FLOW CHARACTERISTICS OF FRACTAL FRACTURE WITH DIFFERENT FRACTAL DIMENSION AND DIFFERENT FRACTURE WIDTH

by

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Original scientific paper
https://doi.org/10.2298/TSCI2106477L

Single fracture is the most basic element in complex fracture network of rock mass. Therefore, the study of flow characteristics of single fracture is an important way to reasonably predict the complex flow state in engineering rock mass. In order to study the flow characteristics of fractal single fracture, fracture models with different fractal dimension and different fracture width are established in this paper. The results show that: the blocking effect of rough structure on fluid is obviously enhanced under high pressure. In addition, it is weakened and reaches a steady-state with the increase of fracture fractal dimension. The larger the fracture width is, the more obvious the phenomenon is. The hydraulic gradient index tends to 0.5 with the increase of fracture width when fractal dimension is greater than 1.3. It also could tend to 0.5 with the increase of fractal dimension when fracture width is greater than 1 mm.

Key words: *single fracture, fractal dimension, fracture width, hydraulic gradient index*

Introduction

The natural rock mass is complex and has the characteristics of non-uniformity and anisotropy. One of the reasons for this phenomenon is that there are various fractures in the rock mass. Single fracture is the most basic element in complex fracture network of rock mass. The geometric characteristics of single fracture, such as the width, direction, roughness and filling property, are closely related to the permeability of rock mass [1-3]. Therefore, the study on fluid-flow characteristics of single fracture is an important way to reasonably predict the complex flow state in rock mass [4, 5].

Witherspoon [6] proposed the cubic law derived from the ideal smooth parallel plate model, which has been widely used in the study of flow law of fractured rock mass [7, 8]. Scholars have carried out flow experiments of smooth parallel plate fractures, and the experimental results have proved the rationality of the cubic law, that is, the fracture discharge is proportional to the cubic power of the crack width [9, 10]. However, the natural fracture is usually rough and irregular with obvious fractal characteristics, so the geometric characteristics of the real fracture are difficult to meet the basic assumptions of the plate fluid model [10, 11]. Then,

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scholars have carried out a variety of semi-theoretical and semi-empirical research work, and put forward the corresponding modified cubic law [11-13]. However, there are some differences among various empirical formulas [14], and it is necessary to further explore the unified flow law and mechanism of natural rough single fracture.

The study of single fracture flow focuses more on the analysis and summary of phenomena, and numerical simulation has obvious advantages in this aspect. The numerical simulation methods of fracture flow include finite difference method [15], boundary element method [16], meshless method [17], but the most popular method is finite difference method. Wang [18] described the basic composition of fracture network with graph theory and established the numerical model of fracture network flow. Chai [19] established a numerical model for coupled analysis of unsteady flow field and stress field in dam foundation fracture network. Based on the Signorini inequality formula for unconfined flow in continuous media [20], Jiang *et al.* [21, 22] proposed a variational inequality method for solving stable and unstable flow in fracture networks. In recent years, this method has been extended to solve 3-D fracture seepage problems [23].

In this paper, firstly, the fractal fractures with different fractal dimension are generated by fractal function. Then fracture models with different fracture width are established. Finally, the model is imported into COMSOL for calculation. Based on the results of numerical simulation,the influence of fracture roughness (fracture fractal dimension), average fracture width (hereinafter referred to as fracture width), and fluid pressure on fracture flow characteristics is studied in detail.

