APPLICATION OF DIFFERENTIAL TRANSFORMATION METHOD FOR AN ANNULAR FIN WITH VARIABLE THERMAL CONDUCTIVITY

by

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The thermal analysis of the annular fin is performed by applying the differential transformation method. The thermal conductivity of the annular fin has been considered as a function of temperature. The effects of non-dimensional parameters, namely thermal conductivity and thermo-geometric fin parameters on the fin efficiency and temperature distribution are determined. Obtained results from the differential transformation method are also compared with the exact analytical results and the results of the finite difference method in the constant thermal conductivity condition. It has been concluded that the differential transformation method provides accurate results in the solution of non-linear problems.

Key words*: annular fin, differential transformation method, variable thermal conductivity*

Introduction

Fins or extended surfaces are used to augment the heat transfer between the fin surface and surrounding fluid by increasing convective heat transfer surface area. Heat transfer always takes place from the hot temperature region to the cold temperature region in three different modes, namely conduction, convection, and radiation. While heat is transferred inside the fin by conduction, heat is transferred from the fin surface to the surrounding fluid by convection. Finned surfaces have been used in a wide variety of thermal applications. Some examples are the cooling of electronic components, air-cooled motors or condensers, evaporators, heater pipes, heat exchangers*, etc.* A detailed review of this topic is given by Kraus *et al.* [1]. Thermal analysis of fins with variable thermal properties are highly non-linear problems and the exact solutions of these types of problems are generally difficult. Various semi-analytical methods have been used in solving non-linear problems or differential equations. Homotopy perturbation method, homotopy analysis method, adomian decomposition method, variational iteration method*, etc.*, are some semi-analytical methods. Many studies have been conducted in the literature on the application of these methods. Ganji *et al.* [2] applied the homotopy perturbation method to determine the temperature distribution for annular fins. Kumar *et al.* [3] presented a new non-integer method for convective straight fins with temperature-dependent thermal conductivity. In their analysis, the homotopy perturbation method coupled Laplace transform method is successfully used. Chowdhury *et al.* [4] compared with homotopy perturbation method and adomian decomposition method to

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determine the temperature distribution of a straight rectangular fin with temperature-dependent heat flux. Arslanturk [5] performed the thermal analysis and optimization of convective straight fins with a step-change in thickness and temperature-dependent thermal conductivity by using the homotopy perturbation method. The other studies, Domairry and Fazeli [6] applied the homotopy analysis method to determine the fin efficiency of straight fins, Khani and Aziz [7] developed an analytical solution for the thermal performance of a straight fin of the trapezoidal profile, Arslanturk [8] and Aksoy [9] analyzed the thermal performance of annular fins with temperature-dependent thermal properties by using the homotopy analysis method. Chiu and Chen [10] have evaluated the efficiency and the optimal length of a convective rectangular fin with variable thermal conductivity and determined the temperature distribution within the fin using Adomian decomposition method. Chang [11] also performed a decomposition solution for a straight rectangular fin with temperature-dependent surface heat flux. Besides these methods, the differential transformation method (DTM) is also one of the most powerful analytical methods to solve non-linear differential equations. The DTM is applied for obtaining the analytic Taylor series solution of differential equations. The DTM was firstly introduced by Zhou [12]. It was used for solving linear and non-linear initial value problems in electrical circuits. After that, DTM is applied to obtain analytical solutions of non-linear engineering problems. There are some advantages to using DTM over other analysis methods. For example, unlike other analytic perturbation techniques, DTM is independent of small or large quantities. Also, as in the homotopy analysis method, there is no need to calculate the auxiliary parameter, to estimate an initial guess and auxiliary linear operator. Equations can be solved directly by using DTM [13]. Joneidi *et al.* [14] applied the DTM to determine the temperature distribution and fin efficiency of convective straight fins with temperature-dependent conductivity. They also verified the obtained results from DTM by compared with those from the exact and numerical solution. They also analyzed the effects of some physical parameters. Moradi and Ahmadikia [15] investigated three profiles of the straight fin namely convex, rectangular and exponential, that have a temperature-dependent thermal conductivity by using DTM and compared with numerical solutions. They found that the exponential profile fin has higher performance and efficiency than the convex and rectangular profiles. Torabi *et al.* [16] performed a thermal analysis in a longitudinal step fin with variable thermal conductivity by applying DTM. In the analysis, they obtain two non-linear heat transfer equations. After that to define boundary conditions, these two equations have been solved by using the DTM. Ghasemi *et al.* [17] used DTM to compute the temperature distribution equation in straight fins. They consider two different conditions. In the first condition, while thermal conductivity is considered constant, heat generation inside the fin changed with temperature, in the second condition, the heat generation and conductivity both varied with temperature. Their results showed that the DTM is very convenient and effective. Kundu *et al.* [18] used the DTM to analyze the thermal performance of exponential fin by considering humidity ratio under wet surface conditions and temperature dependence of thermal conductivity. They obtained dissimilar effects on fin efficiency with the variation of some design constants. Lin and Chen [19] used the double-decomposition method and the differential transform method to analyze the hyperbolic profile annular fins with variable conductivity. They estimated that the DTM is more precise than the double-decomposition method for large thermal conductivity parameter values. Shedzad *et al.* [20] investigated the effect of fin orientation on the natural-convection of aqueous-based nanoencapsulated PCM in a heat exchanger equipped with wing-like fins. The thermal performance of the heat exchanger is analyzed in terms of average and local heat transfer coefficients. They found that the configuration and orientation of the fins are among the prime factors influencing the performance of these heat exchangers. Ambreen *et al.* [21] carried out a performance analysis of hybrid nanofluid in a heat sink equipped with sharp and streamlined micro pin-fins. The fin efficiency is investigated by analyzing elliptical, diamond, and circular fins in the staggered arrangement. Torabi and Yaghoobi [22] determined the temperature distribution and efficiency of a straight fin with a step-change in thickness using the DTM and variation iteration method. They introduced that, the DTM results are more accurate than the variation iteration method results.

