THERMAL ANALYSIS AND PARAMETRIC OPTIMIZATION OF PLATE FIN HEAT SINKS UNDER FORCED AIR CONVECTION

by

Zulfiqar KHATTAK[a*] ***and Hafiz Muhammad ALI***[b]

[a] Department of Engineering, COMSATS University, Islamabad, Pakistan
[b] Mechanical Engineering Department, King Fahd University of Petroleum and Minerals, Dhahran, Saudi Arabia

Original scientific paper
https://doi.org/10.2298/TSCI201227081K

Heat dissipation is becoming more and more challenging with the preface of new electronic components having staggering heat generation levels. Present day solutions should have optimized outcomes with reference to the heat sink scenarios. The experimental and theoretical results for plate type heat sink based on mathematical models have been presented in the first part of the paper. Then the parametric optimization (topology optimization) of plate type heat sink using Levenberg-Marquardt technique employed in the COMSOL Multiphysics® software is discussed. Thermal resistance of heat sink is taken as objective function against the variable length in a predefined range. Single as well as multi-parametric optimization of plate type heat sink is reported in the context of pressure drop and air velocity (Reynolds number) inside the tunnel. The results reported are compared with the numerical modeled data and experimental investigation establish the conformity of results for applied usage. Mutual reimbursements of greater heat dissipation with minimum flow rates are confidently achievable through balanced, heat sink geometry as evident by the presented simulation outcome. About 12% enhancement in pressure drop and up to 51% improvement in thermal resistance is reported for the optimized plate fin heat sink as per data manifested.

Key words: *plate-fin heat sink, COMSOL Multiphysics®, objective function, heat transfer, Levenberg-Marquardt*

Introduction

In present era, the cooling of electronic chips to have optimum performance is becoming more and more challenging. Various heat dissipation solutions in the form of heat sinks are being presented with innovative design, having better metallurgical options. Most widely accepted heat sink designs are plate and pin fin heat sink with aluminum as the feasible material choice.

Typically, performance augmentation of heat sinks attributes to lucid parameters in the domain of pumping power, sizing, cost, and ease of embedding in particular space. To have futuristic performance goals achievement, the optimization of available heat sink geometries is the utmost prerequisite. The optimization of thermal design can produce the desired results of less size and mass, better heat dissipation capabilities to meet the contest of inventing higher speed processing machines. The current research focusses on the parametric optimization of plate fin type, aluminum structured heat sink, using innovative computation-

* Corresponding author, e-mail: Zulfiqarkt@cuiatd.edu.pk

al and simulationol, COMSOL Multiphysics® software. The experimental results for same heat sink geometry put forward by Kim *et al*. [1] have been considered as present model validation. Thermal resistance and pressure drop model for variable length of the plate fin heat sink are regenerated and compared for validation. Finally, parametric optimization using Levenberg-Marquardt technique is presented for better thermal capabilities of given heat sink geometry. The sole drive of the current work is to understand the complex thermal performance attributes of heat sink using modelling, simulation, and optimizationol in framework with the COMSOL Multiphysics®. Instead of experimental techniques adoption, this FEM tool is used for the purpose as it caters for better user graphical user interface (GUI) designing as well as improved controlled environment for heat sink testing.

Optimization techniques overview

To come up with a best solution for a given problem in accordance with the set parameters is always endured a constant quest. The traditional, hit and trial based, design approach proved to be quite valuable till recent past in the absence of power full mathematical and simulation tools.

Optimization generally is the optimum outcome of an engineering problem satisfying all the constraints of design domain. It can be regarded as the tool to solve the engineering design problems in the form of mathematical format tolerable for any given optimization algorithm. Generally thermal resistance and drop in pressure across heat sink are considered as the governing parameters to be addressed carefully to have the required equilibrium which can lead towards the optimal design to cater for the thermal capabilities. Heat transfer maximization was studied Khalid [2] using proper heat flux distribution technique with maximum recordable enhancement of 21.35% for exponential distributionpology under predefined temperature. Khan [3] reported the thermal competence of pin type heat sink adapting the entropy generation minimization principle. About 40% heat removal enhancement was achieved by Jeng and Tzeng [4] in case of staggered square pin type heat sinks using transient-single blow topology. Thermal performance of elliptical heat sink in assisting flow regime under mixed convection by Deshmukh and Warkhedkarby [5].

