POWER TAKE-OFF DAMPING CONTROL PERFORMANCE ON THE POWER CONVERSION OF OSCILLATING-BUOY WAVE ENERGY CONVERTER

by

Weijie SUN[a], Yujie ZHOU[b], Zhengang WAN[a], Xiaoguo ZHOU[b], and Wanchao ZHANG[b,c*]

[a]School of Electronic Information, Jiangsu University of Science and Technology, Zhenjiang, China
[b]School of Naval Architecture and Ocean Engineering, Jiangsu University of Science and Technology, Zhenjiang, China
[c]The PLA Troops 92228, Beijing, China

Original scientific paper
https://doi.org/10.2298/TSCI2106107S

The power take-off (PTO) damping mechanism is very important to the motion and power conversion for the wave energy converters. Based upon the potential flow theory, the series expression with unknown Fourier coefficients of velocity potential function of the basin where the cylindrical floating buoy is located is obtained by using the eigenfunction expansion method. According to the characteristics of the PTO damper, the motion and wave energy conversion characteristics of the float under the linear and non-linear PTO damping are studied, respectively, and the over-damping problem under the linear PTO damping is emphatically explored. The results show that the influence of PTO system with low velocity index on the motion of the device is mainly reflected in the PTO damping coefficient. With the increase of damping coefficient, the resonance frequency of the wave energy device decreases gradually, but the decrease amplitude is very small. The non-linear characteristics of PTO system cannot change the optimal capture width ratio of the float, but the large velocity index can effectively improve the damping capacity of PTO system. At lower and higher frequencies, the optimal PTO damping obtained by the analytic algorithm will make the device in an over-damped state. The highest frequency in the low frequency part and the lowest frequency in the high frequency part which need to be modified will gradually decrease with the increase of radius and draught.

Keywords*: wave energy conversion, analytical solution, non-linear PTO, over-damped modification*

Introduction

With the increasing global demand for energy and electricity [1-4], in the past few decades, more and more attention has been paid to the problem of extracting energy from ocean waves and maximizing their capture width ratio through theoretical or experimental research.

*Corresponding author, e-mail: zhangwanchao@just.edu.cn

There have been a lot of studies on improving the capture width ratio of the simple oscillating float by analytical methods [5-9]. Goggins and Finnegan [7] presented a methodology for the structural geometric configuration optimization considering given deployment site. They obtained the optimal damping coefficient of the considered PTO mechanism. Mavrakos and Katsaounis [8] further studied the shape's effect on the power conversion of a point absorber. They creatively put forward some kinds of absorber shapes such as the skirt-like and piston type. The calculation through analytical solution show that the shape's effect of the buoy should be considered for the initial design of a wave energy converter (WEC). Similar works [10-13] are still going on. However, most of them only consider the linear direct drive load system. In fact, if the energy transmission and conversion loss are considered, there is a non-linear relationship between the obtained and usable wave energy and the motion of the floating body [14, 15]. Sheng *et al*. [14] studied different PTO dampers and their optimizations for a point absorber WEC. The comparisons between the linear and non-linear PTO damping are studied by assuming the buoy fixed on the sea floor with only heave mode considered.

In such case, the hydrodynamic interactions and PTO damping coefficients are both important factors for the power conversion optimization. The control of the damping coefficients should also be considered. In this paper, the hydrodynamic performance and the PTO damper types and coefficients control are considered. For the PTO system considering the non-linear exponential relationship of the velocity of the body, the hydrodynamic characteristics of the simple oscillating floating body are studied analytically by using the linear micro amplitude wave assumption. The interaction between the velocity index and damping coefficient under different radius and draft conditions and their effects on the motion of the floating body and the wave energy capture are systematically studied, which provides a reference for the wave energy generation of the oscillating floating body it provides the basis for optimization.

Mathematical model

Wave power conversion model

It is assumed that a single floating body is connected by a fixed base (seabed) through a damper. Its vibration model can be expressed as shown in fig. 1. Where c_h and k_h represent the equivalent damping and stiffness coefficients generated by hydrodynamic forces, respectively, and c_p is the damping coefficients of the damper.