Model building

The Weierstrass-Mandelbrot function is given [24-26]:

$$W(t)=\sum_{n=-\infty}^{\infty}\left(1-\mathrm{e}^{ib^{n}t}\right)\mathrm{e}^{i\varphi_n}\,/\,b^{(2-D)n} \tag{1}$$

where b is a real number greater than 1, reflecting the degree of deviation of the curve from the straight line, φ_n – the any phase angle, and fractal dimension $D \in (1, 2)$. The real part of the function $W(t)$ is regarded as the fractal control function $C(t)$, which is continuous and non-differentiable everywhere:

$$C(t)=\mathrm{Re}\,W(t)=\sum_{n=-\infty}^{\infty}\left(1-\cos b^{n}t\right)/\,b^{(2-D)n} \tag{2}$$

The fractal dimension satisfies the relations:

$$D_{\mathrm{HB}}-\frac{B}{b}\le D\le D_{\mathrm{HB}} \tag{3}$$

where B is a constant and represents the Hausdorff-Besicovitch (HB) dimension.

Firstly, the parameter b is set to 1.4 [27], and fractal curves with different fractal dimension are generated by MATLAB. The fractal curves are shown in fig. 1.

According to the fractal curves, the fracture models with different fracture width are established, and finally the fractal fracture models with different fractal dimension and different fracture width are obtained.

In order to study the flow characteristics of single fracture, three variables are set in this paper. There are nine kinds of working conditions for inlet pressure, P, which are 50 Pa, 100 Pa, 150 Pa, 200 Pa, 300 Pa, 400 Pa, 500 Pa, and 600 Pa, respectively. There are five kinds of working conditions of fracture width, d, which are 1 mm, 2 mm, 3 mm, 4 mm, and 5 mm,

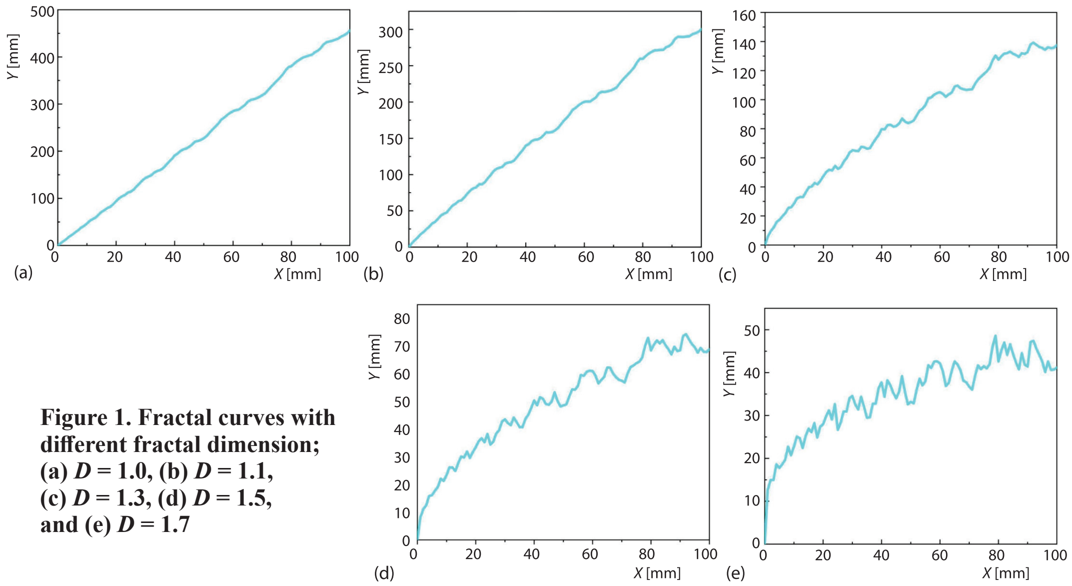

Figure 1. Fractal curves with different fractal dimension; (a) *D* = 1.0, (b) *D* = 1.1, (c) *D* = 1.3, (d) *D* = 1.5, and (e) *D* = 1.7

respectively. The fracture fractal dimension is set in five kinds of working conditions: $D = 1.0$, 1.1, 1.3, 1.5, and 1.7. Therefore, the fracture flow under 225 working conditions is simulated in this paper. The size of fracture fractal dimension reflects the roughness of fracture. The larger the fractal dimension is, the rougher the fracture is. The fluid studied in this paper is water, and it is assumed that the flow of water is laminar [27]. The initial end of the fracture is set as the entrance, the end of the fracture is set as the outlet, and the fractal curve is set as the fracture boundary.