Various designed geometries having rectangular*, etc.* holes of heat sinks were physically optimized and reported by Ahmed and Ahmed [6]. Similarly, Pandit *et al.* [7] extended the thermal performance optimization work to some more physical geometries of heat sink including spherical, tri-angular, hexagonal*, etc.* by using transient liquid crystal thermography technique with 0.5 as the most apt proportion. Plate type heat sink geometry optimization was reported based on heat flux dissipation enhancement rubrics by Karvinen and Karvinen [8]. Constructal trees phenomenon was applied by Bejan and Dan [9] to investigate the volume and comparative fin thickness for design optimization. The FEM based techniques was reported by Linen *et al.* [10] for optimization of PCM filled thermal management arrangements for various fin height range.

Annular fins optimization results were presented by Sharqawy and Zubair [11] . Optimization of longitudinal fin systems were investigated using fuzzy logic topology for better heat dissipation outcome by Liu [12]. Genetic algorithm technique was employed to ascertain the convective heat transfer of fins by Shasikumar and Balaji [13]. Cavazuti and Corticelli [14] and Rao and Patel [15] adapted multi-objective genetic-algorithm and improved training learning-based optimization algorithm for optimizing heat exchangers with higher thermal efficiency and lesser cost. Taguchi method was reported to optimize for enhancement of heat dissipation of the fin spacing for a fin-tube heat-exchanger by Yang *et al.* [16]. Hybrid Taguchi-genetic algorithm for global-numerical optimization was proposed by Tsai *et al.* [17]

The ANN approach based on Levenberg-Marquardt algorithm was experimentally investigated for twisted rectangular micro-channels to envisage friction-factor by Rahimi *et al.* [18]. The validity of Prey-Predator algorithm and Neural network was reported by Hamadneh *et al.* [19] for prediction of optimized data (0.134 and 2.79) for thermal resistance and pumping power for governing attributes of a micro-channel heat sinks. Genetic algorithm (GA) trained network topology was used by Balachander *et al.* [20] with the findings of better heat transfer rate and almost 90% weight drop in hollow pins in comparison the solid pins of a heat sink.

Discussed literature reveals that apart from conventional numerical and computational techniques, present day topologies like ANN, GA, and DOE, Taguchi method*, etc.* are also frequently addressed for single as well as multi-parametric optimization for thermal solutions in general and for heat sink optimal design in particular.

Optimization COMSOL perspective

As more and more computational and visualization simulationols are being presented to meet the requirements of the researchers, COMSOL Multiphysics, with the capabilities of unpretentious and strong GUI for convenient integrating PDE model along with powerful simulation and visualization outcomes based on FEM, is getting acceptance worldwide. Many investigators now acknowledge the said simulationol as the better and less time consumed alternative especially in thermal performance optimization of heat sinks as evident through the recent published research.

The adaption of COMSOL Multiphysics for a 2-D heat sink under multi-field density technique through constant heat flux was reported by Haertel *et al.* [21]. The optimized results for heat sink temperature against pre-defined range of pressure drop and heat flux. Same FEM tool with *mode-FRONTIER* was studied for multi-objective optimization of plate fin heat sink parameters to enhance its thermal efficiency by Clarich *et al.* [22]. Gradient based topology optimization using MMA optimizer of a standard heat conduction problem in COMSOL Multiphysics was presented by Dede [23]. It was reported that such optimizer is a better tool for handling multiple constraints optimization problems. Investigation regarding the legitimacy of COMSOL Multiphysics for multi-objective, GA based algorithm for assessment of thermal competence of plate type heat sink under parallel and impingent air-flow conditions has been reported by Chen and Chen [24]. Parallel air-flow regime was found more efficient to make use of subjected FEM tool for study and optimization of PCM filled heat sink geometry by Srinivas and Ananthasuresh [25]. About 35% thermal enhancement was reported for parametric optimization. Structural topology optimization was implemented to validate COMSOL Multiphysics as a suitable tool for state and optimization equations successively or at the same time.