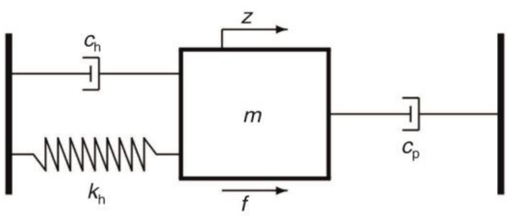

Figure 1. The oscillating model for the WEC

Under the action of incident wave with frequency, ω, the float moves vertically. Taking the balance position of the float as the co-ordinate system, the motion equation can be obtained:

$$m\ddot{z} = F_e + F_r + F_s + F_p \tag{1}$$

where m and $\ddot{z}$ denote the mass and vertical acceleration of the float, respectively, and F_e and F_r represent the vertical wave exciting and radiation force, respectively.

When the external load of the float is periodic, the displacement, velocity and external load of the float both contain the time factors. Therefore, for the convenience of description, they can be expressed as:

$$z = \hat{z} \cdot \exp[i(\omega t + \beta_D)], \quad F_e = \hat{F}_e \cdot \exp[i(\omega t + \beta_F)]$$

where β denotes the phase of displacement and force, and colon $\wedge$ above the variable represents the complex amplitude.

The radiation force can be decomposed into the additional mass force and damping force, which can be expressed as:

$$F_r = -\mu_{33}\ddot{z} - \lambda_{33}\dot{z}$$

where μ_{33} and λ_{33} represent the added mass and damping coefficient, respectively, and F_s – the vertical hydrostatic restoring force. Among them, k_h is the stiffness coefficient of hydrostatic restoring force, ρ – the water density, and g – the acceleration of gravity.

The output load, F_p, is the main form to output energy which is mainly represented by the damper. For the direct drive PTO, considering the coil winding and energy loss, the load is generally described as speed exponential or piecewise function. This paper mainly studies that the PTO load is a linear and exponential function of speed, which can be expressed as:

$$F_p = -c_p \left|\dot{z}\right|^{\alpha} \dot{z}$$

where c_p is the damping coefficient of the damper and α – the velocity index. Then we can get the following results:

$$\{-\omega^2[m+\mu_{33}(\omega)] - i\omega[\lambda_{33}(\omega) + c_p\left|\omega\hat{z}\right|^{\alpha}]\}\hat{z}(\omega)e^{i\beta_D} + k\hat{z}(\omega)e^{i\beta_D} = \hat{F}(\omega)e^{i\beta_F} \tag{2}$$

Bring the float vibration speed $v = \dot{z}$ into the previous equation, the average wave power obtained by the float in a period can be expressed as:

$$P(\omega) = \frac{1}{\mathrm{T}}\int_t^{t+T} \mathrm{Re}(F_p)\,\mathrm{Re}(v)\mathrm{d}t = \frac{1}{2}c_p\left|\hat{v}\right|^{\alpha}\hat{v}\hat{v}^* = \frac{1}{2}c_p\omega^{\alpha+2}\left|\hat{z}\right|^{\alpha}\hat{z}\hat{z}^* \tag{3}$$

The velocity has no effect on the optimal wave power conversion, but has great effect on the displacement and velocity amplitude. If the velocity index is 0, the complex amplitude of velocity and displacement can be obtained quickly. The ratio of the motion amplitude $\hat{z}$ to the amplitude ξ_0 of the incident wave is the amplitude of the heave response (RAO), thus, we can get that the power converted by the float is:

$$P(\omega) = \frac{c_p\hat{v}\hat{v}^*}{2} = \frac{\left(\dfrac{c_p}{2}\right)(\hat{F}_e\hat{F}_e^*)}{(\lambda_{33}+c_p)^2 + \left(\omega m + \omega\mu_{33} - \dfrac{k_h}{\omega}\right)^2} \tag{4}$$

It can be seen from the formula that when the incident wave frequency is given, the energy that can be converted by the wave energy device mainly depends on the damping of the PTO. Because when the damping coefficient c_p is 0 or ∞, $P(\omega)$ is zero, and when $0 < c_p < \omega$ then $P(\omega)$ is over zero.