In this study, temperature distribution and fin efficiency variations for the rectangular profile annular fin are determined by applying the DTM. In the analysis, the fin's thermal conductivity is considered linearly dependent on temperature. The effects of non-dimensional parameters, namely thermal conductivity and thermo-geometric fin parameters on the fin efficiency and temperature distribution are obtained. The DTM results are also verified with the exact solution with constant thermal conductivity and numerical finite difference method (FDM) results.

Formulation of the problem

The schematic representation of the annular fin considered in thermal analysis is given in fig. 1. Annular fin with rectangular profile is formulating by considering the following assumptions:

- The 1-D steady heat conduction in the radial direction.
- No heat generation inside the fin.
- Constant fin thickness, t and base temperature, T_b.
- The thermal conductivity of the fin is linearly dependent on temperature.
- Insulated fin tip.
- Constant surrounding temperature and heat transfer coefficient.
- Convective heat transfer from the fin surface.

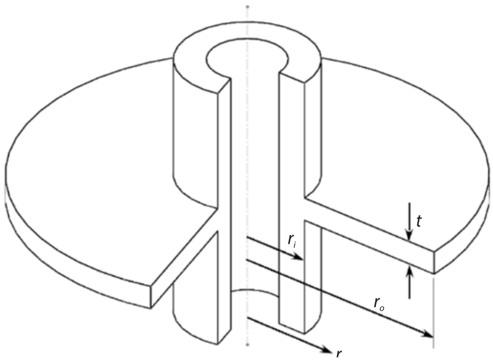

Figure 1. Schematics of annular rectangular profile fin

The energy equation and the boundary conditions under these assumptions:

$$\frac{d}{dr}\left[k(T)A(r)\frac{dT}{dr}\right]dr - 2h(T - T_a)dA_s = 0 \text{ for } r_i \le r \le r_o \tag{1}$$

$$T = T_b \text{ at } r = r_i \tag{2}$$

$$\frac{dT}{dr} = 0 \text{ at } r = r_o \tag{3}$$

where $k(T)$ is the temperature-dependent thermal conductivity, h – the convective heat transfer coefficient, T_b – the fin base temperature, T_a – the ambient temperature, $dA_s = 2\pi rt dr$ – the differential surface area, $A(r) = 2\pi rt$ – the fin's cross-sectional area, t – the fin thickness, and r_o and r_i are, respectively, the outer and inner radius of the annular fin.

Temperature-dependent thermal conductivity of annular rectangular profile fin can be defined:

$$k(T)=k_a\left[1+\lambda\left(T-T_a\right)\right] \tag{4}$$

where k_a is the thermal conductivity of the fin material at surrounding temperature and λ – the coefficient explaining the linear variation of thermal conductivity with temperature.

