BRIGHT, DARK, AND SINGULAR OPTICAL SOLITON SOLUTIONS FOR PERTURBED GERDJIKOV-IVANOV EQUATION

by

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This study consider Gerdjikov-Ivanov equation where the perturbation terms appear with full non-linearity. The Jacobi elliptic function ansatz method is implemented to obtain exact solutions of this equation that models pulse dynamics in optical fibers. It is retrieved some bright, dark optical and singular solitons profile in the limiting cases of the Jacobi elliptic functions. The constraint conditions depending on the parameters for the existence of solitons are also presented.

Key words: *Jacobi elliptic functions, optical solitons, Gerdjikov-Ivanov equation*

Introduction

Optical solitons keep the fiber optic industry afloat. For that reason it is necessary to look deeper into these soliton dynamics. The most familiar equation is the non-linear Schrodinger equation (NLSE) in the field of optics. In the present paper, we consider a NLSE that has a quintic non-linearity, namely the perturbed Gerdjikov-Ivanov (pGI) equation. It is studied to govern the dynamics of soliton propagation through optical fibers, and metamaterials. It also has many important applications in photonic crystal fibers. Some effective methods have been proposed for solving exact solutions of the Gerdjikov-Ivanov equation [1-11].

The Gerdjikov-Ivanov equation in the dimensionless form:

$$iq_t + aq_{xx} + b|q|^4 q + icq^2 q_x^* = 0 \tag{1}$$

where $q = q(x, t)$ is the complex values function and q^* denotes the complex conjugation of q. The parameters a, b, and c are real-valued constants. Moreover a and b represent the coefficient of group velocity dispersion and quintic non-linearity, respectively.

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In presence of perturbation terms, the GI equation extends to:

$$iq_t + aq_{xx} + b|q|^4 q + icq^2 q_x^* = i[\alpha q_x + \lambda(|q|^{2m} q)_x + \mu(|q|^{2m})_x q] \tag{2}$$

where α ithe inter-modal dispersion, λ explain self-steepening with short pulses, μ is the coefficient of the higher-order dispersion, and m is the full non-linearity parameter. In this work we will consider pGI equation which is denoted by eq. (2).

Mathematical analysis

We use the following wave transformation to obtain solutions of eq. (2):

$$q(x,t) = u(\xi)\mathrm{e}^{i\phi(x,t)} \tag{3}$$

$$\xi = x - vt, \quad \phi = -\kappa x + wt + \theta \tag{4}$$

where $u(\xi)$ represent shape of the pulse, the function $\phi(x,t)$ is the phase component of the soliton, v and κ are the velocity and frequency of the soliton, respectively, w is the wave number, and θ is the phase constant.

Substituting hypothesis (3) into eq. (2) and then decomposing into real and imaginary parts we get imaginary part:

$$v = -2a\kappa - \alpha + cu^2 - [(2m+1)\lambda + 2m\mu]u^{2m} \tag{5}$$

while real part is:

$$au'' - (w + a\kappa^2 + \alpha\kappa)u - \kappa\lambda u^{2m+1} - c\kappa u^3 + bu^5 = 0 \tag{6}$$

The hypothesis:

$$u(\xi) = A sn^p(B\xi, \ell) \tag{7}$$

where sn is the Jacobi elliptic function, ℓ is the modulus of Jacobi elliptic function and $0 < \ell < 1.$ Also A represents the amplitude and B represents inverse width while unknown index p will be determined. Using the eq. (2), eq. (6) reduces to:

$$a(p-1)pAB^2 sn^{p-2}(B\xi,\ell) - ap(p + \ell - \ell^2 + p\ell^2)AB^2 sn^p(B\xi,\ell) +$$
$$+ a\ell p(\ell p + 1)AB^2 sn^{p+2}(B\xi,\ell) - (w + a\kappa^2 + \alpha\kappa)A sn^p(B\xi,\ell) -$$
$$-\kappa\lambda A^{2m+1} sn^{(2m+1)p}(B\xi,\ell) - c\kappa A^3 sn^{3p}(B\xi,\ell) + bA^5 sn^{5p}(B\xi,\ell) = 0 \tag{8}$$