Parametric Optimization of plate fin heat sinks

Problem formulation

Basic geometry of the plate type heat sink with L, W, and H as length, width, and height, W_c and W_w as fin spacing and fin thickness under parallel, forced air convection along with all the nomenclature is shown in fig. 1. Similarly, all the implicit values of designed parameters and operating settings are reported in tab. 1. Some

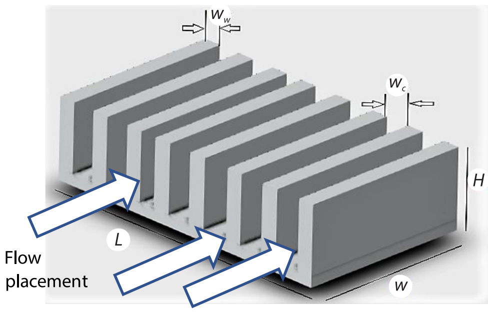

Figure 1. Plate type fin heat sink

assumptions were marked to keep the results consistent with the available literature data including Chen *et al*. [26] and Chen and Chen [24] :

– Incompressible flow with constant thermophysical properties.
– Perfect contact between base and fins.
– Only inline/parallel air-flow.
– Adiabatic conditions at each fin tip.
– Uniform inlet velocity of air.
– Homogeneously distributed heat generation from heat-source.

Table 1. Physical characteristics and operating condition for focused heat sink

Heat sink data	Unit	Value
Length	[mm]	60
Width, W	[mm]	50
Height, H	[mm]	30
Fin spacing, W_c	[mm]	01
Fin thickness, W_w	[mm]	01
No of plate fins	[Nos]	25
Total area of the base plate, A	[m^2]	0.0025
Area of the dins, A_{fin}	[m^2]	0.00325
Exposed area of base plate, A_{base}	[m^2]	0.00325
Sink material	[–]	Aluminum-6063
Thermal conductivity of sink	[Wm^{-1}K^{-1}]	170
Operating conditions		
Working fluid	[–]	Air
Thermal conductivity of air	[Wm^{-1}K^{-1}]	0.0267
Density of air	[Kgm^{-3}]	1.1614
Air-flow type	[–]	Parallel/through flow
Heat load	[W]	10
Ambient temperature	[K]	309

Mathematical model

Thermal performance models for plate type heat sinks in developing and fully developed flow scenarios are separately available in literature by Kim and Kim [27] and Saini and Well [28]. However as both type of flows occur in forced flow regime, it is imperative to have developed model based on the correlations addressing combined effect of these types of flows. Such correlations for pressure drop and thermal resistance for plate type heat sinks were presented by Chulham and Muzychka [29] using optimization technique of entropy minimizationpology incorporating the friction factor and Nusselt number correlations produced by Muzychka and Yovanovich [30] and Teertstra *et al*. [31]. These correlations are reported below and used for numerical scheming:

$$R_{\text{th}} = \frac{1}{h_T\left(A_{\text{b}} + \eta A_{\text{fin}}\right)} \tag{1}$$

$$f=\left[\left(\frac{3.44}{\sqrt[2]{L^*}}\right)^2+\left(f\,\mathrm{Re}_{D_h}\right)^2\right]^{1/2} \tag{2}$$

where

$$L^*=\frac{L}{D_\mathrm{h}\mathrm{Re}_{D_\mathrm{h}}}$$

$$f\,\mathrm{Re}_{D_\mathrm{h}}=\left[24-32.527\left(\frac{W_\mathrm{c}}{H}\right)+46.721\left(\frac{W_\mathrm{c}}{H}\right)^2-40.829\left(\frac{W_\mathrm{c}}{H}\right)^3+22.594\left(\frac{W_\mathrm{c}}{H}\right)^4-6.089\left(\frac{W_\mathrm{c}}{H}\right)^5\right] \tag{3}$$

Problem validation

The proposed model for plate type heat sink was validated using mathematical models based on the friction factor co-relations presented by Muzychka and Yovanovich [30] and Nusselt number co-relation by Teertstra *et al*. [31].The numerical results for the current plate type heat sink geometry has been presented and compared with the available models data as shown in the tab. 2 and fig. 2. The consistency of outcome in terms of thermal resistance and pressure drop validates the numerical model.

