# SEEING WITH A SINGLE SCALE IS ALWAYS UNBELIEVING
## From magic to two-scale fractal

by

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*A same phenomenon can lead to debating laws when observed on different scales, two-scale observation is needed for a discontinuous problem, the large scale matches with the continuum assumption, while the smaller one is to reveal the discontinuous properties. The basic properties of two-scale fractal are discussed in a layman way.*

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### Introduction

We all are familiar with one-scale observations, for example, we can say a clothing is smooth, this is of course right on a large scale, but it is not smooth at all on a much smaller scale. If we study the clothing's friction property, we have to use the two-scale observation, the large-scale observation leads to a macro friction law like Coulomb friction law, which cannot depict the effect of the fabric's porous structure on the friction force. When we study the friction property on a fiber's scale, a fractal friction law can be obtained [1].

To show the importance of two-scale concept, we consider the Moon's motion on different scales. Its motion is determinative when observed on the Earth or on the Sun, but if we observe the Moon at an infinite far place, Heisenberg-like uncertainty principle is found, that is the Moon's motion becomes totally probabilistic [2]. Similarly when we obverse an extragalactic celestial object, the two-scale concept has to be adopted to elucidate its physical law.

### Seeing is not always believing

The same phenomenon can lead to debating theories or laws when measured on different scales, for example, there are debating laws for the metabolic rate in biology [3]. If a cell is considered as a continuum, we have Rubner's 2/3 law, if we consider the cell surface is not smooth, we obtain the Kleiber's 3/4 law [4], and all disputes stopped when a fractal cell is adopted [5]. The well-known wave-particle dualism arises also in different scale observations.

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A magician is good at this two-scale skill. Our eye can recognize 24 photos per second, a faster motion cannot be determinately identified by eyes. The magician can move very fast an object, leading to a magic effect. Anything will become magic and inconceivable when we obverse a phenomenon on a large scale while it arises in a smaller scale observation. For example, the turbulence in fluid mechanics might arise initially in a molecule's perturbation, any a continuum model based on Navier-Stokes equations cannot give a closed turbulence model and artificial assumptions have to be made to make the model closed. The two-scale concept might be a possible mathematical tool to solving the turbulence model, the large scale leads to the Navier-Stokes equations, while the smaller scale, i.e., the molecule scale leads to a fractal turbulence model.

## Dimension or scale is everything

A 2-D model can never describe the 3-D properties, and a 4-D eye can see all things inside a house from its outside, which is considered impossible in view of the 3-D world. The dimension or the scale is everything for the all physical laws. There is a science fiction, saying there was a 4-D alien, who stole a heart from a man without dissection. This phenomenon is unbelievable in a 3-D space, but it can happen in a 4-D space. A physical law on a large scale becomes invalid to describe any phenomena arising in a much smaller observation. The grand unified field theory should use different scales to unify strong, electroweak, and gravitational forces together.

## Two-scale approach

All physical laws are scale-dependent, especially for nanoscience, which bridge the Newton's mechanics and quantum mechanics, the former is on a macro scale, while the latter is on the quantum scale. Between the two scales, there is a nanoscale, which leads to quantum-like property beside conservation laws.

The tracking of a single molecule through a membrane or a porous medium has a coastline-like property. If we use a scale of space to measure the length of a coastline between two points, the result depends upon the scale. A smaller scale results in a longer length of the coastline, and when the scale tends to zero, it becomes infinity [6].

When a particle, saying a small molecular dye, a quantum dot, or a nanoparticle, goes through a membrane or a porous medium, its trajectory is similar to a coastline, its average velocity depends upon its size and the porous structure . For a same porous medium, a particle transmission velocity scales as [7]:

$$u \propto d^{\alpha} \tag{1}$$

$$u \propto t^{\beta} \tag{2}$$

where $u$ is the average velocity, $d$ – the particle diameter, $t$ – the measured time scale, $\alpha$ and $\beta$ – are two-scale fractal dimensions in moving direction and time respectively. When the measured time scale is less than $u/d$, uncertainty arises.

Two-scale method is a new approach to observation of a same phenomenon using two different scales [8-11]. To understand the two-scale concept, we consider a drop of red ink in water [12]. Water is a continuum if a large scale is used, and the motion of the red ink follows laws in fluid mechanics, however, fluid mechanics cannot elucidate the mechanism of the red ink's diffusion. We, therefore, need a smaller scale, saying a molecule's scale. Under such a small scale, water becomes discontinuous, and fractal calculus has to be adopted [13-22].

With proper understanding of the two-scale mathematics, or the fractal calculus, we can understand many phenomena arising in discontinuous problems.

