HEAT ENHANCEMENT ANALYSIS IN A DIFFERENTIALLY HEATED INCLINED SQUARE ENCLOSURE WITH WATER AND ETHYLENE GLYCOL BASED Al_2O_3 NANOFLUID

by

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This paper investigates the analysis of natural-convection heat enhancement in an inclined square enclosure when filled with water-based nanofluids with left edge wall undergoing heating with consistent heat flux while the right edge wall being cold and other remaining walls are kept adiabatic. The parameters used in this analysis include: solid fraction volume (range from 0-20%), length of the heaters (0.25 cm, 0.50 cm, and 1.0 cm from the left edge), and Rayleigh number (range from 10^4 to 10^6). The nanolayer thickness ratio was kept at a constant value of 1.0 throughout the analysis. The heat source is found at the center of the left wall. Polynomial differential quadrature equations have been adopted for this analysis for various angles range from 0-90°. As the Rayleigh numbers and particle volume fraction got the much-needed raise, the average count of the heat transfer rate improved too. The length of the heat flux heater has become more prominent parameter that has been affecting the calculated temperature and the flow fields. When the heat flux heater length is pushed to an increasing limit, the heat enhancement rate essentially starts to decline. This happens at the smaller inclination angle, though.

Key words: *nanofluid, numerical simulation, Rayleigh number, phase change, multi-phase flow*

Introduction

The multiple industries like electronics, automotive, and even aerospace have been demanding a lot of varieties of the exchange devices. Thus, to manage that, the heat exchanger devices eventually had to be smaller, lighter, brimming with much higher performance than the earlier ones. Common fluids in these exchanger devices such as, ethylene glycol (EG), water or oil have lower thermal conductivity to transfer heat conventionally. This has been the one core limitation for improving the electronic devices performances and compactness for engineering and scientific applications to solve complex modern world issues. Therefore, there is a strong encouragement and motivation in engineers to develop better fluids with higher rate in properties related to fluid and heat exchange and get over this impediment. Such fluids will also have

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higher conductivities for a longer duration. The core way to enhance thermal conductivity of the substance is to suspend the available metallic nanoparticles within the fluid chosen for any study, experiment, or engineering application. In fact, these nanofluids, which are the resulting mixtures, have higher thermal conductivity than those which were conventional fluids and not even mixed together.

Choi and Eastman [1] was the first person coin the word *nanofluid*. It was for the reference of the fluid where nanoparticles were suspended. So, the nanofluid does not specifically mean only a solid-liquid mixture. It has to possess certain characteristics to be called that. For example, even and stable suspension of nanoparticles, durable suspension of the same, no chemical change in the fluid or particles, and lower agglomeration of the nanoparticles. Keblinski *et al.* [2] found the increase of thermal conductivity in nanofluids with respect to the Brownian motion of the aforesaid particles. They studied the molecular-level layering of these liquid particles along with the heat transport nature of the nanoparticles and their effect on nanoparticle clustering. Nanofluids heat transfer enhancement holds value because of the effect of the thermal conductivity of the same on the nanofluids. But there is enough lack of studies that can easily predict the thermal conductivity effect on a nanofluid. However, we cannot deny that several researchers have constantly attempted to predict the possible correlation between the thermal conductivity of multi-phase mixtures.

Hamilton and Crosser [3] also proposed a model. This was along with other models by Maxwell-Garnett [4], Bruggeman [5], and Wang *et al*. [6]. They all constantly attempted to find the efficient thermal conductivity of the chosen nanofluid. But none of them predicted the effect with higher accuracy. But the results obtained from experimental investigations have shown better and greater conductivity results than those models by these researchers. Then, Yu and Choi [7] introduced an alternative model. This was for calculating the thermal conductivity of nanofluids or liquid-solid mixtures. They claimed that such a nanofluid must contain bulk liquid, solid nanoparticles along with the nanolayers which were in solid form. These layers act as a thermal bridge between the proposed bulk liquid and solid nanoparticles. This is one of the modern models that were used to determine the thermal conductivity property of the nanofluid effectively.

