PERFORMANCE INVESTIGATION OF A TWO-STAGE THERMOELECTRIC COOLER WITH INHOMOGENEOUS MATERIALS

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A novel model of two-stage thermoelectric cooler with inhomogeneous thermal conductivity in steady-state operating condition is established. The modification of the constant properties model allows controlling the distribution of Joule heat. Considering internal irreversibilities of the thermoelectric cooler, expressions for the cooling capacity, COP, and exergy efficiency are derived. By utilizing numerical methods, the temperature profile along the thermoelectric legs is presented. The optimal operating regions are explored. The COP vs. cooling capacity describing optimal operating regions in inhomogeneity materials are plotted. Meanwhile, the influence of the main parameters such as the variation of thermal conductivity distribution, cold-end temperature and the number of thermoelectric modules on the cooling performance is discussed in detail. Results indicate that the cooling capacity, COP and exergy efficiency are improved compared to those of homogeneous two-stage thermoelectric coolers when an appropriate inhomogeneous property parameter is applied. The work can provide guidance on design of actual two-stage thermoelectric coolers with inhomogeneous materials.

Key words: *two-stage thermoelectric cooler, inhomogeneous thermal conductivity*

Introduction

As the world strives to pay great attention environment protection, the applications of thermoelectric materials in refrigeration are attracting significant attention [1, 2]. Thermoelectric material is a key enabler for sustainable development, which can realize the direct conversion of thermal energy and electric energy. Compared with traditional refrigeration devices, thermoelectric coolers (TEC) have drawn attention for its absence of moving parts, silence in operation, small-scale applications, and continuous stability [3, 4]. Moreover, no refrigerant is required [5, 6]. Due to their advantages, TEC have great appeal in the fields of national defense, aerospace, medical, agriculture and microelectronic [7, 8]. For example, TEC provide a potential for temperature stabilization of semiconductor lasers [1]. However, its main problem is that the value of the dimensionless thermoelectric figure-of-merit (ZT) is relatively low [9-11]. Solving this problem of finding materials with high ZT has always been a challenge [2, 12]. Efforts have been made to investigate some approaches which can reinforce the cooling performance. Several strategies have been mainly focused on improving the geometrical structure of thermoelectric systems and the development of more efficient materials. Various forms of the thermoelectric devices and different types of the thermoelectric materials are investigated to offer better cooling performance.

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Sharma *et al.* [13] not only confirmed the most appropriate number of the thermoelectric elements for multi-stage TEC, but also analyzed the advantages compared with single TEC. Energy, exergy and exergoeconomic are considered as target functions of the performance evaluation for single- and two-stage thermoelectric device in different mode were compared by Nami *et al.* [10]. On the other hand, concerns about inhomogeneous or graded thermoelectric materials are growing. Some studies have found that the asymmetry of Joule heat dissipation has a significant effect on thermoelectric devices. A new model, a single-stage thermoelectric cooling with inhomogeneous materials, was proposed by Lu *et al.* [14]. In the model, it was founded that optimizing the linear correlation coefficient of thermal conductivity play a significant role to improve the maximum cooling capacity and temperature difference. Bian *et al.* [15] offered the graded material and did numerical optimization. The results proved that it could achieve a 27% cooling enhancement compared to the best homogeneous material. The performance of thermoelectric coolers with the spatial- and temperature-dependent properties for a given the graded carrier concentration was investigated by Hu *et al.* [16]. They found that the asymmetric distribution of Joule heat is an effective way to enhance the performance of thermoelectric coolers. Huang *et al.* [17] studied a new type of parallel two-stage thermoelectric cooler (TTEC) by utilizing the space-dependent electrical conductivity of inhomogeneous materials. It was found that the spatial variation of conductivity would enhance the corresponding cooling capacity and COP. Lam *et al.* [18] discussed the effect of pulse current on the properties of tapered non-uniform thermoelectric materials, and obtained the thermoelectric temperature distribution profiles.

However, no open literature has investigated a TTEC constructed by inhomogeneous material with the spatial-dependent thermal conductivity. In order to break the symmetrical distribution of Joule heat and reinforce the cooling performance of thermoelectricity, a TTEC model with inhomogeneous thermal conductivity is theoretically established.