Table 2. Modelled parameters for the current study plate fin geometry

Velocity, [ms^{-1}]	Reynolds number	Pressure drop, ΔP	Thermal resistance, R_th
2.02	249.33	25.33	0.902
3.24	400	31.39	0.53
4.05	500	33.167	0.49
8.1	1000	40.89	0.39

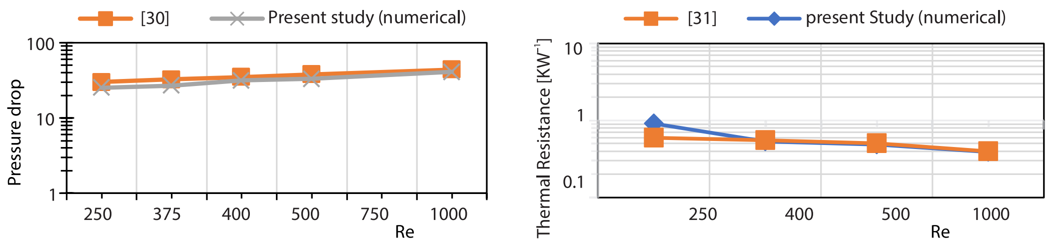

Figure 2. Pressure drop and thermal resistance modelling for plate fin heat sink

Experimental set-up

The experimental set-up consists of 24 × 3 inches rectangular wind tunnel made up of properly insulated ploy vinyl chloride sheet. The DC blower of 12 V, 1.9 W has been installed at one end to induce the air-flow. A total of 06 Nos of sensors type BMP180 with pressure range of 300~1100 hPa and temperature range of –40~90 °C are being installed inside the test tunnel at desired data point as shown in the figure. Inside the tunnel, electric heaters (Tungsten wrapped wire type) are fitted below the specimen heat sink with adequate protection of Bakelite to curtail heat losses.

The test facility consists of two portions, upper potion has test-tunnel and a small Vero-board with connectors, push buttons and LED's for operation and data transfer from sensors to main console. The lower part of test bench has properly coded Arduino m ega 2560 micro-controller which displays temperature and pressure reading from sensors to a (16×2) LCD display. Also, a serial port connectivity has been facilitated for direct display on a laptop GUI. A voltage regulating circuit has been provided at the same Vero board to energize DC blower, Arduino and LCD. The model as well as existing test bench is shown in figs. 3 and 4, respectively.

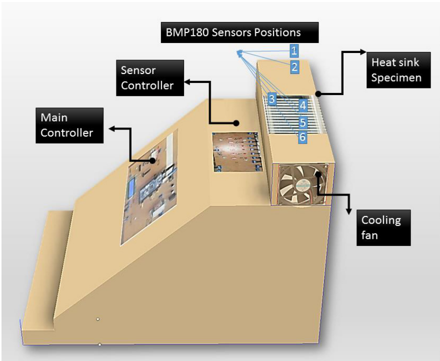

Figure 3. Perspective diagram of test bench facility

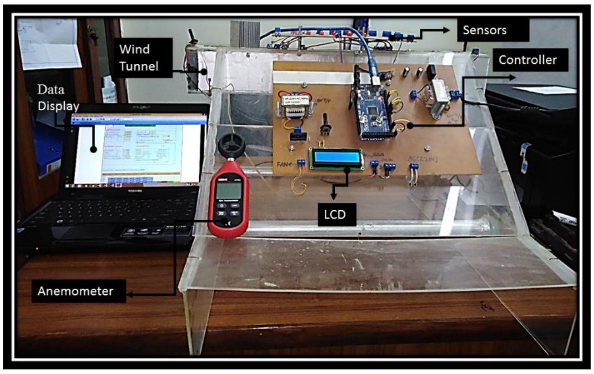

Figure 4. Wind - tunnel test bench assembly

Test procedure

Plate type fin heat sink made of Al with desired dimensions as given in tab. 1, was clamped inside the test tunnel with blocked sides to control any by-pass flow. With all connectors and button in place, heater was powered on provide 10 W of heat at heat sink base through adjustment of potentiometer. Blower was started to provide desired air-flow/Reynold number measured by the flow sensors BMP 180 displayed directly at the LCD through Arduino. Steady-state conditions were achieved and data from selected data points was recorded to calculate thermal resistance and pressure drop.