There has also been a trend of studies for surveying the convective heat transfer in nanofluids. Khanafer *et al.* [8] was the first group to examine the heat transfer enhancement for the nanofluids in a 2-D enclosure in buoyancy-driven mode. Then, Jou and Tzeng [9] investigated the performance of the heat transfer in nanofluids with the use of the 2-D rectangular enclosures. As the volume fraction increased in their study, the heat transfer coefficient showed an average enhancement as well. Santra *et al.* [10] conducted a very similar experiment using up to liquid layer thickness of 10% of the nanoparticle radius. They used models by Maxwell-Garnett and Bruggeman. Their results proved that the Bruggeman model predicted significantly higher heat transport rates than those predicted by the Maxwell-Garnett models.

Hwang *et al.* [11] proposed a theoretical investigation for alumina-based nanofluids' thermal characteristics in the natural-convection method. They used the rectangular cavity that was heated from beneath. They used Jang and Choi's model for predictions related to the thermal conductivity of these nanofluids. But they also used other various models for predicting enough data on the viscosity. Then, Oztop and Abu-Nada [12] studied and examined heat transfer and fluid-flow. This happened when buoyancy forces were present in the enclosures, which were partially heated. They used nanofluids containing different nanoparticles. Their results showed that at a lower aspect ratio, there was an enhancement of the heat transfer rate of a nanofluid. The same was not as much present when the aspect ratio was higher.

A partially heated enclosure having natural-convection heat transfer has been a practical issue. Air-cooling has been chosen as a method for cooling computing devices, instead. That is because of its lower maintenance cost and the simple mechanism. Often, electronic components act as sources for heat supply in case of flat surfaces. In many scenarios, the only possible method or model is the natural-convection for cooling such heat sources. Literature has examined and stated that conventional fluids use the partially heated enclosure with convective heat transfer. Then, numerical studies on the glass-melting tank have been performed for a better understanding of the natural-convection. This was a study by Sarris *et al.* [13]. Then, Calcagni *et al.* [14] studied free convective heat transfer where a square enclosure was used. It was characterized by another discrete heater on the lower wall. It got cooled down by the lateral walls. Aydin and Yang [15] had published their investigative and numerical study too. They studied air's natural-convection in a square enclosure with the isothermal heating source from below. Then, it obtained systematic cooling from sidewalls. The top wall of the enclosure, along with the non-heated parts, were adiabatic for this numerical study.

Sharif and Mohammad [16] had concluded the same configuration as by Aydin and Yang [15]. But they replaced the isothermal bottom heating with a heat flux source having a consistent flow. This was a more realistic scenario for cooling the electronic components. Their study investigated the aspect ratio effect and the inclination angles of the cavity for the accurate heat transfer process. Cheikh *et al.* [17] then studied the cooling of the natural-convection with the help of the heated plate that was localized. It was systematically placed at the bottom of the square enclosure filled with air. But the problem related to the constant flux in an enclosure to study the natural-convection of heat transfer hasn't been formalized or analyzed as of now.

Sadeghi *et al.* [18] reviewed five different geometries of enclosures such as square, circular, triangular, trapezoidal, and unconventional geometries. The relation between the thermophysical properties and the way they affect each other was demonstrated for different geometries of enclosures. Various numerical methods, such as finite difference, finite volume, and finite element methods, as well as different microscopic models, such as single-phase and two-phase models were also considered for review. Sharifpur *et al.* [19] investigated experimentally on the natural-convection of deionized water (DIW)- based ZnO nanofluid in a rectangular cavity at various volume concentrations (0.10, 0.18, 0.36, 0.50, and 1.0 vol.%). The natural-convection of ZnO/DIW nanofluid was performed at Rayleigh number range of 7.45×10^7 and 9.20×10^8. At 0.10 vol.% and temperature difference of 32 °C, the ZnO/DIW nanofluid was observed to enhance the heat transfer coefficient by 9.14% relative to DIW. Further increase in volume concentration resulted in the attenuation of heat transfer. It was also found that the Nusselt number and heat transfer rate were augmented by 8.42% and 6.75% at 0.10 vol.%, respectively.

