \nabla^2 v + 2\omega \frac{\partial}{\partial y} \left( v \frac{\partial^2 v}{\partial x^2} + u \frac{\partial^2 v}{\partial x \partial y} \right) + \right.$$

$$+\omega \frac{\partial}{\partial x} \left[ u \left( \frac{\partial^2 u}{\partial x \partial y} + \frac{\partial^2 v}{\partial x^2} \right) + v \left( \frac{\partial^2 v}{\partial x \partial y} + \frac{\partial^2 u}{\partial y^2} \right) \right] \quad (12)$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{1}{\text{Pr}} \frac{\partial^2 T}{\partial y^2} - \omega \left[ u^2 \frac{\partial^2 T}{\partial x^2} + v^2 \frac{\partial^2 T}{\partial y^2} + 2uv \frac{\partial^2 T}{\partial x \partial y} + \right. \\ \left. + \left( u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) \frac{\partial T}{\partial x} + \left( u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} \right) \frac{\partial T}{\partial y} \right] \quad (13)$$

$$v \frac{\partial j}{\partial y}(y) = \frac{1}{\text{Sc}} \frac{\partial^2 j}{\partial y^2}(y) - k_1 j(y) s^2(y) \quad (14)$$

$$v \frac{\partial s}{\partial y}(y) = \frac{\delta}{\text{Sc}} \frac{\partial^2 s}{\partial y^2}(y) + k_1 j(y) s^2(y) \quad (15)$$

$$u(0) = x, \quad v(0) = 0, \quad \frac{\partial T}{\partial y}(0) = -\text{Bi}(1 - T) \\ D_A \frac{\partial j}{\partial y}(0) = k_s \sqrt{\frac{v}{c}} j(0), \quad D_B \frac{\partial s}{\partial y}(0) = -k_s \sqrt{\frac{v}{c}} j(0) \quad (16)$$

$$u(\infty) = \frac{a}{c} x + \gamma_1 y, \text{ as } y \rightarrow \infty. \quad T(\infty) = 0, \quad j(\infty) \rightarrow 1, \quad s(\infty) \rightarrow 0 \quad (17)$$

where  $\lambda = [g_1 \beta_T (T_f - T_\infty)] / (c \sqrt{v} c)$  is the mixed convection parameter,  $\gamma_1 = b/c$  – the obliqueness of the flow,  $\omega = \lambda_2 c$  – the non-Newtonian Jeffery parameter,  $\text{Sc} = v D_A$  – the Schmidt number,  $\text{Pr} = \mu c_p / k$  – the Prandtl number,  $\text{Bi} = h / k \sqrt{v/c}$  – the Biot number, and  $k_1 = (k_c c^2 o) / c$ ,  $k_2 = [k / (D_A c o)] / \sqrt{c/v}$  – the amount of chemical reactions with properties of homogeneity and heterogeneity.

Using stream function transformation with simplification, obtained system is:

$$f''' + \omega(f''^2 - ff^{iv}) + (1 + \lambda_1) \left[ ff'' - (f')^2 + \left( \frac{a}{c} \right)^2 \right] = 0 \quad (18)$$

$$h'' + \omega(hf''' - f'h'' + f''h' - fh''') + (1 + \lambda_1) \left( fh' - f'h - \frac{\lambda}{\gamma_1} \theta - A \right) = 0 \quad (19)$$

$$\theta'' + \text{Pr}[f\theta' - \omega(f^2\theta'' + ff'\theta')] = 0 \quad (20)$$

$$J'' + \text{Sc}[ff' - k_1 j(1 - j^2)] = 0 \quad (21)$$

$$\begin{aligned} f(0) = h(0) = 0, \quad f'(0) = 1 \\ \theta'(0) = -\text{Bi}[1 - \theta(0)], \quad j'(0) = k_2 j(0) \\ f'(\infty) = \frac{a}{c}, \quad h'(\infty) = 1, \quad \theta(\infty) = 0, \quad j(\infty) \rightarrow 1 \end{aligned} \quad (22)$$

Skin friction coefficient and local heat flux from the surface are physical quantities of attention and stagnation point,  $x_s$ , in non-dimensional form are:

$$\tau_w = \left( \frac{1}{1 + \lambda_1} \right) [x(1 + \omega)f''(0) + \gamma_1 h'(0)], \quad q_w = -\theta'(0), \quad x_s = \frac{-\gamma h'(0)}{(1 + \omega f''(0))} \quad (23)$$