From eq. (8), matching the exponents $sn^{p+2}(B\xi,\ell)$ and $sn^{(2m+1)p}(B\xi,\ell)$ yields:

$$(2m+1)p = p + 2 \tag{9}$$

which gives:

$$p = \frac{1}{m} \tag{10}$$

If we equate the coefficients of these two functions to each other and equal the coefficients of the other functions to zero in (8), as functions are linearly independent, yields:

$$A=\left\{-\frac{\ell(\ell+m)(w+a\kappa^2+\alpha\kappa)}{\kappa\lambda[m\ell(1-\ell)+\ell^2+1]}\right\}^{\frac{1}{2m}} \tag{11}$$

$$B=\left\{\frac{-(w+a\kappa^2+\alpha\kappa)m^2}{a[m\ell(1-\ell)+\ell^2+1]}\right\}^{\frac{1}{2}} \tag{12}$$

which requires the constraints:

$$(w+a\kappa^2+\alpha\kappa)\kappa\lambda[m\ell(1-\ell)+\ell^2+1]<0 \tag{13}$$

and

$$(w+a\kappa^2+\alpha\kappa)a[m\ell(1-\ell)+\ell^2+1]<0 \tag{14}$$

Considering (3) and (7), the Jacobi elliptic function solution of (2) is obtained:

$$q(x,t)=\left(\sqrt{-\frac{\ell(\ell+m)(w+a\kappa^2+\alpha\kappa)}{\kappa\lambda[1+m\ell(1-\ell)+\ell^2]}}sn\left\{m\sqrt{\frac{-(w+a\kappa^2+\alpha\kappa)}{a[1+m\ell(1-\ell)+\ell^2]}}(x-vt),\ell\right\}\right)^{\frac{1}{m}}\mathrm{e}^{i(-\kappa x+wt+\theta)} \tag{15}$$

when modulus $\ell\to 1$, solution (15) becomes:

$$q(x,t)=\left\{\sqrt{-\frac{(m+1)(w+a\kappa^2+\alpha\kappa)}{2\kappa\lambda}}\tanh\left[m\sqrt{\frac{-(w+a\kappa^2+\alpha\kappa)}{2a}}(x-vt),\right]\right\}^{\frac{1}{m}}\mathrm{e}^{i(-\kappa x+wt+\theta)} \tag{16}$$

For the dark optical soliton solution (16), the conditions (13) and (14) reduce to

$$2\kappa\lambda(w+a\kappa^2+\alpha\kappa)<0 \tag{17}$$

and

$$2a(w+a\kappa^2+\alpha\kappa)<0 \tag{18}$$

If we use the hypothesis:

$$u(\xi)=Acn^p(B\xi,\ell) \tag{19}$$

From eq. (19), eq. (6) reduces to:

$$a(1-\ell^2)(p-1)pAB^2cn^{p-2}(B\xi,\ell)+ap[\ell+\ell^2(2p-1)-p]AB^2cn^p(B\xi,\ell)-$$

$$-a\ell p(\ell p+1)AB^2cn^{p+2}(B\xi,\ell)-(w+a\kappa^2+\alpha\kappa)Acn^p(B\xi,\ell)-$$

$$-\kappa\lambda A^{2m+1}cn^{(2m+1)p}(B\xi,\ell)-c\kappa A^3cn^{3p}(B\xi,\ell)+bA^5cn^{5p}(B\xi,\ell)=0 \tag{20}$$

Equating the exponents and coefficients $cn^{p+2}(B\xi,\ell)$ and $cn^{(2m+1)p}(B\xi,\ell)$, and setting coefficients of rest of functions to zero in (20) gives the same value in (10) and:

$$A=\left\{-\frac{\ell(\ell+m)(w+a\kappa^2+\alpha\kappa)}{\kappa\lambda[m\ell(1-\ell)+2\ell^2-1]}\right\}^{\frac{1}{2m}} \tag{21}$$

$$B=\left\{\frac{-(w+a\kappa^2+\alpha\kappa)m^2}{a[m\ell(1-\ell)+2\ell^2-1]}\right\}^{\frac{1}{2}} \tag{22}$$

with conditions:

$$(w+a\kappa^2+\alpha\kappa)\kappa\lambda[m\ell(1-\ell)+2\ell^2-1]<0 \tag{23}$$

and

$$(w+a\kappa^2+\alpha\kappa)a[m\ell(1-\ell)+2\ell^2-1]<0 \tag{24}$$

Hence, we get the Jacobi elliptic function solution for pGI equation:

$$q(x,t)=\left(\sqrt{-\frac{\ell(\ell+m)(w+a\kappa^2+\alpha\kappa)}{\kappa\lambda[m\ell(1-\ell)+2\ell^2-1]}}cn\left\{m\sqrt{\frac{-(w+a\kappa^2+\alpha\kappa)}{a[m\ell(1-\ell)+2\ell^2-1]}}(x-vt)\right\},\ell\right)^{\frac{1}{m}}\mathrm{e}^{i(-\kappa x+wt+\theta)} \tag{25}$$

when modulus $\ell\to1$, (25) becomes following bright optical soliton solution

$$q(x,t)=\left\{\sqrt{-\frac{(m+1)(w+a\kappa^2+\alpha\kappa)}{\kappa\lambda}}\sec h\left[m\sqrt{\frac{-(w+a\kappa^2+\alpha\kappa)}{a}}(x-vt)\right]\right\}^{\frac{1}{m}}\mathrm{e}^{i(-\kappa x+wt+\theta)} \tag{26}$$

with conditions:

$$(w+a\kappa^2+\alpha\kappa)\kappa\lambda<0 \tag{27}$$

and

$$(w+a\kappa^2+\alpha\kappa)a<0 \tag{28}$$

Now, we use the initial hypothesis:

$$u(\xi)=Ans^p(B\xi,\ell) \tag{29}$$

$$a(p-1)p\ell^2AB^2ns^{p-2}(B\xi,\ell)-ap^2(1+\ell^2)AB^2ns^p(B\xi,\ell)+ap(p+1)AB^2ns^{p+2}(B\xi,\ell)-$$

$$-(w+a\kappa^2+\alpha\kappa)Ans^p(B\xi,\ell)-\kappa\lambda A^{2m+1}ns^{(2m+1)p}(B\xi,\ell)-$$

$$-c\kappa A^3 ns^{3p}(B\xi,\ell)+bA^5 ns^{5p}(B\xi,\ell)=0 \tag{30}$$

From eq. (30), matching the exponents $(2m+1)p$ and $p+2$ we obtain the same value in (10). By performing similar operations as above yields:

$$A=\left[-\frac{(m+1)(w+a\kappa^2+\alpha\kappa)}{\kappa\lambda(\ell^2+1)}\right]^{\frac{1}{2m}} \tag{31}$$

$$B=\left[\frac{-(w+a\kappa^2+\alpha\kappa)m^2}{a(\ell^2+1)}\right]^{\frac{1}{2}} \tag{32}$$

with restrictions:

$$(w+a\kappa^2+\alpha\kappa)\kappa\lambda<0 \tag{33}$$

and

$$(w+a\kappa^2+\alpha\kappa)a<0 \tag{34}$$

Thus another Jacobi elliptic function solution of (2) is given by:

$$q(x,t)=\left\{\sqrt{-\frac{(m+1)(w+a\kappa^2+\alpha\kappa)}{\kappa\lambda(1+\ell)}}ns\left[m\sqrt{\frac{-(w+a\kappa^2+\alpha\kappa)}{a(\ell^2+1)}}(x-vt),\ell\right]\right\}^{\frac{1}{m}}\mathrm{e}^{i(-\kappa x+wt+\theta)} \tag{35}$$

when modulus $\ell\to 1$ we have singular optical soliton solution which is given by:

$$q(x,t)=\left\{\sqrt{-\frac{(m+1)(w+a\kappa^2+\alpha\kappa)}{2\kappa\lambda}}\coth\left[m\sqrt{\frac{-(w+a\kappa^2+\alpha\kappa)}{2a}}(x-vt)\right]\right\}^{\frac{1}{m}}\mathrm{e}^{i(-\kappa x+wt+\theta)} \tag{36}$$

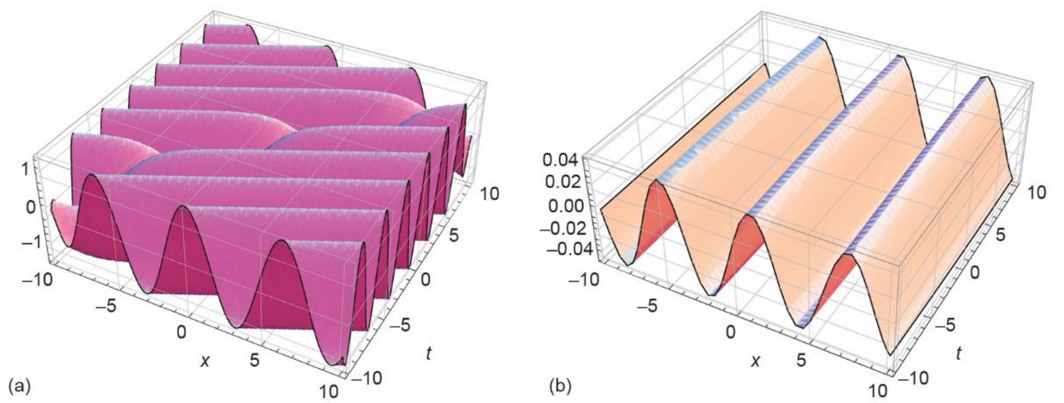

Figure 1. (a) profile of the solution of eq. (16) and (b) profile of the solution of eq. (26)