Model description

Schematic diagrams of the TTEC considering inhomogeneous thermal conductivity in the electrically series configuration, which consists of *p*-type and *n*-type semiconductor elements, has been illustrated by fig. 1. Here, the colder and hotter stage with N_c and N_h numbers of thermoelectric elements are cascaded with $N = N_c + N_h$ being the total number thermocouples. The $r = N_h/N_c$ is the ratio of number of modules between the two-stages, respectively. The T_{cc} and T_{ch} denote the temperatures of the cold-side and hot-side of colder end, T_{hc} and T_{hh} are the temperatures of the cold-side and hot-side of hotter end, respectively.

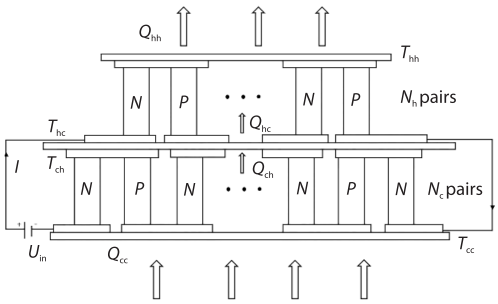

Figure 1. Two-stage series TEC

Certain hypotheses were adopted in modelling of TTEC systems [16, 19-23]:

- The TTEC cooler is operated at a steady-state condition.
- The Thomson effect in the thermoelectric devices is assumed negligible.
- Heat losses generated by radiation and convection are not considered.
- The seebeck coefficient, thermal conductance and electrical resistance are independent of temperature.
- Only a 1-D model that the heat flows through the element along the length direction of thermoelectric leg is considered.

Spatial-dependent thermal conductivity

In the steady-state condition, Domenicali's equation [24] for the energy balance and the definition of heat flow $q(x)$ are formulated here in steady-state:

$$\frac{\mathrm{d}}{\mathrm{d}x}\left[\lambda(x)\frac{\mathrm{d}T(x)}{\mathrm{d}x}\right]=-\frac{I^2\rho(x)}{A^2}+\frac{I}{A}T(x)\frac{\mathrm{d}\alpha(x)}{\mathrm{d}x} \tag{1}$$

$$q(x)=\alpha(x)IT(x)-\lambda(x)\frac{\mathrm{d}T(x)}{\mathrm{d}x} \tag{2}$$

where A is the cross-sectional area, α – the seebeck coefficient, x – the distance from the cold end, $T(x)$ – the temperature profile, and I – the electric current density.

The study utilizes explicit spatial dependent thermal conductivity with an aim to improve cooling performance. For simplicity, it is assumed that seebeck coefficient and resistivity are independent of temperature and space, no thermal or electrical contact resistances, and no heat losses. A novel model is proposed that the inhomogeneous materials with the power function form $\lambda(x) = \lambda_0(x/L)^c$. Here, c is a power exponent and λ_0 is the initial thermal conductivity at hot end. Note that if $x = 0$ implies $\lambda(0) = 0$, there is no physical meaning, and we may exclude this possibility:

$$\frac{\mathrm{d}^2T}{\mathrm{d}x^2}+\frac{cx}{L}\frac{\mathrm{d}T}{\mathrm{d}x}=-\frac{I^2\rho}{\lambda(x)A^2} \tag{3}$$

Based on Dirichlet boundary condition, the cold ($x = 0$) and hot junction ($x = L$) of the TTEC temperatures are constant. The boundary conditions are expressed:

$$\begin{aligned}&x=0,\quad T=T_\mathrm{c}\\&x=L,\quad T=T_\mathrm{h}\end{aligned} \tag{4}$$

where is the length of the model. Considering the boundary conditions, eq. (4) yields the expression for the differential of temperature with respect to x with a finite $\lambda(x)$:

$$\frac{\mathrm{d}T}{\mathrm{d}x}=-\frac{I^2\rho x}{\lambda_0\left(\dfrac{x}{L}\right)^c A^2}+(1-c)\frac{T_\mathrm{h}-T_\mathrm{c}}{x^cL^{1-c}}+\frac{I^2R(1-c)}{\lambda_0\left(\dfrac{x}{L}\right)^c A(2-c)} \tag{5}$$