This special issue focuses on the study of phenomena arising in unsmooth boundary or unsmooth space, where the continuum assumption becomes invalid and fractal calculus has to be adopted. Many experiments have revealed that nanoscale surface morphology can greatly enhance mass, energy and charge transmission through the unsmooth boundary, hierarchical porous medium behaves excellently in thermal property for advanced applications, and nanoparticles in a fluid can enhance remarkably the thermal property of micro/nano devices.

## References

[1] Kong, H. Y., He, J.-H., A Novel Friction Law, *Thermal Science, 16* (2012), 5, pp. 1529-1533
[2] He, J.-H., Frontier of Modern Textile Engineering and Short Remarks on Some Topics in Physics, *International Journal of Nonlinear Sciences and Numerical Simulation, 11* (2010), 7, pp. 555-563
[3] Riisgard, H. U., No Foundation of a "3/4 Power Scaling Law" for Respiration in Biology, *Ecology Letters, 1* (1998), 2, pp. 71-73
[4] Li, X. X., He, J.-H., Along the Evolution Process Kleiber's 3/4 Law Makes Way for Rubner's Surface Law: A Fractal Approach, *Fractals, 27* (2019), 2, 1950015
[5] Li, L. J., Thermal Therapy for Eye Diseases, *Thermal Science, 24* (2020), 4, pp. 2319-2324
[6] Mandelbrot, B. B., The Fractal Geometry of Nature, W. H. Freeman and Company, USA, 1982
[7] He, J.-H., Fractal Calculus And Its Geometrical Explanation, *Results in Physics, 10* (2018), Sept., pp. 272-276
[8] Ain, Q. T., He, J.-H., On Two-Scale Dimension and its Application, *Thermal Science, 23* (2019), 3B, pp. 1707-1712
[9] He, J.-H., Ji, F. Y., Two-Scale Mathematics and Fractional Calculus for Thermodynamics, *Thermal Science, 23* (2019), 4, pp. 2131-2133
[10] He, J.-H., Ain, Q. T., New Promises and Future Challenges of Fractal Calculus: From Two-Scale Thermodynamics to Fractal Variational Principle, *Thermal Science, 24* (2020), 2A, pp. 659-681
[11] He, J.-H., Thermal Science for the Real World: Reality and Challenge, *Thermal Science, 24* (2020), 4, pp. 2289-2294
[12] Zhou, C.-J., *et al.*, Highly Selective Penetration of Red Ink in a Saline Water, *Thermal Science, 23* (2019), 4, pp. 2265-2270
[13] Liu, H.-Y., *et al.*, A Fractal Rate Model for Adsorption Kinetics at Solid/Solution Interface, *Thermal Science, 23* (2019), 4, pp. 2477-2480
[14] Wang, Q. L., *et al.*, Fractal Calculus and its Application to Explanation of Biomechanism of Polar Hairs (Vol. 26, 1850086, 2018), *Fractals, 27* (2019), 5, 1992001
[15] Wang, Q. L., *et al.*, Fractal Calculus and its Application to Explanation of Biomechanism of Polar Hairs, *Fractals, 26* (2018), 6, 1850086
[16] He, J.-H. A Fractal Variational Theory for One-Dimensional Compressible Flow in a Microgravity Space, *Fractals, 28* (2020), 2, 2050024
[17] He, J.-H., A Simple Approach to One-Dimensional Convection-Diffusion Equation and its Fractional Modification for E Reaction Arising in Rotating Disk Electrodes, *Journal of Electroanalytical Chemistry, 854* (2019), 113565
[18] Ji, F. Y., *et al.*, A Fractal Boussinesq Equation for Nonlinear Transverse Vibration of a Nanofiber-Reinforced Concrete Pillar, *Applied Mathematical Modelling, 82* (2020), June, pp. 437-448
[19] Shen, Y., He, J.-H., Variational Principle for a Generalized Kdv Equation in a Fractal Space, *Fractals, 28* (2020), 4, 2050069
[20] Li, X. J., *et al.*, A Fractal Two-Phase Flow Model for the Fiber Motion in a Polymer Filling Process, *Fractals,* On-line first, https://doi.org/10.1142/S0218348X20500930, 2020
[21] Xu, L. Y., *et al.*, Detection of Cigarette Smoke Using a Fiber Membrane Filmed with Carbon Nanoparticles and a Fractal Current Law, *Thermal Science, 24* (2020), 4, pp. 2469-2474
[22] Yang, Z. P., *et al.*, A Fractal Model for Pressure Drop Through a Cigarette Filter, *Thermal Science, 24* (2020), 4, pp. 2653-2659

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