If non-dimensional parameters are defined:

$$\theta=\frac{T-T_a}{T_b-T_a},\ \xi=\frac{r_o-r}{r_i},\ \beta=\lambda\left(T_b-T_a\right),\ \gamma=\frac{r_o}{r_i},\ \psi^2=\frac{2hr_i^2}{tk_a} \tag{5}$$

where θ is the non-dimensioning temperature, β – the non-dimensional thermal conductivity parameter, ξ – the non-dimensional co-ordinate, ψ – the non-dimensional thermo-geometric fin parameter, and γ – the radius ratio of the annular fin. Then, the general energy equation and boundary conditions are re-written in non-dimensional form:

$$\left(1+\beta\theta\right)\frac{d^2\theta}{d\xi^2}+\frac{\left(1+\beta\theta\right)}{\left(\xi-\gamma\right)}\frac{d\theta}{d\xi}+\beta\left(\frac{d\theta}{d\xi}\right)^2-\psi^2\theta=0,\ \ 0\le\xi\le\gamma-1 \tag{6}$$

$$\theta=1\ \text{ at }\ \xi=\gamma-1 \tag{7}$$

$$\frac{d\theta}{d\xi}=0\ \text{ at }\ \xi=0 \tag{8}$$

Principles of differential transformation method

For understanding the concept of the DTM, let us assume that $f(x)$ is an analytic function, and $x = x_o$ represents any point in the D domain. The function $f(x)$ is then expressed by a power series centered at x_o. The Taylor series of the function $f(x)$:

$$f(x)=\sum_{k=0}^{\infty}\frac{\left(x-x_o\right)^k}{k!}\left[\frac{d^k f(x)}{dx^k}\right]_{x=x_o},\ \ \forall x\in D \tag{9}$$

The Maclaurin series of $f(x)$ can be represented by setting $x_o = 0$, then eq. (9) is re-written:

$$f(x)=\sum_{k=0}^{\infty}\frac{x^k}{k!}\left[\frac{d^k f(x)}{dx^k}\right]_{x=0},\ \ \forall x\in D \tag{10}$$

The differential transformation of the function $f(x)$ as explained in [12] is defined:

$$F(k)=\sum_{k=0}^{\infty}\frac{H^k}{k!}\left[\frac{d^k f(x)}{dx^k}\right]_{x=0} \tag{11}$$

where $f(x)$ and $F(k)$ are the original and transformed functions, respectively. The differential spectrum of $F(k)$ confined within the interval $x \in [0, H]$, where H is defined as a constant. The original function is expressed from the inverse transform of $F(k)$:

$$f(x)=\sum_{k=0}^{\infty}\left(\frac{x}{H}\right)^k F(k) \tag{12}$$

The basic idea of differential transformation is based on the Taylor power series expansion. The function values of $F(k)$ at values of k argument are named as discrete, *i.e.*, $F(0)$, $F(1)$, … are known as the zero, first, second discrete, …, respectively. The more discrete available, the more accurate it is possible to restore the unknown function. The function $f(x)$ consists of the transformed function $F(k)$, and its value is given by the sum of the transformed function with $(x/H)^k$ as its coefficient. In the case of actual applications, the discrete of spectrum reduces rapidly for the large values of argument k with the proper choice of constant H. The $f(x)$ function is defined by a finite power series and eq. (12) could be re-written:

$$f(x)=\sum_{k=0}^{n}\left(\frac{x}{H}\right)^{k}F(k) \tag{13}$$

where n is the approximation order of infinite series.

Some fundamental operations of the 1-D differential transform method are given in tab. 1.