The distribution of temperature is determined:

$$T(x)=-\frac{I^2\rho L^c x^{2-c}}{(2-c)\lambda_0A^2}+\left[\frac{T_\mathrm{h}-T_\mathrm{c}}{L^{1-c}}+\frac{I^2R}{L^{1-c}\lambda_0A(2-c)}\right]x^{1-c} \tag{6}$$

Substituting eq. (5) into eq. (2), the absorption and released of heat are calculated:

$$Q_\mathrm{c}=\alpha IT_\mathrm{c}-K\beta(T_\mathrm{h}-T_\mathrm{c})-\omega RI^2 \tag{7}$$

$$Q_\mathrm{h}=\alpha IT_\mathrm{h}-K\beta(T_\mathrm{h}-T_\mathrm{c})+(1-\omega)RI^2 \tag{8}$$

where $K = \lambda_0 A/L$ denotes the thermal conductivity, $R = L\rho/A$ – the electrical resistance, $\beta = 1 - c$ – the defined as the normalized thermal conducted by presuming a homogeneous material with λ_0 , $\omega = (1 - c)/(2 - c)$ – the distribution of the Joule heat, which flows to the cold end, and $1 - \omega$ – the partial Joule heat flowing to the hot end. In the generalized model,

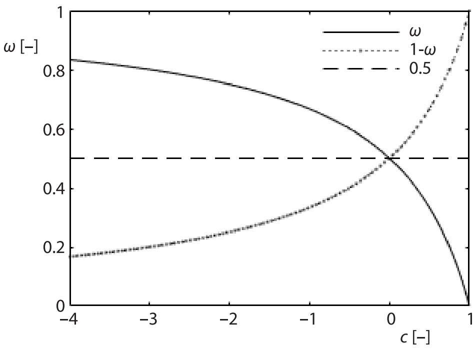

Figure 2. Variation of Joule heat flow to hot or cold end *vs. c*

$c \le 1$ is required. Note that for $c = 0$, this situation presents that the dumping of Joule heat into each end is symmetrical. For $c = 1$, $\omega = 0$ is obtained, which implies the whole of Joule heat is dumped in the hot end, and when $c \to \infty$, the Joule heat flowing to the hot end is zero. As shown by fig. 2, the value of ω in the model with inhomogeneous materials lies between 0 and 1. What is more, it is clear that ω is a monotonically increasing function of c. Whether c increases or decreases, it would increase the unevenness of Joule heat distribution.

Analysis of TTEC

For a TTEC considering inhomogeneous thermal conductivity, the heat absorbed at the cold side of the colder stage, Q_{cc}, the heat released at the hot side of the hotter stage, Q_{hh}, the heat released at the hot side of colder end, Q_{ch}, and the heat absorbed at the cold side of hotter end, Q_{hc}, based on the theory of non-equilibrium thermodynamics:

$$Q_{cc} = N_c\left[\alpha_c I_c T_{cc} - K_c\beta_c(T_{ch} - T_{cc}) - \omega_c R_c I_c^2\right] \tag{9}$$

$$Q_{ch} = N_c[\alpha_c I_c T_{ch} - K_c\beta_c(T_{ch} - T_{cc}) + (1-\omega_c)R_c I_c^2] \tag{10}$$

$$Q_{hc} = N_h\left[\alpha_h I_h T_{hc} - K_h\beta_h(T_{hh} - T_{hc}) - \omega_h R_h I_h^2\right] \tag{11}$$

$$Q_{hh} = N_h[\alpha_h I_h T_{hh} - K_h\beta_h(T_{hh} - T_{hc}) + (1-\omega_h)R_h I_h^2] \tag{12}$$

Assume that the materials of both two-stages possess the same properties:

$$\alpha_c = \alpha_h = \alpha,\ R_c = R_h = R,\ K_c = K_h = K,\ c_c = c_h = c \tag{13}$$