Hydrodynamic model

As previously mentioned, it is assumed that a cylinder with radius, R, and draft, D, is connected by a PTO system and floats on the wave surface with water depth, h. A cylindrical co-ordinate system is established at the center of the water line plane of the float in still water, as shown in fg. 2.

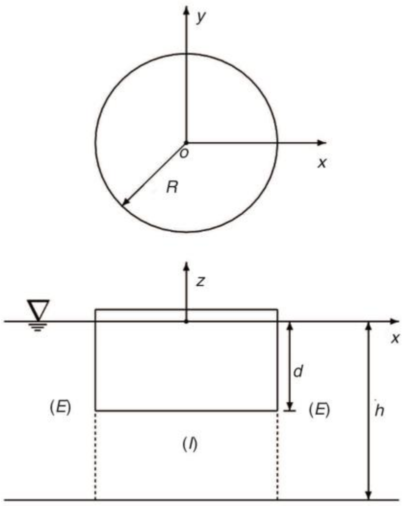

Figure 2. Sketch of the oscillating buoy and the definition of fluid subdomains

In the cylindrical co-ordinate system, the velocity potential $\phi(r,\theta,z,t)$ is introduced to describe the flow field, it can be expressed as:

$$\phi(r,\theta,z,t)=\mathrm{Re}[\Phi(r,\theta,z)e^{-i\omega t}]$$

According to the assumption of linear wave theory, velocity, Φ, can be decomposed into two parts in cylinder co-ordinate system:

$$\begin{aligned}&\Phi(r,\theta,z)=\\&=\Phi_0(r,\theta,z)+\Phi_7(r,\theta,z)+\sum_{j=1}^{6}-i\omega\xi_j\Phi_j(r,\theta,z)=\\&=\Phi_D(r,\theta,z)+\sum_{j=1}^{6}-i\omega\xi_j\Phi_j(r,\theta,z)\end{aligned}\tag{5}$$

where $\Phi(r,\theta,z)$ represents the velocity potential of incident wave, $\Phi_7(r,\theta,z)$ – the scattering velocity potential, which together constitute the diffraction velocity potential $\Phi_D(r,\theta,z)$, $\Phi_j(r,\theta,z)$ – the radiation velocity potential in j-modal motion, and ξ_j – the amplitude of the body of j-modal. In this paper, only the case of heave motion of floating body is considered, that is, j is 3 in eq. (5).

The aforementioned velocity potentials need to satisfy the Laplace equation, free surface condition, seafloor impenetrable condition, object surface condition and far-field radiation condition, expressed as:

$$\begin{aligned}&\nabla^2\Phi=0\\&\omega^2\Phi-g\partial_z\Phi=0\quad z=0,\ \ r\geq R\\&\partial_z\Phi=0\quad z=-h\\&\partial_n\Phi_j=V_n\ \ ,\ \ \partial_n\Phi_D=0\\&\lim_{r\to\infty}\sqrt{kr}\,(\partial\Phi/\partial r-ik\Phi)=0\end{aligned}\tag{6}$$

The solutions of the velocity potentials can be found in [9].

Motion characteristics

It is found in fig. 3 that for a float with a given draft, its heave RAO change sharply with the radius. When the radius increases, the peak value and the frequency at the peak value gradually decrease. When the radius of the float is given and the draft is changed, it can be found that the peak RAO of the float increases with the increase of the draft, but the resonance frequency shows the opposite trend.

It can be found from the heave response amplitude of the float that for a given float, the heave RAO has a peak at a specific frequency, which is the resonance frequency, and the corresponding RAO is the resonance response amplitude. Therefore, by setting the cylinder with different radius and draft, the resonance frequency and the response amplitude are shown in tabs. 1 and 2. It can be seen from the table that the resonance frequency of the float decreases with the increase of radius and draft, but its motion response increases with the increase of draft and decreases with the increase of radius.