## Results and discussion

The governing set of eqs. (18)-(22) are solved numerically by shooting technique with RK Fehlberg method having tolerance of 0.001. Flow behavior of governed fluid problem is being discussed here for all affecting physical parameter. These displays are intrigued to learn the influence of ratio of relaxation to retardation times,  $\lambda_1$ , mixed convection,  $\lambda$ , Biot number, homogeneous heterogeneous reactions,  $k_1$  and  $k_2$ , on both components of velocity  $f'(y)$ ,  $h(y)'$ , temperature,  $\theta(y)$ , and concentration of chemical species  $j(y)$ . Figure 2 shows that  $f'(y)$  and momentum boundary-layer width drops with increase in ratio of relaxation to retardation times  $\lambda_1$ . Figure 3 is plotted for ratio of relaxation to retardation times  $\lambda_1$ , tangential velocity showed increasing behavior near the wall but reverses its conduct as far from the wall. Figure 4 is plotted for mixed convection parameter,  $\lambda$ , which shows increasing effect of tangential velocity close to the surface but far away from the surface behavior is upturned. Figure 5 is plotted to shown that temperature of fluid that is increased for increase in Biot number. Figures 6-8 are designed for concentration of chemical species,  $j(y)$ , for proportion of relaxation to retardation times,  $\lambda_1$ , and quantity chemical reaction with properties of homogeneity and heterogeneity,  $k_1$  and  $k_2$ . Concentration profile,  $j(y)$ , shrinkages by increasing in ratio of relaxation to retardation times,  $\lambda_1$ , see fig. 6, and for quantity chemical reaction with properties of homogeneity and heterogeneity,  $k_1$  and  $k_2$ , concentration profile declines as mentioned respectively in figs. 7 and 8. Table 1 provides computational results for both components of skin friction,  $-f''(0)$ , -

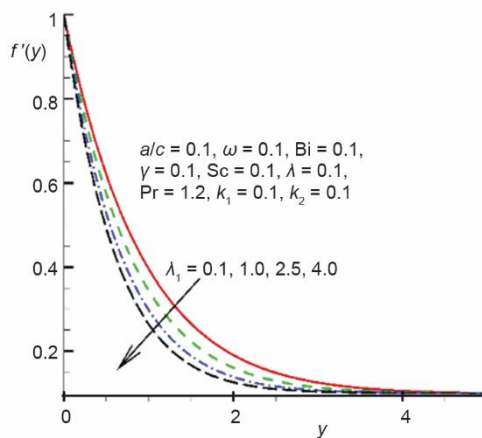


Figure 2. Velocity outline  $f'(y)$  for  $\lambda_1$

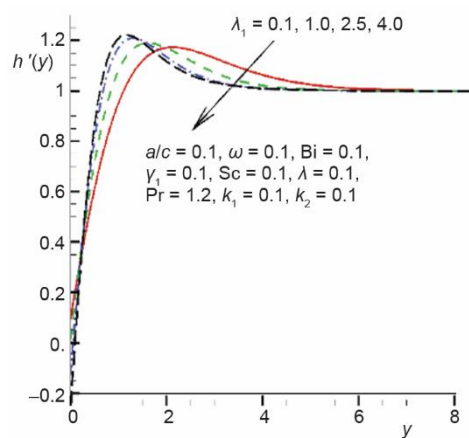
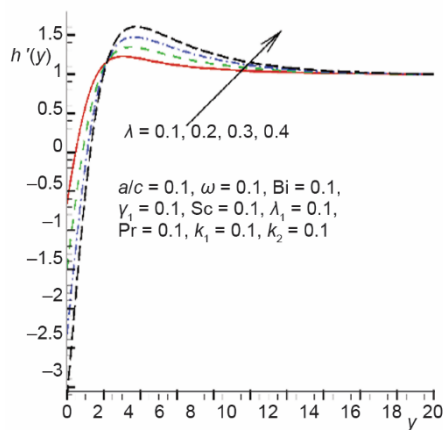
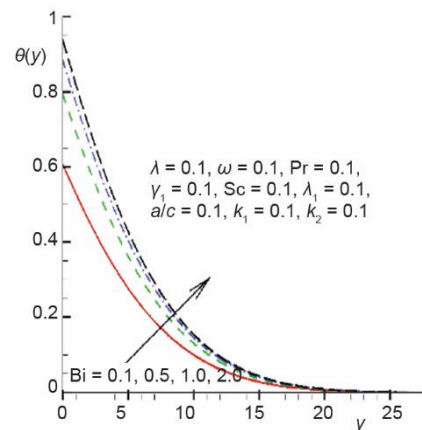
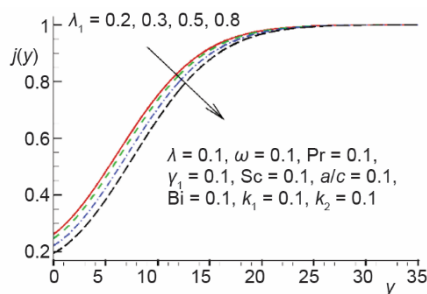
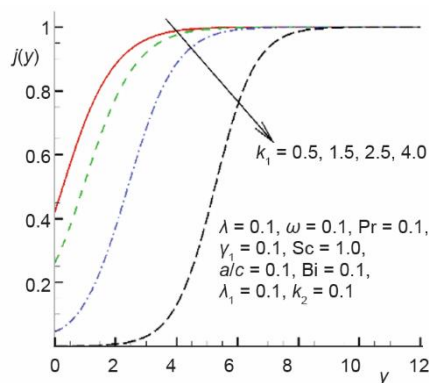
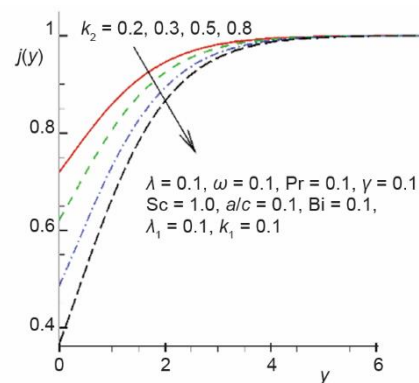