Finite element model

Geometry and grid

An inclined 2-D square cavity, filled with Al_2O_3 nanofluid, having a length of 8 cm is investigated for the analysis of the heat transfer (or heat removal) effect. The square cavity is analysed at four different inclinations, $\theta = 0°$, $\theta = 30°$, $\theta = 45°$, and $\theta = 60°$. The top and bottom walls of the cavity are insulated, while the lateral walls are isothermal. The volume concentration of Al_2O_3 nanoparticle in the base fluid is kept constant at 3%. The nanofluid is analyzed for three different base fluids: water, EG and water-EG mixture. The hot wall is analysed for four temperatures: 306 K, 312 K, 318 K, and 324 K, while the cold wall is kept constant at 300 K.

Mesh generation

Figure 1 shows the square cavity with boundary conditions. The finite element technique is incorporated to simulate the governing equations. The mixture of base fluid and nanoparticles is assumed to be homogenous and behave as a single-phase system. The direction of the gravitation vector used for the simulation model is shown in fig. 1. The gravitation vector incorporates the buoyancy effect due to lateral heating in the walls. Figure 2 shows the mesh grid used for the analysis and consists of square elements.

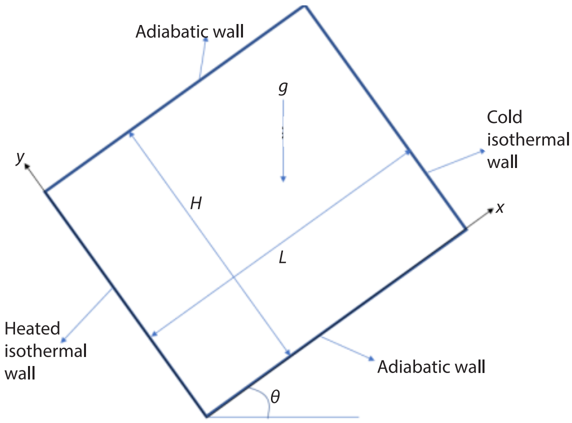

Figure 1. Model geometry with boundary conditions

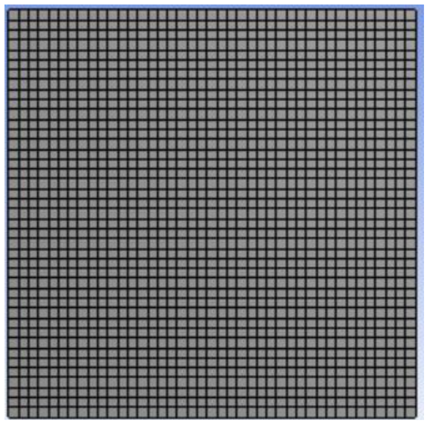

Figure 2. Finite element mesh with square elements

Governing equations and boundary conditions

In the present work, the flow has been assumed to be incompressible. The nanoparticles are assumed to be easily fluidized in the base fluid, hence the nanofluid can be assumed to exist in a single-phase system. The present work's main aim is to investigate the heat enhancement rate due to the introduction of a horizontal fin and understand the resulting flow dynamics of nanofluids. Therefore, any effect arising due to external force such as magnetic field is not considered. Boundary conditions for the model are illustrated in figs. 1 and 2 are represented (*the south-west corner of the cavity is taken as origin and all values are in cm*)

$$u = v = 0,\ T = T_{\mathrm{h}},\ \text{at } x = 0, \text{ and } 0 \le y \le 8$$

$$u = v = 0,\ T = T_{\mathrm{c}},\ \text{at } x = 8, \text{ and } 0 \le y \le 8$$

$$u = v = 0,\ \partial T/\partial y,\ \text{at } y = 0,\ y = 8, \text{ and } 0 \le x \le 8$$

The properties of the nanofluids like density, ρ_{nf}, viscosity, μ_{nf}, thermal volume expansion, β_{nf}, thermal conductivity, k_{nf}, and specific heat capacity, $C_{p,\mathrm{nf}}$, are evaluated using previous studies by Khanafer *et al.* [20], and the proposed equations are:

– Density

$$\rho_{\mathrm{nf}} = \Phi\rho_{\mathrm{s}} + (1-\Phi)\rho_{\mathrm{bf}} \tag{1}$$