Table 1. Operations of 1-D differential transform method

Original function	Transformed function
$f(x)=\alpha g(x)\pm\beta s(x)$	$F(k)=\alpha G(k)\pm\beta S(k)$
$f(x)=\alpha g(x)$	$F(k)=\alpha G(k)$
$f(x)=g(x)s(x)$	$F(k)=\sum_{i=0}^{k}G(i)S(k-i)$
$f(x)=\frac{\mathrm{d}g(x)}{\mathrm{d}x}$	$F(k)=(k+1)G(k+1)$
$f(x)=\frac{\mathrm{d}^n g(x)}{\mathrm{d}x^n}$	$F(k)=\frac{(k+n)!}{k!}G(k+n)$
$f(x)=x^m$	$F(k)=\delta(k-m)=\begin{cases}1, & k=m\\ 0, & k\neq m\end{cases}$
$f(x)=g_1(x)g_2(x)\ldots g_{n-1}(x)g_n(x)$	$F(k)=\sum_{k_{n-1}=0}^{k}\sum_{k_{n-2}=0}^{k_{n-1}}\ldots\ldots\sum_{k_2=0}^{k_3}\sum_{k_1=0}^{k_2}G_1(k_1)G_2(k_2-k_1)\ldots$ $\ldots G_{n-1}(k_{n-1}-k_{n-2})G_n(k-k_{n-1})$

Solution of energy equation with the differential transformation method

By multiplying both sides of eq. (6) by $(\xi-\gamma)$ and then taking differential transformation according to tab. 1, the following transformed equation is obtained:

$$\sum_{i=0}^{k}\Big(\delta[i-1](k-i+1)(k-i+2)\Theta[k-i+2]-\beta\gamma\Theta[i]\cdot$$
$$\cdot(k-i+1)(k-i+2)\Theta[k-i+2]+\beta\Theta[i](k-i+1)\Theta[k-i+1]-$$
$$-\beta\gamma(i+1)\Theta[i+1](k-i+1)\Theta[k-i+1]-\psi^2\delta[i-1]\Theta[k-i]+$$
$$+\beta\sum_{j=0}^{i}\Big(\delta[j-1]\Theta[i-j](k-i+1)(k-i+2)\Theta[k-i+2]+$$
$$+\delta[j-1](i-j+1)\Theta[i-j+1](k-i+1)\Theta[k-i+1]\Big)\Big)-$$
$$-\gamma(k+1)(k+2)\Theta[k+2]+(k+1)\Theta[k+1]+\psi^2\gamma\Theta[k]=0 \tag{14}$$

Boundary conditions given in eq. (8) is transformed:

$$\Theta(1)=0 \tag{15}$$

Assuming that $\Theta(0) = a$, in this case, other terms can be obtained from transformed equation, eq. (14):

$$\Theta(2)=\frac{a\psi^2}{2(1+a\beta)},\ \Theta(3)=\frac{a\psi^2}{6\gamma(1+a\beta)}$$
$$\Theta(4)=-\frac{a\psi^2\left(-3-6a\beta-3a^2\beta^2-\gamma^2\psi^2+2a\gamma^2\beta\psi^2\right)}{24\gamma^2(1+a\beta)^3}$$
$$\Theta(5)=-\frac{a\psi^2\left(-6-12a\beta-6a^2\beta^2-\gamma^2\psi^2+4a\gamma^2\beta\psi^2\right)}{60\gamma^3(1+a\beta)^3} \tag{16}$$

and so on. Substitution all terms of the transformed function into eq. (13) for $H = 1$, the dimensionless temperature distribution along the fin:

$$\theta(\xi)=a+\frac{a\psi^2}{2(1+a\beta)}\xi^2+\frac{a\psi^2}{6\gamma(1+a\beta)}\xi^3-$$
$$-\frac{a\psi^2\left(-3-6a\beta-3a^2\beta^2-\gamma^2\psi^2+2a\gamma^2\beta\psi^2\right)}{24\gamma^2(1+a\beta)^3}\xi^4-$$
$$-\frac{a\psi^2\left(-6-12a\beta-6a^2\beta^2-\gamma^2\psi^2+4a\gamma^2\beta\psi^2\right)}{60\gamma^3(1+a\beta)^3}\xi^5+\ldots \tag{17}$$

where the unknown parameter a may be calculated by substituting the boundary condition given in eq. (7) into eq. (17). In the calculation of the unknown parameter a, finite number of terms is considered in eq. (17). Correctness of the approximate solution increases with increasing number of terms taken in eq. (17).