The retrieved data has been tabulated in tab. 3 and compared with the experimental result of Kim *et al*. [1] as shown in fig. 5 with satisfactory conformity for the model validation in FEM tool.

Table 3. Experimental data for plate-type heat sink with varying flow under constant heat flux

Study type	Velocity [ms^{-1}]	Re	Q [W]	T_1/P_1	T_2/P_2	T_3/P_3	T_4/P_4	T_5/P_5	T_6/P_6	ΔT [°C]	ΔP	R_{th}
Experimental	7	9350	10	27.7/871.24	28.2/871.76	43.7/872	42.7/873.9	36.9/874.23	34.8/874.8	7.1	3.56	0.71
	8.5	11400	10	28.5/871.6	28.9/872	44.8/872.55	43.2/874.01	37.3/874.8	35.2/875.36	6.3	3.76	0.63
	10	13400	10	29.5/872	29.9/875.45	45.5/874.7	44.1/874.7	38/875	34.8/875.3	4.9	3.3	0.49

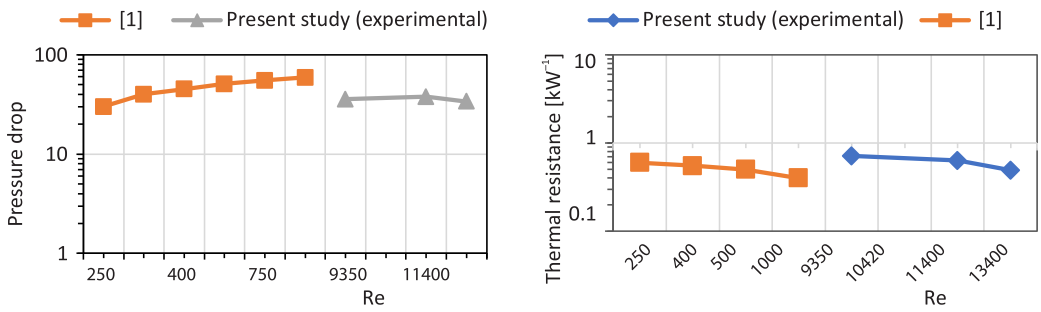

Figure 5. Pressure drop and thermal resistance *vs.* Reynolds number experimental evaluation

Results and discussion

The COMSOL-model generation

Plate Fin heat sink with discussed geometry was modelled in COMSOL Multiphysics® as shown in fig. 6. In actual cases, an air gap is always present between sink base and heat source, so a thin layer is used to overcome this effect to keep results factual. Also, for heat sink thermal performance enhancement, model is kept inside wind tunnel with variable air velocity, allowing maximum fin temperatures. The MATLAB® interface to the COMSOL Multiphysics®. Was utilized for numerical calculation through the available model presented in section *Mathematical model*. In first set of simulation, as shown in fig. 7, the heat sink was subjected to a velocity of 0.1 m/s (laminar flow domain) and contours for temperature difference and pressure difference inside wind tunnel across the heat sink were presented and analyzed. As observed velocity is not reduced exactly at the heat sink face causing less heat dissipation. Similarly, pressure distribution is not even which pertains to less pressure drop across heat sink leading towards low efficiency of the present operating conditions.