– Dinamic viscosity

$$\mu_{\mathrm{nf}} = \frac{\mu_{\mathrm{bf}}}{(1-\Phi)^{2.5}} \tag{2}$$

– Thermal volume expansion

$$\left(\rho\beta\right)_{\mathrm{nf}} = \Phi\rho_{\mathrm{s}}\beta_{\mathrm{s}} + \left(1-\Phi\right)\rho_{\mathrm{bf}}\beta_{\mathrm{bf}} \tag{3}$$

– Thermal condustivity

$$\frac{k_{\mathrm{nf}}}{k_{\mathrm{bf}}} = \frac{k_{\mathrm{s}} + 2k_{\mathrm{bf}} + 2\left(k_{\mathrm{s}} - k_{\mathrm{bf}}\right)\Phi}{k_{\mathrm{s}} + 2k_{\mathrm{bf}} - 2\left(k_{\mathrm{s}} - k_{\mathrm{bf}}\right)\Phi} \tag{4}$$

Specific heat capacity

$$\left(\rho C_p\right)_{\mathrm{nf}} = \Phi\rho_{\mathrm{s}}C_{p,\mathrm{s}} + \left(1-\Phi\right)\rho_{\mathrm{bf}}C_{p,\mathrm{bf}} \tag{5}$$

A square cavity filled with a nanofluid is investigated for four different inclinations such as θ = 0°, θ = 30°, θ = 45°, and θ = 60°. The nanofluid in the cavity is analysed for three base fluids such as water, EG and water-EG mixture and compared in the present analysis at a constant Al_2O_3 nanoparticle concentration level (Φ = 3%). The water-EG mixture is investigated at four compositions: 20-60%, 40-60%, 60-40%, and 80-20%, respectively, and the thermo-fluid properties of these base fluids are presented in tab. 1. The temperature of the cold wall is kept constant at 300 K while the hot isothermal wall temperature is investigated for four temperatures: 306 K, 312 K, 318 K, and 324 K. Therefore, the system is investigated for three variables (*varied one at a time*):
– inclination of the square cavity,
– base fluid composition, and
– temperature difference between hot wall and cold wall.

Table 1. Thermophysical properties of base fluid and Al_2O_3 nanoparticle

Property	Water	80-20% water-EG	60-40% water-EG	40-60% water-EG	20-80% water-EG	Ethylene--glycol	Al_2O_3
ρ [kgm^{-3}]	997.1	1027.93	1057.60	1083.87	1107.40	1132	3970
C_p [J kg^{-1}K^{-1}]	4180	3826	3485	3106	2690	2349	765
k [Wm^{-1}K^{-1}]	0.613	0.498	0.408	0.336	0.283	0.258	40
μ [mPas^{-1}]	0.891	1.46	2.57	4.52	8.29	15.1	–
$\alpha\times10^7$ [m^{2}s^{-1}]	1.47	1.266	1.106	0.998	0.95	0.97	131.7
β [K^{-1}]	0.00021	0.00028	0.00035	0.00042	0.00049	0.00057	0.000024
Pr	6.07	11.21	21.95	41.78	78.79	137.48	–

For the numerical investigation, the value of density is approximated by Boussinesq approximation, as its value changes linearly with temperature in the buoyancy term of the momentum equation. The rest of the properties are assumed to remain constant during the numerical simulation. Under the above consideration, the prevailing continuity, momentum, and energy equations for the incompressible flow can be presented:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial x} = 0 \tag{6}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{1}{\rho_{\mathrm{nf}}}\left[-\frac{\partial P}{\partial x} + \mu_{\mathrm{nf}}\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right)\right] + \left\{\left(\mathrm{g}\sin\theta\right)\left(\rho\beta\right)_{\mathrm{nf}}\left(T - T_{\mathrm{c}}\right)\right\} \tag{7}$$

$$u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=\frac{1}{\rho_{\mathrm{nf}}}\left[-\frac{\partial P}{\partial y}+\mu_{\mathrm{nf}}\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)\right]+\left\{(\mathrm{g}\cos\theta)(\rho\beta)_{\mathrm{nf}}(T-T_{\mathrm{c}})\right\} \tag{8}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha_{\mathrm{nf}}\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right) \tag{9}$$

Numerical method and analysis

The FLUENT 16.0 software is used to conduct CFD analysis for free convection problem in the square cavity. Using default values of the relaxation factor, SIMPLE algorithm is employed. A convergence criterion of 10^{-6} was assigned for residuals of x-velocity, y-velocity, continuity, and energy equations.

