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COMMENTS ON "A COMPREHENSIVE NOTE ON THERMALLY STRATIFIED FLOW AND NON-FOURIER HEAT FLUX THEORY"

by

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The $\theta(\eta)$ equation is invalid.

Khan *et al*. [1] presented a comprehensive note on theory of non-Fourier heat flux and thermally stratified flow. The researchers considered Cattaneo-Christov heat flux theory. They presented the equation of energy, eq. (10) in [1] as:

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial r}+\lambda_1\left[\begin{array}{c}u^2\dfrac{\partial^2 T}{\partial x^2}+v^2\dfrac{\partial^2 T}{\partial r^2}+2uv\dfrac{\partial^2 T}{\partial x\partial r}+u\dfrac{\partial u}{\partial x}\dfrac{\partial T}{\partial x}+\\ +u\dfrac{\partial v}{\partial x}\dfrac{\partial T}{\partial r}+v\dfrac{\partial u}{\partial r}\dfrac{\partial T}{\partial x}+v\dfrac{\partial v}{\partial r}\dfrac{\partial T}{\partial r}\end{array}\right]=\frac{1}{\rho c_p r}\frac{\partial}{\partial r}\left[k(T)r\frac{\partial T}{\partial r}\right] \tag{1}$$

From eq. (1), T depends on x, r.

Khan *et al*. [1] expressed the similarity variable (η), eq. (4) in [1]:

$$\eta=\sqrt{\frac{u_0}{\nu l}}\left(\frac{r^2-R^2}{2R}\right) \tag{2}$$

Khan *et al*. [1] expressed the temperature $\theta(\eta)$, eq. (13) in [1]:

$$\theta(\eta)=\frac{T-T_\infty}{T_w-T_0} \tag{3}$$

From eq. (2), η depends on r only. From eq. (3), RHS $[T(x,r)-T_\infty]/(T_w-T_0)$ depends on x, r whereas $\theta(\eta)$ depends on r only. Therefore, eq. (3) is invalid.

Pantokratoras [2-6] elucidated that the $\theta(\eta)$ equation is invalid. He presented in [6] the correct η equation in Minkowycz and Sparrow [7]. The correct η equation can also be found in the recent paper in this journal by Hussanan *et al*. [8], eq. (16) in [8].

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In 2021, Awad [9-12] elucidated that the $\theta(\eta)$ equation is invalid.

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