The following conditions are well observed in the electrically series configuration:

$$U_{in} = U_c + U_h,\ I_c = I_h = I \tag{14}$$

Without regard to the heat leakages, the thermal current balance equation is expressed:

$$Q_{ch} = Q_{hc} \tag{15}$$

One can presume that there is an intermediate junction temperature, T_m. Hence, $T_{ch} = T_{hc} = T_m$, the value of T_m comes out:

$$T_m = \frac{K\beta(N_h T_{hh} + N_c T_{cc}) + [N_c(1-\omega) + N_h\omega]RI^2}{\alpha I(N_h - N_c) + K(N_c T_{cc} + N_h T_{hh})} \tag{16}$$

The voltage and power output of the colder stage are calculated:

$$U_c = \frac{Q_{hc} - Q_{cc}}{I} = \alpha N_c(T_m - T_{cc}) + N_c RI \tag{17}$$

$$P_c = (Q_{hc} - Q_{cc}) = \alpha I N_c(T_m - T_{cc}) + N_c RI^2 \tag{18}$$

The voltage and power output of the hotter stage are written:

$$U_h = \frac{Q_{hh} - Q_{hc}}{I} = \alpha N_h (T_{hh} - T_m) + N_h RI \tag{19}$$

$$P_h = Q_{hh} - Q_{hc} = \alpha I N_h (T_{hh} - T_m) + N_h RI^2 \tag{20}$$

The voltage V_{in} and power output P_{in} of the TTEC model are, respectively:

$$U_{in} = \frac{Q_{hh} - Q_{cc}}{I} = \alpha \left[N_c (T_m - T_{cc}) + N_h (T_{hh} - T_m) \right] + (N_h + N_c) RI \tag{21}$$

$$P_{in} = (Q_{hh} - Q_{cc}) = \alpha I [N_c (T_m - T_{cc}) + N_h (T_{hh} - T_m)] + (N_h + N_c) RI^2 \tag{22}$$

The COP exergy and exergy efficiency, ε, of the TTEC model are determined, respectively [10, 25]:

$$COP = \frac{Q_{cc}}{P_{in}} \tag{23}$$

$$Ex_{Qcc} = Q_{cc} \frac{T_o}{T_{cc}} - 1 \tag{24}$$

$$\varepsilon_{TTEC} = \frac{Ex_{Qcc}}{P_{in}} \tag{25}$$

Input parameters for numerical calculation are summarized in tab. 1 [14].

Table 1. Parameters used in modelling

Parameters	Value
Seebeck coefficient, α [$\mu V^{-1}K^{-1}$]	200
Electrical resistivity, R [Ωm^{-1}]	10^{-5}
Thermal conductivity, λ_0 [$Wm^{-1}K^{-1}$]	1.7
Length, L [m]	$5 \cdot 10^{-3}$
Area, A [m^2]	$4 \cdot 10^{-6}$
Cold end temperature, T_{cc} [K]	260
Hot end temperature, T_{hh} [K]	300
Ambient temperature, T_0 [K]	298.15
Number of the colder stage thermocouples, N_c	15
Number of the hotter stage thermocouples, N_h	15

Validation

The validation of the present model is executed by using the numerical and experimental results of Liu *et al*. [26] under identical operating conditions, where its hot end is fixed at 343.2 K and cooling capacity is set at 5.5 W. All the parameters and input data are identical with those reported by Liu *et al*. [26]. Figure 3 illustrates a comparison of the present model results and the numerical and experimental results of Liu *et al.* [26] for COP of the TTEC varying with current. A fairly good consistency between the model predictions and experimental results is observed, which means that the 1-D model adopted here can provide reliability in the calculations.

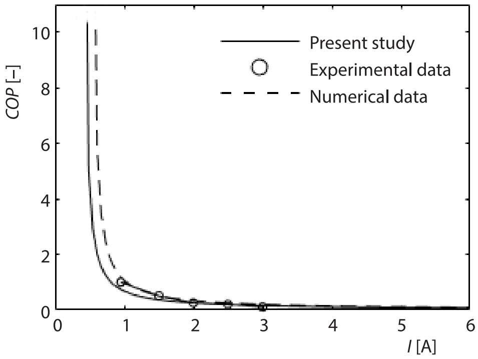

Figure 3. Comparison between the COP of the present simulation results and the experimental data