Figure 3. Velocity outline  $h'(y)$  for  $\lambda_1$

$h'(0)$ , rate of heat and mass transfer,  $-\theta'(0)$ , and  $j(0)$  against certain physical parameters. It is bring into noticed that normal and tangential skin friction components decreases while heat transfer rate and surface mass transfer rate of chemical spices rises for rise in stretching ratio  $a/c$  respectively, normal and lateral skin friction figures increases while heat and mass transfer rates are declined with the rise in ratio of relaxation to

Figure 4. Velocity outline  $h'(y)$  for  $\lambda$ Figure 5. Thermal outline  $\theta(y)$  for  $Bi$ Figure 6. Velocity outline  $j(y)$  for  $\lambda_1$ 

retardation times  $\lambda_1$ . Table 2 is constructed to observe the behavior of local heat flux and absorption of chemical flavors at surface for all related parameters and it is found that it is increased for Prandtl number and Biot number, and chemical reaction with properties of homogeneity and heterogeneity,  $k_1$  and  $k_2$ , on absorption of chemical flavors at surface are decline for quantity of chemical reaction with properties of homogeneity and heterogeneity,  $k_1$  and  $k_2$ .

Figure 7. Velocity outline  $j(y)$  for  $k_1$ Figure 8. Velocity outline  $j(y)$  for  $k_2$

**Table 1. Skin friction coefficient when  $\omega = \gamma_1 = \lambda = k_1 = k_2 = \text{Bi} = \text{Sc} = 0.1$ ,  $\text{Pr} = 1$**

$a/c$	$\lambda_1$	$-f''(0)$	$-h'(0)$	$-\theta'(0)$	$j(0)$
0.1	0.1	0.9708	0.6662	0.0563	0.5461
	0.2	1.0140	0.7373	0.0560	0.5419
	0.3	1.0554	0.8078	0.0556	0.5381
0.1	0.2	0.9708	0.6662	0.0563	0.5461
0.2		0.9212	0.2104	0.0602	0.5901
0.8		0.8898	0.0627	0.6164	0.6058

**Table 2. Rate of heat transfer  $-\theta'(0)$  when  $\omega = \text{Sc} = \gamma_1 = \lambda = \lambda_1 = a/c = 0.1$**

$\text{Pr}$	$\text{Bi}$	$k_1$	$k_2$	$-\theta'(0)$	$j(0)$
0.1	0.1	0.1	0.1	0.0565	
0.5				0.0795	
1.0				0.0859	
0.1	0.1	0.1	0.1	0.0564	
	0.5			0.1026	
	1.0			0.1144	
0.1	0.1	0.1	0.1		0.8562
		0.3			0.8532
		0.5			0.8499
			0.1		0.8562
			0.3		0.6624
			0.5		0.5396

## Conclusions

Non-orthogonal stagnation point flow of a mixed convective Jeffrey fluid has been debated with homogeneous-heterogeneous effects and Cattaneo-Christov heat flux. Normal and tangential velocity contours and momentum boundary-layer width enhances with fraction of relaxation to retardation times,  $\lambda_1$ . Thermal boundary-layer thickness rises with a growth in ratio of relaxation to retardation times,  $\lambda_1$ . Concentration of chemical reactions,  $j(y)$ , decreases by growing values in ratio of relaxation to retardation times,  $\lambda_1$ , and with strength of homogeneous- heterogeneous reactions,  $k_1$  and  $k_2$ . Surface skin friction quantities surges with an rise in ratio of relaxation to retardation times,  $\lambda_1$ . Local heat flux,  $-\theta'(0)$ , increases by increasing in Prandtl number and Biot number. Mass flux on surface,  $j(0)$ , drops for homogeneous-heterogeneous reactions,  $k_1$  and  $k_2$ .

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