Theoretical analysis

Generally, the fluid resistance includes friction resistance and local resistance. The friction resistance mainly comes from the friction between the fluid and the fracture surface along the straight path, and the local resistance mainly comes from the change of fluid-flow direction in the local path. The calculation formula of friction resistance and local resistance is [28, 29]:

$$H_{\mathrm{f}} = \frac{L \,\mathrm{Re}\, \rho v^2}{128 d'} \tag{4}$$

$$H_{\mathrm{l}} = \frac{\varepsilon \rho v^2}{2} \tag{5}$$

$$\varepsilon = \frac{12\theta R}{\Delta p \,\mathrm{Re}\, B} \tag{6}$$

where H_{f} is the frictional resistance along the fluid-flow path, L – the length of the flow path, Re – the Reynolds number, ρ – the density of the liquid, v – the average velocity, d' – the equivalent diameter of the cross-section of flow path, H_{l} – the local resistance, ε – the local resistance coefficient, θ – the bending angle at any part of the flow path, R – the curvature radius of the center line of the local position of the flow path, and Δp – the local pressure difference.

From eqs. (4)-(6) we see that the local resistance would increase with the increase of fracture roughness (fracture fractal dimension), that the friction resistance would decrease with the increase of fracture width, and that Both frictional resistance and local resistance would increase with the increase of pressure.

Analysis of fracture flow characteristics

Influence of inlet pressure and fracture width

The factors affecting the flow of rough fracture include fracture width, fluid pressure and its own rough structure (fractal dimension of fracture). Rough structure and width of fracture can be regarded as the true property of fracture, and the fluid pressure can be regarded as the additional property of fracture. Therefore, the fractal fracture flow characteristics need to be discussed under combined working conditions.

Figure 2 shows the relationship between fracture discharge and inlet pressure when fracture fractal dimension is the same but fracture width is different. The calculation results show that the fracture discharge would increases with the increase of inlet pressure, but the growth rate of fracture discharge would decrease. This phenomenon shows that the flow blocking effect of rough structure on fluid under different pressure is different. The higher the inlet pressure is, the smaller the growth rate of fracture discharge is, which indicates that the blocking effect of rough structure on fluid is obviously enhanced under high pressure.

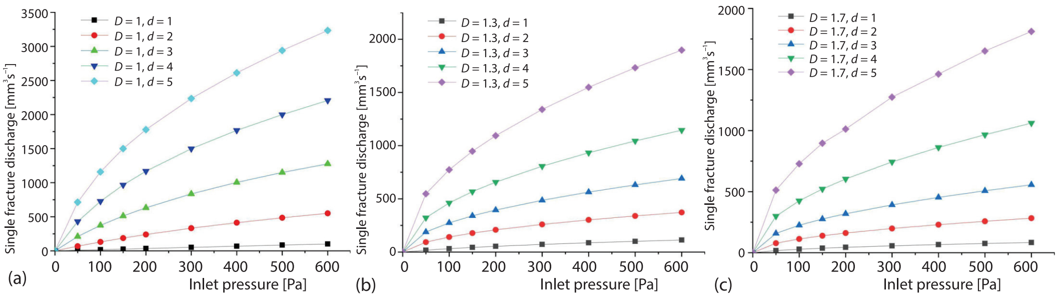

Figure 2. Relationship between fracture discharge and inlet pressure; (a) *D* = 1.0, (b) *D* = 1.3, and (c) *D* = 1.7