Table 2 shows the mesh sensitivity analysis conducted for Al_2O_3-water nanofluid with Φ = 3% and $\Delta T = T_{\mathrm{h}} - T_{\mathrm{c}}$ = 12 K. Figures 3(a) and 3(b) show the mesh grid and plot for grid independence test and sensitivity analysis and Nusselt number variation with number of elements, respectively. It is observed that a mesh grid of 14400 square elements is suitable for numerical investigation as it almost gives constant Nusselt number thereafter.

Table 2. Mesh sensitivity analysis

Number of divisions on the grid	Number of elements	Heat flux [Wm^{-2}]	Nusselt number
41 × 41	1600	3876.92	39.08
81 × 81	6400	3072.95	30.97
121 × 121	14400	2812.9	28.35
161 × 161	25600	2812.6	28.35
321 × 321	102400	2737.5	27.59

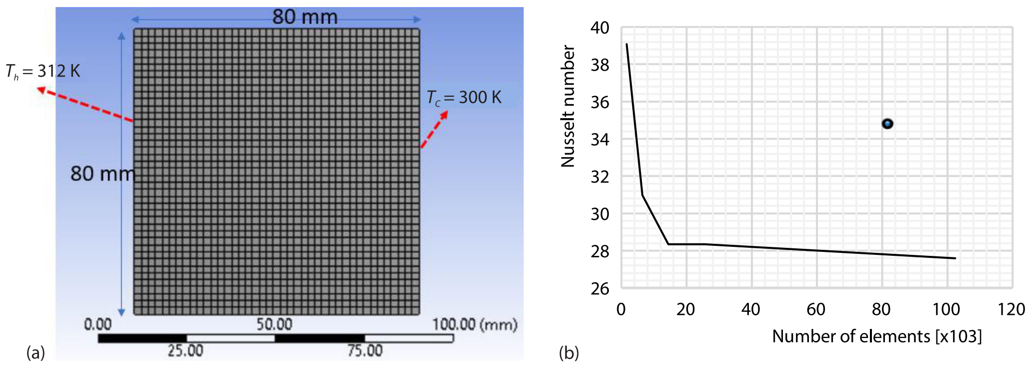

Figure 3. (a) Mesh grid for sensitivity analysis and (b) variation of Nusselt number with mesh size

Data validation

For data validation, results from Ho *et al.* [21] and Saghir *et al.* [22] are used where a square cavity of 8 cm length is employed and filled with Al_2O_3-water nanofluid. The vertical walls of the cavity are isothermal, and the horizontal walls are adiabatic. The hot and cold isothermal walls are maintained at 312 K and 300 K, respectively. The results are tabulated in tab. 3.

Table 3. Data validation

vol.% of nanoparticle	Nusselt number [21]	Nusselt number [22]	Nusselt number (present analysis)
1	32.2037	31.8633	31.46
2	31.0905	31.6085	30.90
3	29.0769	31.2101	30.34

It can be observed that as the concentration of nanoparticle increases, the variation increases from 2.31-4.34%, but this variation is well within permissible limits and hence, the methodology employed for the current numerical investigation is considered validated.

Results and discussion

Figure 4 shows the temperature contour for Al_2O_3-water nanofluid at different inclinations in a square cavity of 8 cm edge length. For all the four inclinations of the square cavity, the vol.% concentration of Al_2O_3 nanoparticle is set at Φ = 3% and the hot wall edge kept at 312 K. Irrespective of the inclination, it can be observed that thermal stratification is exhibited in all square cavities, with more evenly and equally spaced stratification in the case of θ = 0°. The stratification becomes less even as the inclination increases. However, the exact effect of inclination on the heat enhancement remains inconclusive with temperature contours and hence, velocity contour plots are required to get a better understanding, and plotted in fig. 5.