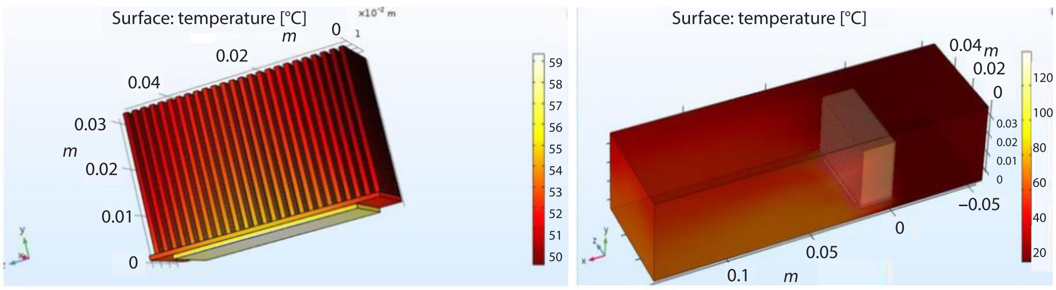

Figure 6. Heat sink-model representation in COMSOL GUI

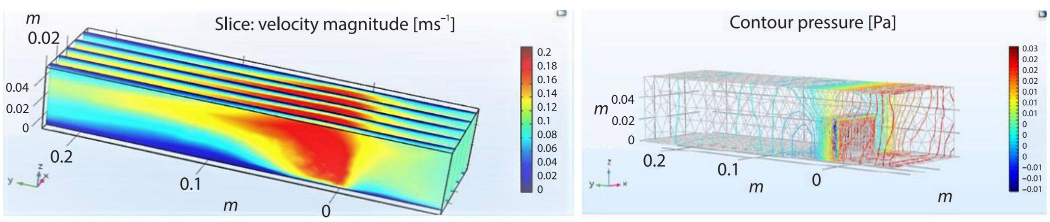

Figure 7. Velocity and pressure contours at 0.1 m/s

To evaluate the effect of velocity in laminar regime, same configuration is tested under the air-flow velocity of 10 m/s as presented in fig. 8. The pressure and velocity profile reports inferior heat dissipation since instead of more pressure drop and velocity difference across heat sink, supplementary concentration is being observed near to the walls of the tunnel.

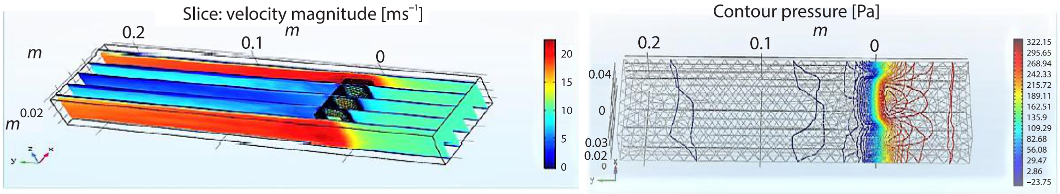

Figure 8. Velocity and pressure contours at 10 m/s

Optimized model using Levenberg-Marquardt technique

The sole purpose of heat sink optimization in current research is to have best optimal sink design in terms of thermal resistance (objective function) against flow velocity (variable). The optimization approach is employed by keeping other design parameters fixed against the changing length and flow rate. Single as well as multiparametric optimization technique, based on Levenberg-Marquardt method, available in COMSOL Multiphysics® was adapted in a quest for best design values in predefined range. The optimized results presented can be taken as the base study for plate fin sinks and any other type of sink geometry in general for practical purposes. Single and multiparametric optimization, while keeping the length variable in a defined range and evaluating the variation trend for governing factors like thermal resistance, pressure drop and Nusselt number for a categorization of plate type heat sink length, obtained through FEM tool has been reported in fig. 9. For sink length in given domain, an optimum value can be predicted in lieu of predominant influential factors.

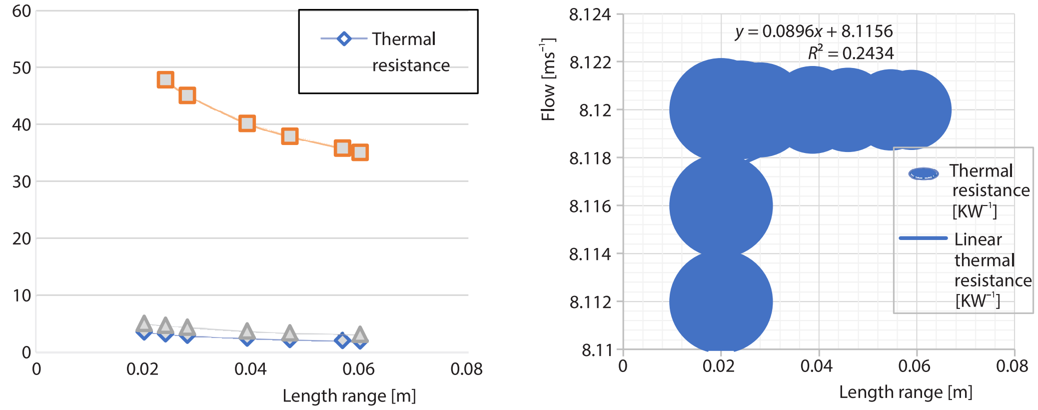

Figure 9. Single and multi-parametric optimization trends for thermal resistance

Although single parametric optimizationpology can present the optimized solution against all other pre-defined design conditions, the value-ability of this *Optimized* solution remain questionable when some or any of the design parameters are unconstrained. For a better hold of heat sink geometry for practical purposes, optimization must be accomplished through multi-parametric optimizationpology, to have concurrent solutions for maximum catering for the un-constrained variables.