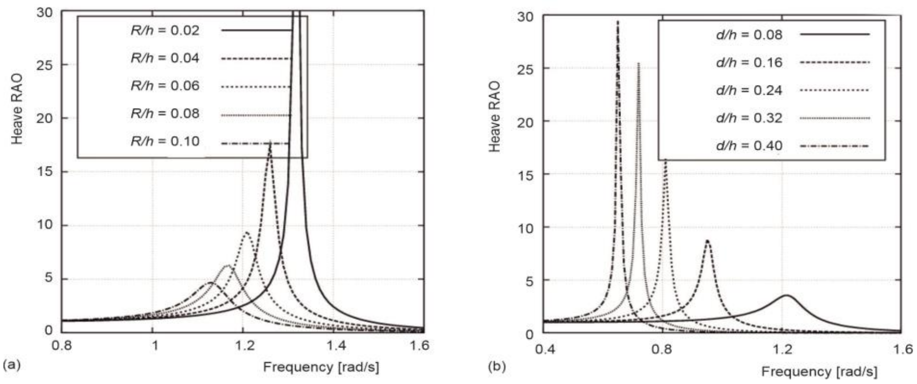

Figure 3. Heave RAO with *d*/*h* = 0.1 (a) and *R*/*h* = 0.1 (b)

Table 1. The resonance frequencies of the buoy

d/*h*	*R*/*h*						
	0.02	0.04	0.06	0.08	0.10	0.12	0.14
0.04	1.94	1.78	1.67	1.58	1.50	1.43	1.37
0.06	1.64	1.54	1.46	1.39	1.34	1.29	1.24
0.08	1.45	1.38	1.32	1.27	1.22	1.18	1.15
0.10	1.32	1.26	1.21	1.17	1.13	1.10	1.07
0.12	1.21	1.17	1.13	1.09	1.06	1.03	1.01
0.14	1.13	1.09	1.06	1.03	1.00	0.98	0.96

Table 2. The heave RAO of the buoy at resonance frequencies

d/*h*	*R*/*h*						
	0.02	0.04	0.06	0.08	0.10	0.12	0.14
0.04	12.61	4.74	2.89	2.12	1.71	1.46	1.30
0.06	23.14	8.15	4.69	3.29	2.55	2.11	1.82
0.08	35.37	12.36	6.82	4.69	3.55	2.87	2.43
0.10	46.95	17.47	9.45	6.26	4.69	3.72	3.10
0.12	43.52	22.10	12.01	8.11	5.95	4.69	3.84
0.14	66.65	27.64	15.56	10.08	7.34	5.70	4.65

Wave energy conversion

Non-linear PTO damping effects

The influence of the speed index of the PTO system on the heave motion is studied when the damping coefficient is given as 5 $\mathrm{kNs}^{\alpha+1}/\mathrm{m}^{\alpha+1}$, and the corresponding speed indexes, α, are given as 0.0, 0.2, 0.4, 0.6, and 0.8, respectively. The calculation results are shown in fig. 4. It can be found that the RAO of float heave is 1 in the low frequency part and approaches zero in the high frequency part. The change of damping coefficient and velocity index almost does not change the peak frequency (resonance frequency) of the buoy. However, the effect on the peak value is obvious. Careful observation shows that when the PTO damping coefficient or speed index is given, the peak value of RAO will decrease with the increase of the corresponding speed index or PTO damping coefficient.

As shown in fig. 5, the dimensionless PTO damping force (damping force divided by float displacement) and captured energy increase in the low damping coefficient part, but as the damping coefficient continues to increase, the PTO damping force changes slowly, and the difference is very small under different speed indexes; The captured energy reaches a peak value and then decreases with the increase of damping coefficient.

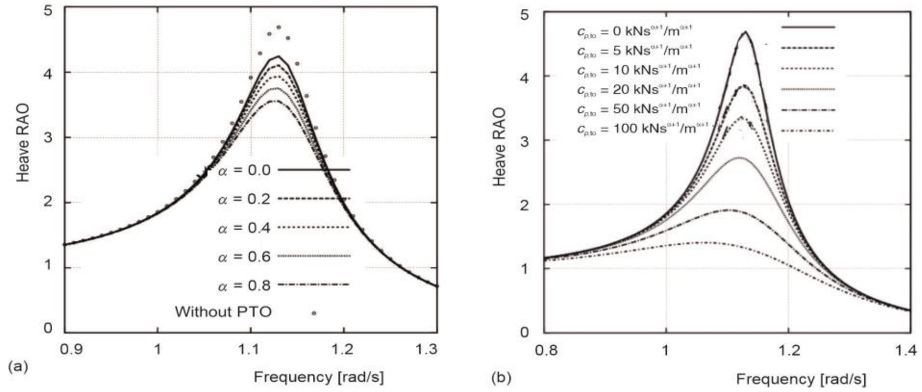

Figure 4. Heave RAO with damping coefficient 5 kNs$^{\alpha+1}$/m$^{\alpha+1}$ and velocity index 0.5, respectively