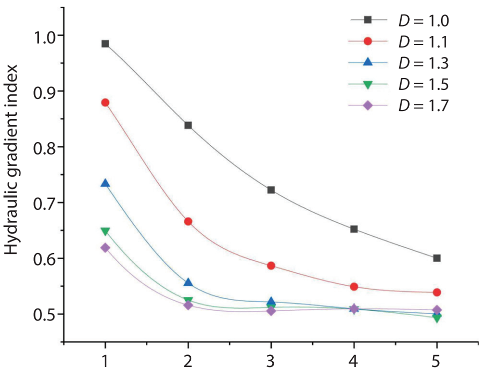

Figure 3. Relationship between hydraulic gradient index and fracture width

It also can be seen from fig. 2 that the fracture discharge increases with the increase of fracture width, and the increasing rate increases. The phenomenon reflects that the increase of the fracture width would expand the effective flow space, at the same time, it also would weaken the blocking effect of the rough structure on fluid. Therefore, the larger the fracture width is, the stronger the fracture flow capacity is. The relation curve of fracture discharge and pressure is fitted into a power function like $y = ax^b$, and the exponent b of this power function is the hydraulic gradient index. Figure 3 shows that the hydraulic gradient index decreases with the increase of fracture width, and the decreasing rate would gradually decrease. When the fractal dimension D is larger than 1.3, the hydraulic gradient index gradually converges to 0.50 with the increase of fracture width. When the fractal dimension is $D = 1.1$, the hydraulic gradient index gradually converges to 0.54. It should be noted that the hydraulic gradient index is close to 1 when fractal dimension $D = 1.0$, fracture width $d = 1$. This means that the hydraulic gradient index is close to 1 when the fracture is very smooth and the width is very small.

Influence of fracture fractal dimension

Figure 1 indicates that the length of fracture flow path with different fractal dimension is different. The total impedance capacity of fracture to fluid increases with the increase of flow path, while the fracture discharge is on the contrary. In order to eliminate the influence of flow path length on the results, the length of flow path or fracture discharge should be modified. Assuming that the impedance capability of the rough fracture to fluid is the same everywhere, and taking the fracture discharge at $D = 0$ as the reference, the fracture discharge correction formula:

$$Q'_{D=i} = Q_{D=i}\frac{L_{D=i}}{L_{D=1.0}},\ \ i = 1.0,\ 1.1,\ 1.3,\ 1.5,\ 1.7 \tag{7}$$

where $Q'_{D=i}$ is the corrected fracture discharge when $D = i$, $Q_{D=i}$ – the fracture discharge when $D = i$, $L_{D=1.0}$ – the length of the flow path when $D = 1.0$, and $L_{D=i}$ – the length of the flow path when $D = i$.

The relationship between the corrected fracture discharge and inlet pressure is shown in fig. 4. With the increase of the inlet pressure, the fracture discharge increases and the increasing rate decreases. The fracture discharge curves almost coincide when the fracture fractal dimension is large. This indicates that the fracture discharge tends to be stable with the increase of fractal dimension. This phenomenon shows that with the increase of fracture fractal dimension, the blocking effect of fracture rough structure on fluid is weakened and reaches a steady-state. The larger the fracture width is, the more obvious the phenomenon is.