Fin efficiency

The ratio of real heat transfer to the maximum possible heat transfer that will happen when the overall fin is at the uniform base temperature, T_b, is defined as the fin efficiency. Fin efficiency can be represented in dimensionless form:

$$\eta = \frac{Q_{\text{actual}}}{Q_{\max}} = \frac{2(1+\beta)}{\psi^2\left(\gamma^2-1\right)}\frac{\mathrm{d}\theta}{\mathrm{d}\xi}\bigg|_{\xi=\gamma-1} \tag{18}$$

Results and discussion

The DTM provides an analytical approximate solution in terms of infinite series. The accuracy of the method depends on the number of terms taken in the infinite power series. In the calculations, it is seen that taking the first 12-15 terms in the infinite series gives good results. To show the validity of the DTM, the obtained results are compared by exact analytic results in the case of constant thermal conductivity, $\beta = 0$. The exact analytical solution of the governing energy equation for 1-D heat dissipation from the rectangular profile annular fins are given in many heat transfer textbooks in terms of Bessel functions. The non-dimensional temperature distribution is given in fig. 2. It is seen that a very good agreement is observed between the results of the DTM and the exact analytical method in the case of constant thermal conductivity.

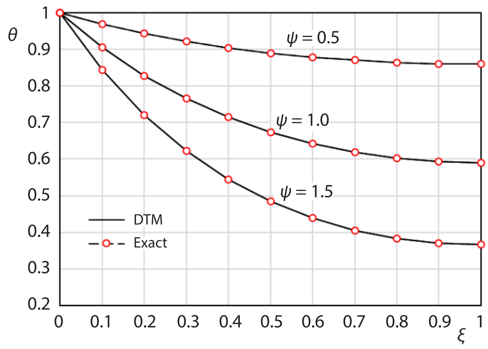

Figure 2. Non-dimensional temperature distribution in the case of $\beta = 0$, $\gamma = 2$

Results from DTM are also compared with numerical FDM results for different β values in tab. 2. The FDM analysis is carried out by using the finite differences MATLAB bvp4c code. The MATLAB's built-in function bvp4c solves the boundary value problems. A very good agreement is observed which confirms the accuracy of the DTM. The mean absolute difference between DTM and FDM results in the case of $\beta = -0.3$ and $\beta = 0.3$ are around

Table 2. The DTM and FDM results ($\psi = 1$ and $\gamma = 2$)

	$\beta = -0.3$		$\beta = 0$			$\beta = 0.3$	
ξ'	DTM	FDM	DTM	FDM	EXACT	DTM	FDM
0	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000
0.1	0.880303	0.880219	0.905818	0.905818	0.905818	0.922019	0.922019
0.2	0.786863	0.786778	0.828514	0.828514	0.828513	0.856586	0.856589
0.3	0.713435	0.713361	0.765365	0.765366	0.765365	0.802119	0.802123
0.4	0.655791	0.655728	0.714278	0.714279	0.714278	0.757349	0.757354
0.5	0.610974	0.610918	0.673622	0.673623	0.673622	0.721246	0.721251
0.6	0.576860	0.576809	0.642118	0.642119	0.642118	0.692969	0.692975
0.7	0.551899	0.551852	0.618758	0.618759	0.618758	0.671826	0.671833
0.8	0.534956	0.534911	0.602751	0.602752	0.602751	0.657250	0.657257
0.9	0.525202	0.525159	0.593481	0.593482	0.593481	0.648775	0.648782
1.0	0.522049	0.522007	0.590475	0.590476	0.590475	0.646022	0.646029

$5.4 \cdot 10^{-5}$ and $4.6 \cdot 10^{-6}$, respectively. The DTM results are nearly the same with FEM and exact analytical results in constant thermal conductivity case, $\beta = 0$.