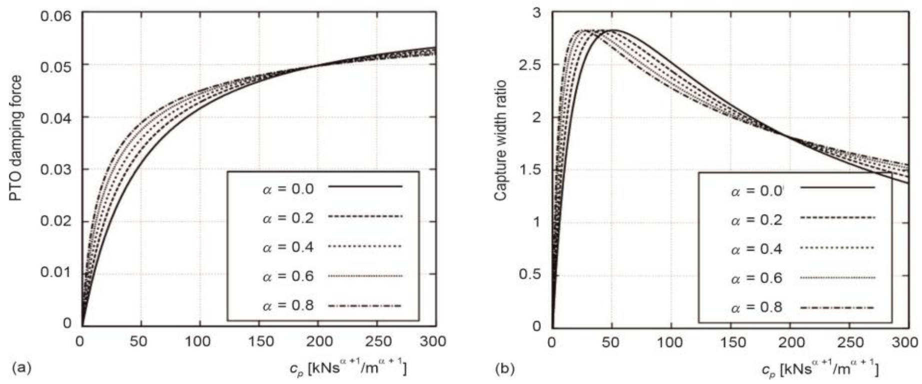

Figure 5. The PTO damping force and captured width ratio with incident wave frequency 1.15 rad/s

In the low damping coefficient part, the damping force increases with the exponential increase of velocity, and the damping coefficient decreases with the exponential increase at the peak value. Comprehensive analysis shows that the PTO system with high-speed index can have ideal output load at low damping coefficient, and the speed index will not change the peak value of capture width ratio, which indicates that if the damping capacity of the designed PTO system is large enough, its nonlinearity will not affect the optimal conversion capacity of the wave energy device.

Linear PTO damping effects

Under the action of linear PTO, the analytical solution can be obtained. However, considering the unavailability over damped state, the optimal damping of the system under damped (UD) state is studied. When the draft of the buoy is given, the variation of the optimal PTO damping with the frequency is shown in the line marking results in fig. 6. The minimum value decreases first and then increases. The optimal PTO damping at both ends of the minimum point increases with the radius increasing. The difference is that with the radius increasing, the minimum value increases, and the frequency corresponding to the minimum point decreases. From the modified optimal results, the PTO damping in the low and high frequency bands is far less than the unmodified damping value. The maximum frequency of the optimal PTO correction in the low frequency part and the minimum frequency of the optimal PTO correction in the high frequency part decrease with the increase of the radius. For a given radius, the optimal PTO damping corresponding to the minimum point decreases slightly with the increase of float draft, while the corresponding frequency decreases obviously.