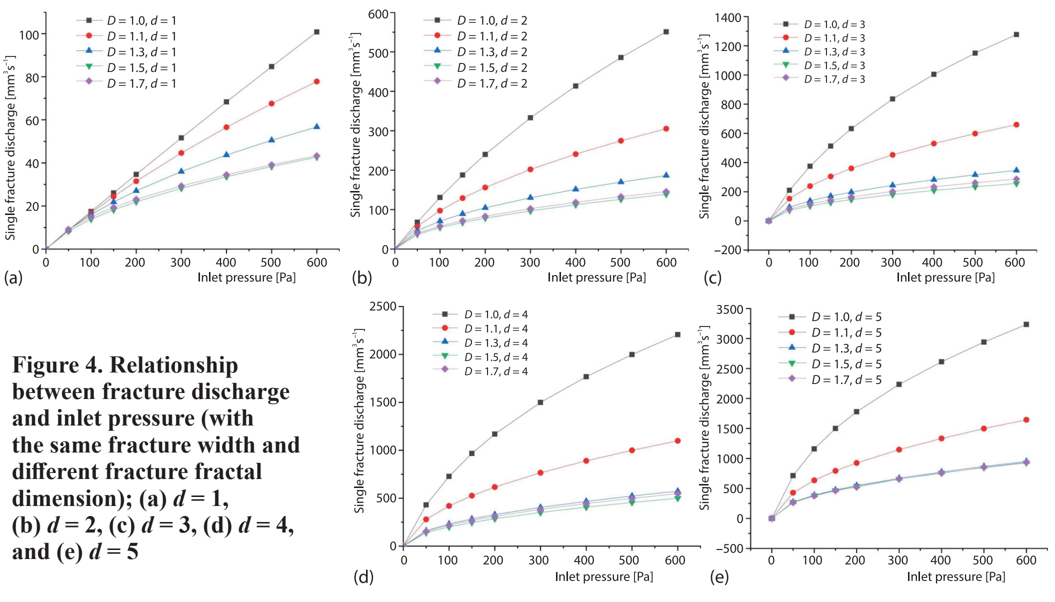

Figure 4. Relationship between fracture discharge and inlet pressure (with the same fracture width and different fracture fractal dimension); (a) *d* = 1, (b) *d* = 2, (c) *d* = 3, (d) *d* = 4, and (e) *d* = 5

Figure 5 shows the relationship between fracture discharge and fracture fractal dimension. With the increase of fractal dimension, the fracture discharge decreases sharply and the decreasing rate becomes smaller. In addition, the fracture discharge also gradually converges, and its convergence value gradually increases with the increase of inlet pressure.

The relationship between hydraulic gradient index and fracture fractal dimension is shown in fig. 6. The hydraulic gradient index decreases and the decreasing rate is gradually decreasing with the increase of fracture fractal dimension. Finally, the hydraulic gradient index converges to a constant value. When the fracture width is $d \geq 1$, the hydraulic gradient index converges to about 0.5. When the fracture width is larger, the hydraulic gradient index is easier to converge.

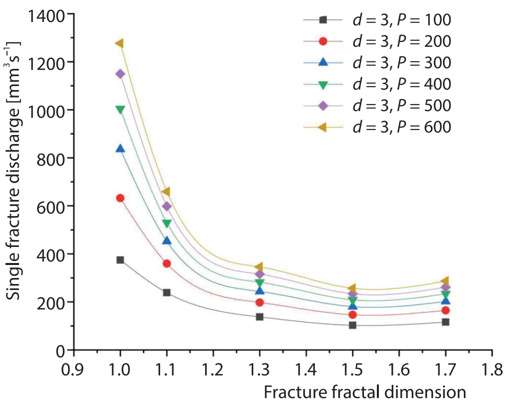

Figure 5. Relationship between fracture discharge and fracture fractal dimension

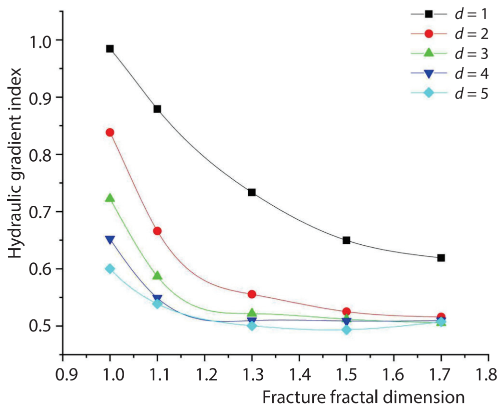

Figure 6. Relationship between hydraulic gradient index and fracture fractal dimension