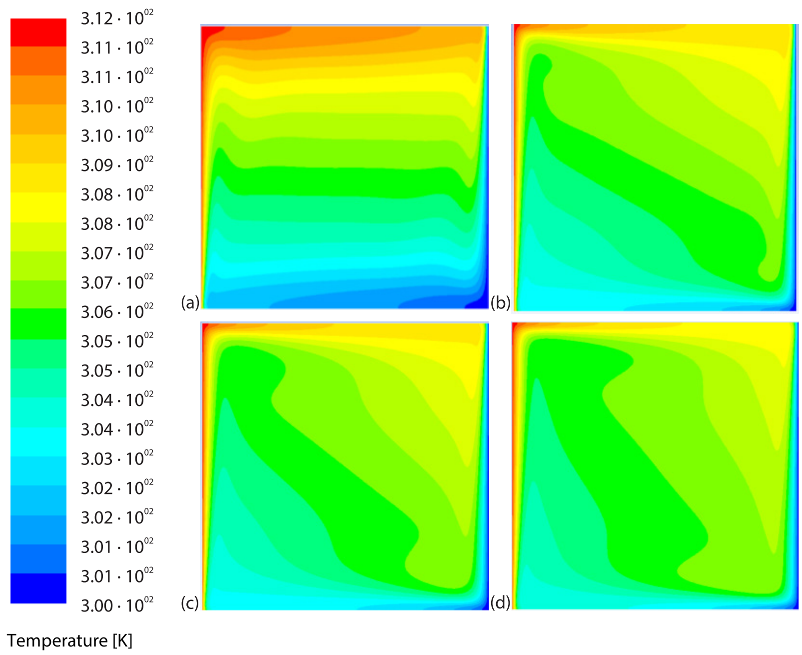

Figure 4. Temperature contour for Φ = 3% Al_2O_3-water and 312 K hot wall temperature; (a) θ = 0° inclination, (b) θ = 30° inclination, (c) θ = 45° inclination, and (d) θ = 60° inclination

Figure 5 shows the velocity contour plot for Al_2O_3-water nanofluid at different inclinations in a square cavity of 8 cm edge length. For all the four inclinations of the square cavity, the vol.% concentration of Al_2O_3 nanoparticle is set at Φ = 3% and the hot wall edge kept at

312 K. It is evident from the contour plot that the magnitude of the activity at the isothermal walls is highest for $\theta = 30°$. This can be attributed to the fact that at $\theta = 30°$, the core region is the largest which forces the fluid stream towards the walls leading to better heat interaction. Increased activity at the isothermal surface implies better heat transfer. This holds good for any base fluid composition, as shown in fig. 6.

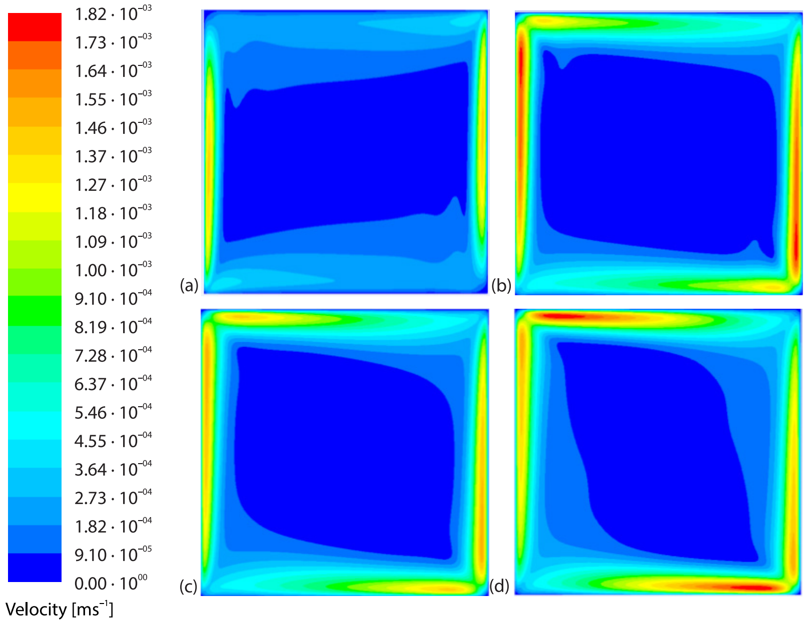

Figure 5. Velocity contour for $\Phi = 3\%$ Al_2O_3-water and 312 K hot wall temperature; (a) $\theta = 0°$ inclination, (b) $\theta = 30°$ inclination, (c) $\theta = 45°$ inclination, and (d) $\theta = 60°$ inclination