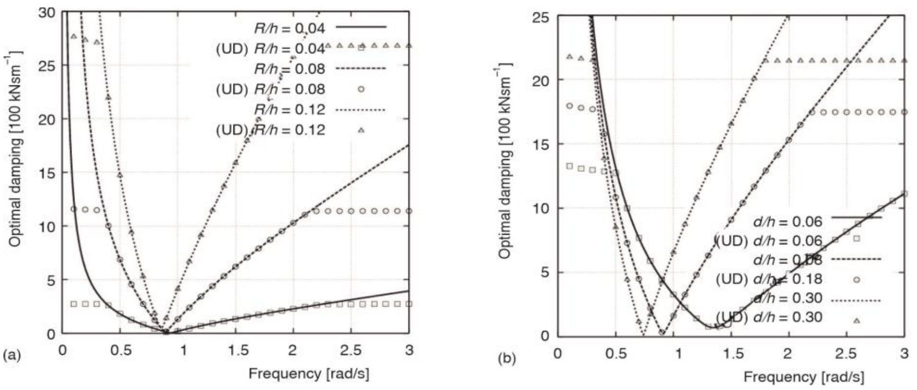

Figure 6. Optimal PTO damping with *d*/*h* = 0.1 and *R*/*h* = 0.1

Figure 7 show that the variation of the optimal capture width ratio considering the PTO over damping correction. In the low frequency part, the optimal capture width ratio after PTO damping correction is less than that without correction, which means in the initial design of the oscillating float wave energy device, if the wavelength is long, the damping of the PTO system needs to be modified to obtain the maximum wave energy capture of the device. With the frequency increases, it is found that whether the correction is carried out has no effect on the optimal wave energy capture of the device, which indicates that the excessive damping state of the PTO only occurs at low frequency will affect the wave energy capture, but no matter whether it is high frequency or low frequency, it is necessary to modify the PTO damping to ensure the PTO system work normally.

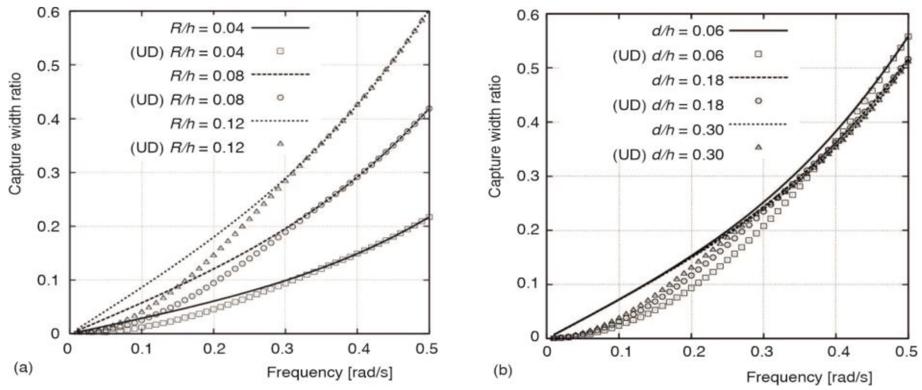

Figure 7. Optimal capture width ratio with *d*/*h* = 0.2 and *R*/*h* = 0.1 at lower frequencies

In the calculation range of size, the change of the resonance frequency is not linear, which means there is a peak value with one of the variables, as shown in fig. 8. When the radius and draught of the float are the minimum, the resonance frequency of the float increases. Under the given float draught, the resonance frequency of the float decreases with the increase of the radius. If the radius R/h is less than 0.08, the resonance frequency decreases with the increase of the draught, but with the increase of the radius, the resonance frequency first increases and then decreases. There is a maximum in the process. The calculation results of optimal capture width ratio show that the change of capture width ratio with float radius and draft is linear and single, and the maximum capture width ratio appears when the float draft radius is the minimum.

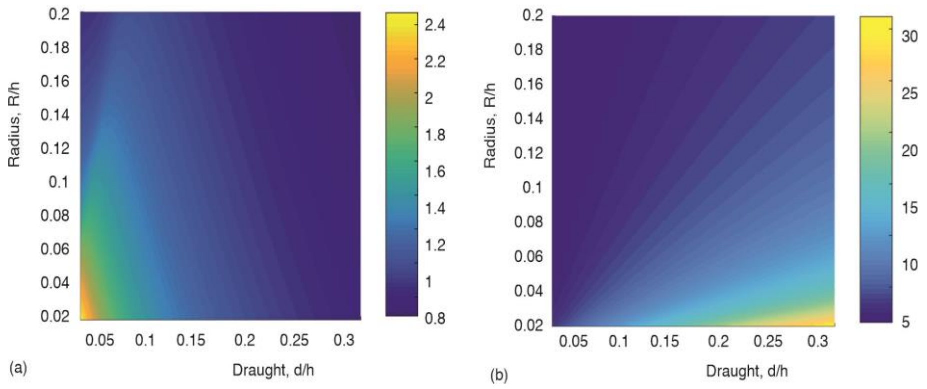

Figure 8. The resonance frequencies and optimal capture width ratio

Discussion and conclusions

Based on the linear wave theory, this paper studies the hydrodynamic characteristics of cylindrical oscillating energy absorber, and focuses on the influence of the PTO system model on the wave energy conversion of floats with different configuration parameters, the conclusions are as follows.