Conclusion

By calculating and analyzing the fractal fracture flow under 225 working conditions, the following conclusions are obtained. The fracture discharge would increase with the increase of inlet pressure, but the growth rate of fracture discharge would decrease, which indicates that the blocking effect of rough structure on fluid is obviously enhanced under high pressure. The fracture discharge increases with the increase of fracture width, and the increasing rate increases. The phenomenon reflects that the increase of the fracture width expands the effective flow space, at the same time, it also weakens the blocking effect of the rough structure on fluid. Therefore, the larger the fracture width is, the stronger the fracture flow capacity is. The fracture discharge would gradually converge with the increase of fracture fractal dimension, and its convergence value would also increase with the increase of inlet pressure. This phenomenon shows that the blocking effect of fracture rough structure on fluid is weakened and reaches a steady-state with the increase of fracture fractal dimension. The larger the fracture width is, the more obvious the phenomenon is. The hydraulic gradient index tends to 0.5 with the increase of fracture width when fractal dimension is greater than 1.3. It also could tend to 0.5 with the increase of fractal dimension when fracture width is greater than 1.

Acknowledgment

This work was financially supported by National Natural Science Foundation of China (52004167; U2013603), the Applied Basic Research Programs of Sichuan Province (No. 2021YJ0411), the Program for Guangdong Introducing Innovative and Entrepreneurial Teams (No. 2019ZT08G315), and the Shenzhen Fundamental Research Project (JCYJ20190808153416970).

Nomenclature

D – fracture fractal dimension, [–]
d – fracture width, [mm]
d' – equivalent diameter of the cross-section of seepage path, [mm]
H_f – frictional resistance, [Nmm⁻1]
H_l – local resistance,[Nmm^{-1}]
L – length of the flow path, [mm]
P – inlet pressure, [Pa]
Re – Reynolds number, [–]
v – average velocity, [mms^{-1}]

Greek symbols

ε – local resistance coefficient, [–]
ρ – density of the liquid, [gcm^{-3}]
φ_n – any phase angle, [°]

References

[1] Xie, J., *et al*., Gas-flow Characteristics of Coal Samples with Different Levels of Fracture Network Complexity under Triaxial Loading and Unloading Conditions, *Journal of Petroleum science and Engineering*, *195* (2020), 107606
[2] Gao, M., *et al*., Principle and Technology of Coring with in-Situ Pressure and Gas Maintaining in Deep Coal Mine (in Chinese), *Journal of China Coal Society*, *46* (2021), 3, pp. 885-897
[3] Wang, D., *et al*., Effect of Cyclic Temperature Impact on Coal Seam Permeability, *Thermal Science*, *21* (2017), S1, pp. S351-S357
[4] Xie, J., *et al*., Experimental Investigation on the Anisotropic Fractal Characteristics of the Rock Fracture Surface and Its Application on the Fluid-Flow Description, *Journal of Petroleum Science and Engineering*, *191* (2020), 107190
[5] Zhang, Z., *et al*., Permeability Evolution of Unloaded Coal Samples at Different Loading Rates, *Thermal Science*, *18* (2014), 5, pp. 1497-1504
[6] Witherspoon, P. A., *et al*., Validity of Cubic Law for Fluid-Flow in a Deformable Rock Fracture, *Water Resources Research*, *16* (1980), 6, pp. 1016-1024
[7] Gao, M., *et al*., Mechanical Behavior of Coal under Different Mining Rates: A Case Study from Laboratory Experiments to Field Testing, *International Journal of Mining Science and Technology*, (2021), On-line first, https://doi.org/10.1016/j.ijmst.2021.06.007
[8] Gao, M., *et al.*, Calculating Changes in Fractal Dimension of Surface Cracks to Quantify how the Dynamic Loading Rate Affects Rock Failure in Deep Mining, *Journal of Central South University*, *27* (2020), 10, pp. 3013-3024
[9] Konzuk, J. S., Kueper, B. H., Evaluation of Cubic Law Based Models Describing Single-Phase Flow through a Rough-Walled Fracture, *Water Resources Research*, *40* (2004), 2, W02402
[10] Klimczak, C., *et al*., Cubic Law with Aperture-Length Correlation: Implications for Network Scale Fluid-Flow, *Hydrogeology Journal*, *18* (2010), 4, pp. 851-862
[11] Wang, Z., *et al*., A Modified Cubic Law for Single-Phase Saturated Laminar Flow in Rough Rock Fractures, *International Journal of Rock Mechanics and Mining Sciences*, *103* (2018), Mar., pp. 107-115
[12] Guo, B., *et al*., A Modified Cubic Law for Rough-Walled Marble Fracture by Embedding Peak Density, *Advances in Civil Engineering*, *2020* (2020), ID9198356
[13] Roslin, A., *et al*., Permeability Upscaling Using Cubic Law Based on the Analysis of Multi-Resolution Micro-CT Images of Intermediate Rank Coal, *Energy and Fuels*, *33* (2019), 9, pp. 8215-8221
[14] Gao, M., *et al*., The Location Optimum and Permeability-Enhancing Effect of a Low-Level Shield Rock Roadway, *Rock Mechanics Rock Engineering*, *51* (2018), 9, pp. 2935-2948
[15] Lu, W., *et al*., Model Experiment and Numerical Simulation of Flow and Heat Transfer for Sand-Filled Fractured Rock Model (in Chinese), *Rock and Soil Mechanics*, *32* (2011), 11, pp. 3448-3454
[16] Li, X., Boundary Element Method for 3-D Fracture Network Seepage Flow and Its Programming (in Chinese), *Journal of China Institute of Water Resources and Hydro power Research*, *4* (2006), 2, pp. 81-87
[17] Li, X., *et al.*, A Study on the Meshless Method on Seepage of Intersected Fractures (in Chinese), *Rock and Soil Mechanics*, *28* (2007), Z, pp. 371-374
[18] Wang, E., Network Analysis and Seepage Flow Model of Fractured Rockmass (in Chinese), *Chinese Journal of Rock Mechanics and Engineering*, *12* (1993), 3, pp. 214-221