Results and discussions

Effect of power exponent c

The power exponent c plays a significant role in affecting the performance of TTEC. The $I – Q_{cc}$, $I – COP$, and $I – \varepsilon$ curves of the two-stage TEC with the power exponent c = –0.51, 0, 0.31, and 0.55 (in order to ensure the step of ω is 0.1) are shown in fig. 4. It is clearly seen from all curves that there exists a maximum cooling capacity and a corresponding current for a given power exponent c. Increasing current improves cooling capacity and consumed electrical power. However, cooling capacity going up is dominant up to the optimal region of current, and then, the consumed power is overcoming, which causes a decrease in cooling capacity. What is more, for different power exponent c, the maximum cooling capacity and corresponding current will be different. Obviously, the peak of the cooling capacity shifts towards right with power exponent c going up. Hence, for better power exponent c, the maximum cooling capacity is obtained at higher current values. With an increase in power exponent c, more Joule heat has poured at the hot end, and consequently, the cooling capacity increases.

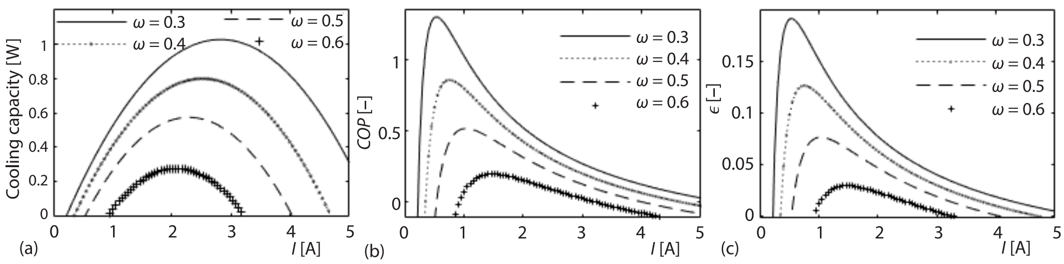

Figure 4. Cooling capacity (a), COP (b), and the exergy efficiency (c) *vs.* the electric current for different value *c*

In order to illustrate the effect of the Joule heat distribution parameter on the performance of the TTEC more clearly, the curves of COP and exergy efficiency varying with current are plotted. As compared in figs. 4(b) and 4(c), COP and exergy efficiency first go up then decrease after attaining an optimum value. For example, the maximum values of COP and exergy efficiency are 1.26 and 0.19 for c = 0.55, which is achieved at currents of 0.51 A. The maximum of COP and exergy efficiency decrease as the value of power exponent decreasing. It is obviously shown that COP and exergy efficiency are highest for c = 0.55 and lowest for c = –0.51. What is more, the maximum COP and exergy efficiency appear at relatively low current in the modelling of TTEC. Fact have been proved that exergy destruction keeps rising with increasement of current as the higher current results in more irreversibilities in the model.

As can be seen from fig. 4, the maximum cooling capacity, COP and exergy efficiency of an inhomogeneous TTEC are 38%, 57%, and 52% higher than those of the homogeneous TTEC for inhomogeneous parameter ω = 0.4. We conclude that the Joule heat flowing to the hot end is enhanced when the thermal conductivity in the vicinity of the hot end is larger than that in the vicinity of the cold end, which leads to a larger fraction of the Joule heat flowing towards the hot end, thus enhancing the cooling performance. The physical explanation for this phenomenon is that the existence of explicit spatial dependent thermal conductivity changes the space conversion syndrome.

The relation between the COP and cooling capacity

Figure 5 describes the curves of the cooling capacity Q_{cc} as a function of the COP for different values of power exponent c. The closed curves going through the origin are presented. There is a negative slope arc segment on which Q_{cc} goes down as COP rises. The optimal regions are situated in the parts of the Q_{cc} – COP curves with negative slope, and we can obtain a maximum cooling capacity $Q_{cc,max}$ whose corresponding coefficient of performance is COP_m and a maximum coefficient of performance is COP_{max} whose corresponding cooling capacity is $Q_{cc,m}$. To make the TTEC operate at the optimal state, the optimal regions of cooling capacity and COP are determined:

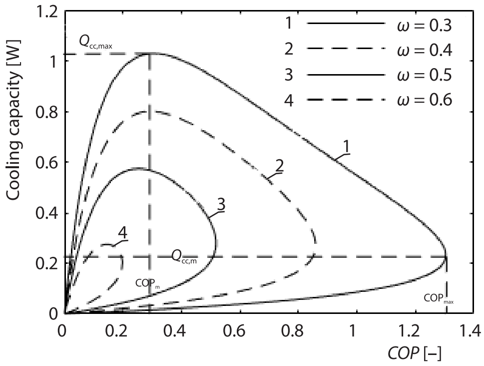

Figure 5. Cooling capacity Q_{cc} *vs.* COP for different value ω

$$Q_{cc,m} \le Q_{cc} \le Q_{cc,max} \tag{26}$$

$$COP_m \le COP \le COP_{max} \tag{27}$$

It is seen clearly that $Q_{cc,max}$ and COP_m determine upper bounds of the cooling capacity and COP, while $Q_{cc,m}$ and COP_m offer the lower boundaries of the optimized the cooling capacity and COP. What is more, it is worth noting that the enhancement of performance is more remarkable with the value of ω decreasing.

Effects of cold end temperature

Figure 6 shows that the cooling capacity, COP, and the exergy efficiency of the two different TTEC designs with three cold end temperatures of 240 K, 250 K, and 260 K. For inhomogeneous materials, $\omega = 0.3$ is taken as an example, while $\omega = 0.5$ is homogeneous materials. Increment in the cold end temperature improves the cooling capacity, COP, and exergy efficiency for both homogeneous and inhomogeneous TTEC. The reason is that the cooling capacity consumes more power by reducing the cold end temperature in a constant hot end temperature, which brings about lower values of COP, and exergy efficiency. For all temperature differences, the cooling capacity, COP, and exergy efficiency of inhomogeneous TTEC are higher than those of homogeneous TTEC. Moreover, based on the changing trend of the three parameters, the variation of cold end temperature has a greater impact on homogeneous TTEC.

Effects of thermocouples configuration on the model

The cooling capacity Q_{cc} is plotted as a function of the ratio of number of modules in the cold stage to the hot stage $r = N_h/N_c$ for various values of power exponent c in fig. 7(a). The trend of the cooling capacity curve is a parabola *vs.* the ratio. There exists an optimum ratio, which contributes to the maximum cooling capacity. Moreover, comparing three curves, it is clearly seen that the cooling capacity are 0.34 for $\omega = 0.4$, 0.22 for $\omega = 0.5$, and 0.12