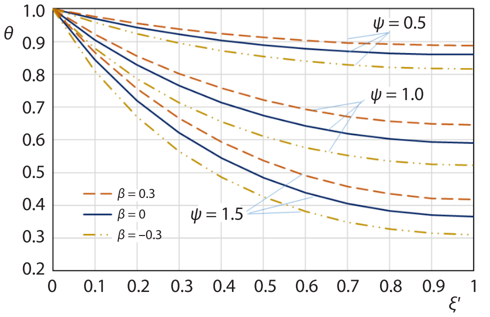

Figure 3. Non-dimensional temperature distribution in the case of $\gamma = 2$

For different values of β and ψ, the dimensionless temperature distribution along the fin is given in fig. 3. It is seen that the non-dimensional temperature gradient decreases along the fin with the increase in thermal conductivity parameter, β. The $\beta > 0$ means that the fin's material thermal conductivity decreases along the fin with the decrease in temperature. Consequently, the fin's inner thermal resistance increases as a result of the decrease in thermal conductivity, and therefore, the temperature gradient decreases along the fin. Conversely, $\beta < 0$ means that the thermal conductivity of the fin increases along the fin with the decreases in temperature. In the case of the non-dimensional thermo-geometric fin parameter $\psi = 1$, dimensionless temperatures at the fin tip for the $\beta = -0.3$, 0, and 0.3 values are 0.646, 0.591, and 0.522, respectively. It should also be expressed that, the non-dimensional temperature gradient increases along the fin with the increase of thermo-geometric fin parameter, ψ. The meaning of this increase is that the thermal conductivity of the fin decreases. In this case, the fin's internal conduction resistance increases, so the magnitude of the temperature gradient increases in the fin. For example, in the case of $\beta = 0.3$, dimensionless temperatures at the fin tip for the $\psi = 0.5$, 1.0, and 1.5 values are 0.888, 0.646, and 0.418, respectively.

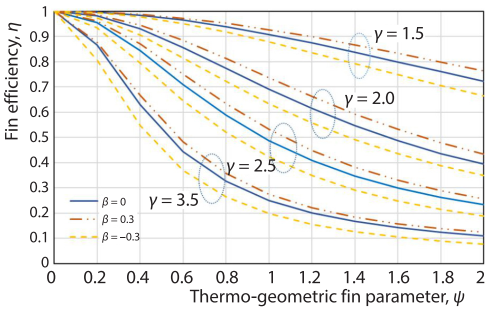

Figure 4. Variation of fin efficiency with ψ parameter

The variation of the fin efficiency with thermo-geometric fin parameter in the case of different values of the fin radii ratios and thermal conductivity parameters is shown in fig. 4. It is seen that the efficiency of the fin η decreases with the increase of thermo-geometric fin parameter ψ value for a specified fin radii ratio γ and β. For example, fin efficiency values in the case of fin radii ratio $\gamma = 2$ and $\beta = -0.3$ for $\psi = 0.2$, 0.6, 1.0, 1.4, and 1.8 are 0.974, 0.815, 0.634, 0.491, and 0.389, respectively. Also, the fin efficiency decreases with an increase in fin radii ratio for a specified ψ and β value. Fin efficiency values in the case of fin radii ratio $\psi = 1$ and $\beta = 0.3$ for $\gamma = 1.5$, 2.0, 0, 2.5, and 3.5 are 0.927, 0.736, 0.531, and 0.275, respectively. Besides, it is noticed that the fin efficiency is affected by the thermal conductivity parameter, β. For a specified fin radii ratio γ and thermo-geometric fin parameter ψ, the efficiency of the fin increases with increasing the β value. For example, fin efficiency values in the case of fin radii ratio $\beta = 2.0$ and $\psi = 0.8$ for $\beta = -0.3$, 0, and 0.3 are 0.722, 0.774, and 0.812, respectively.

Conclusion

The DTM has been used for the analyses of the annular fin by considering variable thermal conductivity. Dimensionless fin efficiency and temperature distribution are calculated to be dependent on dimensionless parameters, namely thermo-geometric fin parameter, fin radii ratio, and thermal conductivity parameter. The effects of dimensionless parameters on the fin efficiency and the temperature distribution are investigated and illustrated with figures. It has been found that dimensionless parameters have significant effects on fin efficiency and temperature distribution. The results calculated with DTM are also compared with the numerical results obtained from exact results on condition of constant thermal conductivity and FDM, and a very close agreement is obtained which validates the accuracy of DTM. Accordingly, DTM can be applied for this type of highly non-linear fin problems and non-linear engineering problems.

Nomenclature

A – area, [m^2]
h – heat transfer coefficient, [Wm^{-2}K^{-1}]
k – thermal conductivity, [Wm^{-1}K^{-1}]
r – radius, [m]
T – temperature, [°C]
t – fin thickness, [m]

Greek symbols

β – dimensionless thermal conductivity parameter
γ – radius ratio
η – fin efficiency
θ – dimensionless temperature
λ – a coefficient related with thermal conductivity
ξ – dimensionless co-ordinate
ξ' – normalized dimensionless co-ordinate
ψ – dimensionless thermo-geometric fin parameter

Subscripts

a – ambient or surrounding
b – base
i – inner
o – outer
s – surface

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