[19] Chai, J., Coupling Analysis of Unsteady Seepage and Stress Fields in Discrete Fractures Network of Rock Mass in Dam Foundation (in Chinese), *Science China Technological Sciences*, *54* (2011), S1, pp. S133-S139
[20] Zheng, H., *et al*., A New Formulation of Signorini's Type for Seepage Problems with Free Surfaces, *International Journal for Numerical Methods in Engineering*, *64* (2005), 1, pp. 1-16
[21] Jiang, Q., *et al.*, Seepage Flow with Free Surface in Fracture Networks, *Water Resources Research*, *49* (2013), 1, pp. 176-186
[22] Jiang, Q., *et al.*, A New Variational Inequality Formulation for Unconfined Seepage Flow through Fracture Networks (in Chinese), *Science China Technological Sciences*, *55* (2012), 11, pp. 3090-3101
[23] Yao, C., *et al.*, The Variational Inequality Formulation for Unconfined Seepage through 3-D Dense Fracture Networks, *Science China Technological Sciences*, *56* (2013), 5, pp. 1241-1247
[24] Mandelbrot, B. B., *The Fractal Geometry of Nature*, W. H. Freeman, New York, USA, 1982
[25] Feder, J., *Fractals,* Plenum Press, New York, USA, 1988
[26] Xie, H., *Fractals in Rock Mechanic*s, A. A. Balkema, Rotterdam, The Netherlands, 1993
[27] Ju, Y., *et al*., An Experimental Investigation on the Mechanism of Fluid-Flow through Single Rough Fracture of Rock, *Science China Technological Sciences*, *56* (2013), 8, pp. 2070-2080
[28] Roberson, J. A., Crowe, C. T., *Engineering Fluid Mechanics*, Houghton Mifflin, Boston, Mass., USA, 1985
[29] Finnemore, E. J., Franzini, J. B., *Fluid Mechanics with Engineering Applications*, McGraw-Hill Companies Inc., New York, USA, 2002

Paper submitted: May 11, 2021
Paper revised: June 16, 2021
Paper accepted: July 27, 2021