For base fluid composition of pure water, pure EG, or, water-EG mixture, the effect of heat enhancement with variation of cavity inclination is analysed and plotted in fig. 6. It is found that irrespective of the base fluid composition, the heat enhancement is highest for

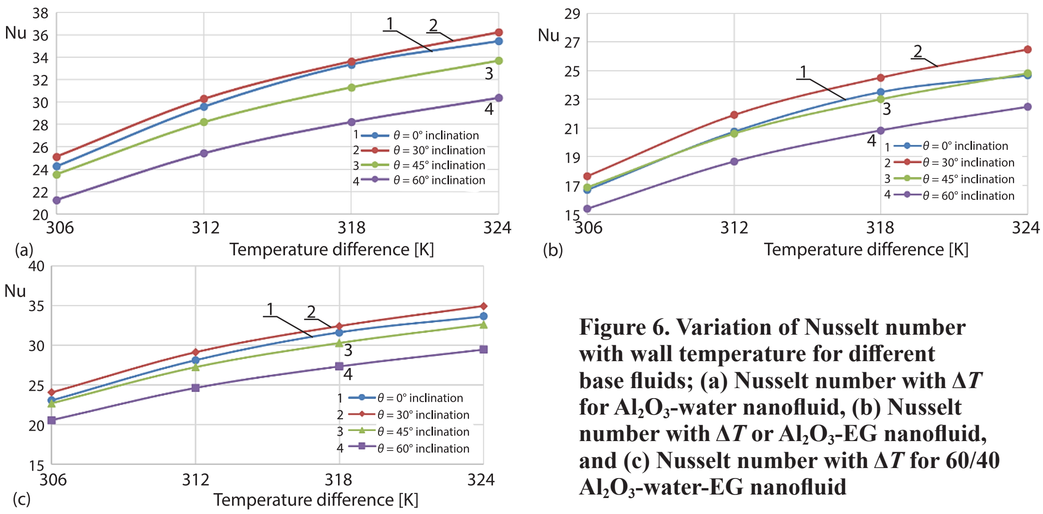

Figure 6. Variation of Nusselt number with wall temperature for different base fluids; (a) Nusselt number with ΔT for Al_2O_3-water nanofluid, (b) Nusselt number with ΔT or Al_2O_3-EG nanofluid, and (c) Nusselt number with ΔT for 60/40 Al_2O_3-water-EG nanofluid

the θ = 30° inclination. This can be attributed to the fact that the activity near the isothermal walls are hughest for this inclination, as also observed in fig. 5. Moreover, with an increase in ΔT (= $T_h - T_c$), it is observed that the Nusselt number value increases. This can be attributed to the fact that with an increase in ΔT, the Rayleigh number increases and increase in Rayleigh number enhances heat convection.

Figure 7 shows the effect of variation of temperature difference between hot and cold wall on the heat enhancement process at different inclinations.

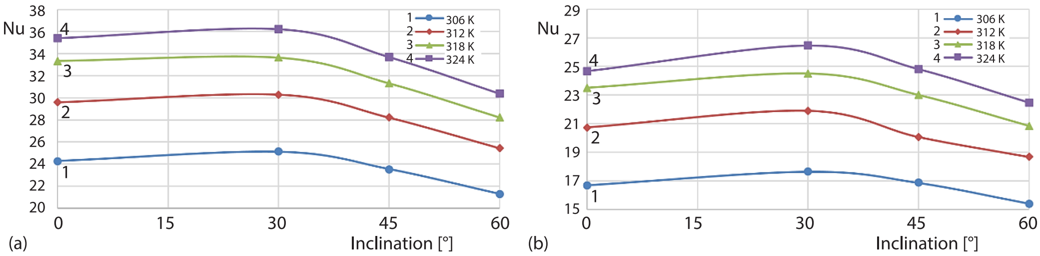

Figure 7. Variation of Nusselt number for different wall temperature at different inclinations; (a) Nusselt number with inclination for different wall temperatures of Al_2O_3-water nanofluid and (b) Nusselt number with inclination for different wall temperatures of Al_2O_3-EG nanofluid