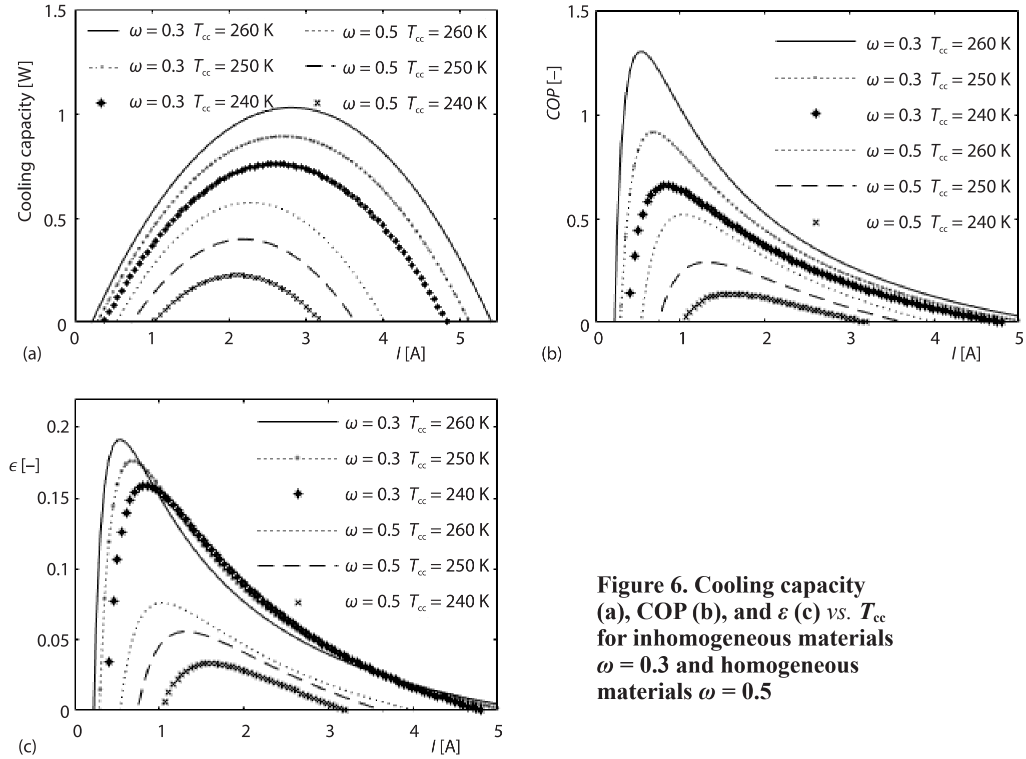

Figure 6. Cooling capacity (a), COP (b), and ε (c) *vs.* T_{cc} for inhomogeneous materials ω = 0.3 and homogeneous materials ω = 0.5

for ω = 0.6. It is obvious that a lower dimensionless factor gives higher cooling capacity. The variation of COP and exergy efficiency are shown in figs, 7(b) and 7(c). These curves signify that two functions go up in the initial r till optimal point and thereafter get reduced considerably. That is to say, the ratio of number of modules between the two-stages r has a significant effect on the design of TTEC.

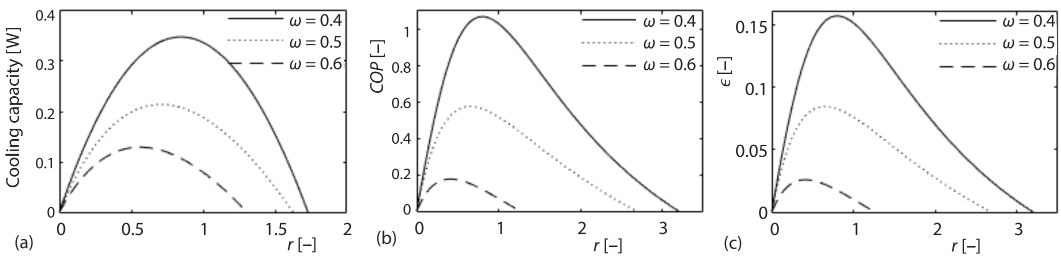

Figure 7. Cooling capacity (a), COP (b), and ε (c) *vs.* r for different value ω

Conclusion

The 1-D model of a TTEC cooler constructed by inhomogeneous material with the spatial-dependent thermal conductivity is developed. The expressions for temperature profile in the thermoelectric material is formulated. In light of a tradeoff between the cooling capacity and COP, the optimum operating regions are determined. The improvement of thermoelectric cooling confirms a fact that the inhomogeneous thermal conductivity conducted spatial dependence form contributes to the asymmetric dissipation of Joule heat when power exponent c is located in the positive region. Thermal rectification can be realized through the modification of the constant properties model which can improve cooling capacity, COP and exergy efficiency. In fact, the asymmetric flow of Joule heat in functionally graded thermoelectric materials can be achieved by varying carrier concentrations. The present analysis lays a foundation for the deeper investigation of practical multi-stage thermoelectric refrigeration devices made of inhomogeneous materials.

Nomenclature

A – area, [m^2]
L – length, [m]
N_c – number of the colder stage thermocouples
N_h – number of the hotter stage thermocouples
R – electrical resistivity, [Ωm^{-1}]
T_{cc} – cold end temperature, [K]
T_{hh} – hot end temperature, [K]
T_0 – ambient temperature, [K]

Greek symbols

α – seebeck coefficient, [μV^{-1}K^{-1}]
β – normalized thermal conducted, [–]
λ_0 – thermal conductivity, [Wm^{-1}K^{-1}]
ω – distribution of the Joule heat, [–]

Subscripts

c – colder stage
h – hotter stage
cc – cold side of colder stage
ch – hot side of colder stage
hc – cold side of hotter stage
hh – hot side of hotter stage

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