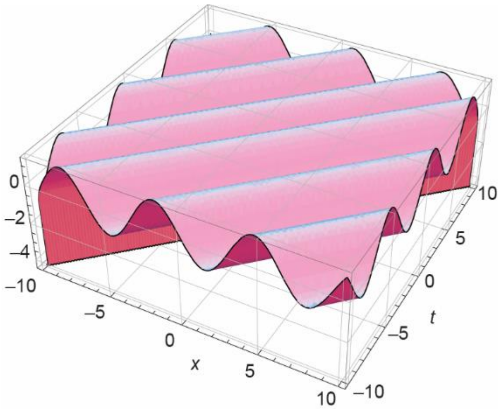

Figure 2. Profile of the solution of eq. (36)

Conclusion

We have studied the generalized pGI equation that is used for description of pulse propagation in the optical fiber. The non-linear terms appear with full non-linearity in the model. We have acquired generalized exact solutions of the pGI equation applying the Jacobi elliptic function ansatz method. We have retrieved bright, dark and singular optical soliton solutions as well as Jacobi elliptic function solutions of this equation. The constraint conditions depending on the parameters for the existence of solitons are also reported. In addition we have present the figures of the optical soliton solutions for arbitrary values of parameters. These solutions are helpful for realising properties of the non-linear Schrodinger type equations and for recognising physical phenomena described by the equation.

References

[1] Biswas, A., *et al.* Optical Soliton Perturbation for Gerdjikov-Ivanov Equation by Extended Trial Equation Method, *Optik, 158* (2018), Apr., pp. 747-752
[2] Arshed, S., Two Reliable Techniques for the Soliton Solutions of Perturbed Gerdjikov-Ivanov Equation, *Optik, 164* (2018), July, pp. 93-99
[3] Kaur, L., Wazwaz, A. M., Optical Solitons for Perturbed Gerdjikov-Ivanov Equation, *Optik, 174* (2018), Dec., pp. 447-451
[4] Biswas, A., Alqahtani, R. T., Chirp-free Bright Optical Solitons for Perturbed Gerdjikov-Ivanov Equation by Semi-Inverse Variational Principle, *Optik, 147* (2017), Oct., pp. 72-76
[5] Arshed, S., *et al.*, Optical Soliton Perturbation for Gerdjikov-Ivanov Equation Via Two Analytical Techniques, *Chin. J .Phys., 56* (2018), 6, pp. 2879-2886
[6] He, B., Meng, Q., Bifurcations and New Exact Travelling Wave Solutions for the Gerdjikov-Ivanov Equatin, *Commun. Non-linear Sci. Numer. Simul., 15* (2010), 7, pp. 1783-1790
[7] Fan, E. G., Darboux Transformation and Soliton-Like Solutions for the Gerdjikov-Ivanov Equation, *J. Phys. A, 33* (2000), 39, ID 6925
[8] Luo, J., Fan, E., ∂ -dressing method for the coupled Gerdjikov–Ivanov equation, *Appl. Math. Lett., 110* (2020), Dec., ID 106589
[9] Tian, S., Zhang, T., Long-Time Asymptotic Behavior for the Gerdjikov-Ivanov Type of Derivative Non-Linear Schrodinger Equation with Time-Periodic Boundary Condition, *Proc. Am. Math. Soc., 146* (2018) 4, pp. 1713-1729
[10] Biswas, A., *et al.*, Conservation Laws for Gerdjikov-Ivanov Equation in Non-Linear Fiber Optics and PCF, *Optik, 148* (2017), Nov., pp. 209-214
[11] Manafian, J., Lakestani, M., Optical Soliton Solutions for the Gerdjikov-Ivanov Model Via tan(ϕ /2)–expansion method, *Optik, 127* (2016), 20, pp. 9603-9620

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