- With the increase of the damping coefficient, the resonance frequency of the wave energy device decreases gradually, but the decrease amplitude is very small; The non-linear characteristics of the PTO system cannot change the optimal capture width ratio of the float, but a larger speed index can effectively improve the damping capacity of the PTO system.
- At lower frequency and higher frequency, the optimal PTO damping obtained by the analytical algorithm will make the device in an over damped working state, and the maximum frequency of the optimal PTO correction in the low frequency part and the minimum frequency of the optimal PTO correction in the high frequency part will gradually decrease with the increase of the radius and draft.
- The resonance frequency of the float reaches the maximum when the radius and draft are the minimum. Except that the resonance frequency reaches the maximum when the radius R/h is greater than 0.08, it changes with the size of the float in other cases.

Acknowledgment

This paper is financially supported by the National Natural Science Foundation of China (No.51909111) and Natural Science Foundation of Jiangsu Province (No. BK20180980).

References

[1] Zhou, F. Q., Renewable Energy Development Strategy in China (in Chinese), *Shanghai Electric Power, 6* (2005), pp. 554-560

[2] Falcao, A. F. O., Wave Energy Utilization: A Review of the Technologies, *Renewable and Sustainable Energy Reviews*, *14* (2010), 3, pp. 899-918
[3] Caraher, S. L., *et al*., Investigation of Fluid Film Bearings for Use in Direct Drive Linear Generators in Submerged Wave Energy Converters, *Proceedings*, 18th International Offshore Polar Engineering Conference, Vancouver, Canada, 2008, pp. 409-416
[4] Ciement, A., Mccullen, P., Wave Energy in Europe: Current Status and Perspectives, *Renewable and Sustainable Energy Reviews*, *6* (2002), 5, pp. 405-431
[5] Wu, B. J., *et al*., Wave Power Conversion Characteristics of a Complex Vertical Cylinder (in Chinese), *Journal of Harbin Engineering University*, *8* (2010), 31, pp. 1023-1028
[6] Pastor, J., Liu, Y. C., Frequency and Time Domain Modeling and Power Output for a Heaving Point Absorber Wave Energy Converter, *International Journal of Energy and Environment*, 5 (2014), Apr., pp. 100-112
[7] Goggins, J., Finnegan, F., Shape Optimization of Floating Wave Energy Converters for a Specified Wave Energy Spectrum, *Renewable Energy*, *71* (2014), Nov., pp. 208-220
[8] Mavrakos, S. A., Katsaounis, G. M., Effects of Floaters' Hydrodynamics on the Performance of Tightly Moored Wave Energy Converters, *IET Renewable Power Generation*, *4* (2010), 6, pp. 531-544
[9] Bachynski, E. E., *et al*., Analysis and Optimization of a Tethered Wave Energy Converter in Irregular Waves, *Renewable Energy*, *48* (2012), Dec., pp. 133-145
[10] Falnes, J., *Linear Interaction Including Wave-Energy Extraction. Ocean Waves and Oscillating System*, Cambridge University Press, Cambridge, UK, 2002
[11] Eriksson, M., *et al*., Hydrodynamic Modelling of a Direct Drive Wave Energy Converter, *International Journal of Engineering Science*, *43* (2005), 17-18, pp. 1377-1387
[12] Fitzgerald, J., Bergdahl, L., Including Moorings in the Assessment of a Generic Offshore Wave Energy Converter: A Frequency Domain Approach, *Marine Structures*, *21* (2008), 1, pp. 23-46
[13] Price, A. A. E., *et al*., On the Capture Width of Wave Energy Converters, *Applied Ocean Research*, *31* (2009), 4, pp. 251-259
[14] Sheng, W. N., *et al*., Optimizing Power Take-Offs for Maximizing Wave Energy Conversions, *Proceedings*, 30th International Workshop on Water Waves and Floating Bodies, Bristol, UK, 2015
[15] Sheng, W. N., Lewis, A., Power Take Off Optimization for Maximizing Energy Conversion of Wave-Activated Bodies, *IEEE Journal of Oceanic Engineering*, *41* (2016), 3, pp. 529-540

Paper submitted: January 4, 2021
Paper revised: July 10, 2021
Paper accepted: July 12, 2021