From fig. 7, it is observed that the heat enhancement is highest for θ = 30°, irrespective of the base fluid composition and ΔT (= $T_h - T_c$) This can be attributed to the conclusions drawn from fig. 6, *i.e.*, the activity at the isothermal wall is highest for θ = 30°. The Nusselt number is observed to increase from θ = 0° to θ = 30°, and then decrease from θ = 30° to θ = 60°. The minima at all the wall temperatures and base fluid compositions are observed at θ = 60°. This minima at θ = 60° can be attributed to the smallest core region as observed in fig. 5. The heat enhancement variation with change in base fluid composition (pure water to water-EG mixture to pure EG) is plotted in fig. 8 for ΔT = 12 K and 3% Al_2O_3-nanofluid.

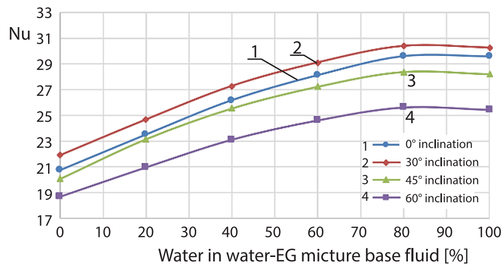

Figure 8. Variation of Nusselt number with change in base fluid composition at different inclinations

As seen earlier, the heat enhancement is highest for θ = 30° and minimum for θ = 60°, irrespective of other conditions. It is observed that as the % of water in the water-EG mixture increases, the Nusselt number increases. Initially, the increase in Nusselt number is almost linear from 0% water to 80% water and almost found to be constant from 80% water to 100% water.

Conclusion

In the present analysis, the numerical investigation of an inclined square cavity filled with a nanofluid for heat enhancement have been conducted. The square cavity edge length is kept at 8 cm. The vertical walls are isothermal (with one wall as hot and the other as cold) and the horizontal walls are kept adiabatic (or insulated). The four inclinations where square cavity is analysed are θ = 0°, θ = 30°, θ = 45°, and θ = 6.0°. The Al_2O_3 nanoparticle in three base fluids such as pure water, water-EG mixture, and pure EG is analyzed for Nusselt number variation with different wall temperatures, and a fixed 3% volume concentration of nanoparticle. It is observed that with an increase in cavity inclination, the Nusselt number increases and then decreases. The most optimal inclination for heat enhancement is $\theta = 30°$. It is also observed that with an increase of wall temperature, the Rayleigh number increases which leads to higher activity and thereby enhanced heat transfer. Further, as the composition of base fluid changes from EG to water, the heat transfer gets enhanced. Thus, the authors conclude that inclination of the square is an important parameter for enhancing the heat transfer process, along with the composition of base fluid and wall temperature.

Nomenclature

C_p – specific heat, [Jkg^{-1}K^{-1}]
g – gravitational acceleration, [ms^{-2}]
H – width, [m]
h – heat transfer coefficient, [Wm^{-2}K^{-1}]
k – thermal conductivity, [Wm^{-1}K^{-1}]
L – length,
Nu – Nusselt number
Pr – Prandtl number
T – temperature, [K]
u, v – dimensional velocities
x, y – dimensional co-ordinates

Greek symbols

α – thermal diffusivity, [m^{2}s^{-1}]
β – thermal volume expansion, [K^{-1}]
θ – cavity inclination, [°]
μ – dynamic viscosity, [kgm^{-1}s^{-1}]
ρ – density, [kgm^{-3}]
Φ – nanoparticle volume concentration, [%]

Subscript

c – cold
bf – base fluid
h – hot
nf – nanofluid
s – solid particle

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