Multiparametric optimization for the discussed plate type heat sink is reported in fig. 9 (right), considering the effect of variable flow rate in combination with the length of sink to attain minimization of objective function *i.e.* thermal resistance offered by the geometry. It is evident that taking care of more unconstrained variables like flow rate in this case, makes design more feasible for applied usage.

Optimized plate heat sink simulation and analysis

In view of user-friendly graphic interface with ease of PDE implementation, COMSOL Multiphysics®, is selected to present the 3-D optimized model for the plate type heat sink. Similarly, optimization module embedded in subjected FEM tool with least square topology is utilized to generate the 3-D temperature/velocity and pressure-based simulations presented in fig. 10. It is evident from the optimized simulated model that pressure is dropped precisely at the face of heat sink allowing more air conduction with very less air particles by-passing the heat sink. Similarly, optimized model velocity distribution contours confirm lesser by-pass flow adaption leading towards the more heat dissipation. The parametric and operating conditions prevailed during iterations are presented in tab. 4.

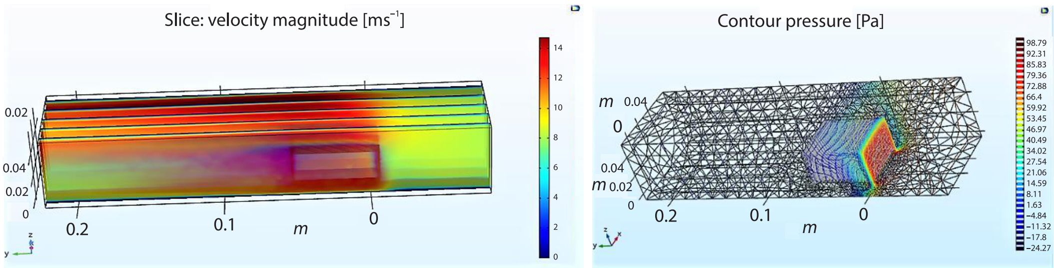

Figure 10. Optimized velocity and optimized pressure distribution contour

Table 4. Simulation settings for focused optimized heat sink geometry

Parameter	Unit	Value
Optimality tolerance	–	0.01
Model evaluation	Nos	1000
Heat transfer coefficient, h	[$Wm^{-2}K^{-1}$]	81.622
Reynolds number	–	629.76
Poiseuille number	–	22.966
Friction factor	–	0.05611
Nusselt number	–	3.1106
Thermal resistance, R_{th}	[Kw^{-1}]	0.38588
Pressure difference, ΔP	Pa	21.496

Figure 11 depicts the performance enhancement in terms of thermal resistance [KW^{-1}] of the multiparametric optimized model against the velocity and pressure distribution. The simulation results presented are consistent with the optimization data obtained through single as well as multi parametric optimization already discussed in section *Optimized model using Levenberg-Marquardt technique*. In the same context, comparison of un-optimized model, figs. 7

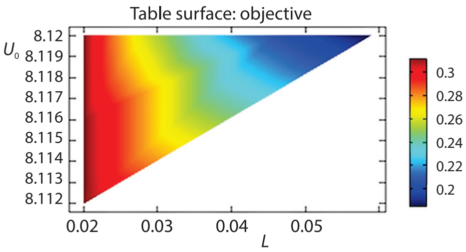

Figure 11. Multi-parametric optimized model performance

and 8, with the optimized contours, fig. 9, clearly demonstrates the success of optimally designed plate type heat sink for applied purposes.

Conclusions

The current work pertains to ascertain the optimal thermal performance characteristics for given geometry of plate type fin heat sink using advanced FEM tool COMSOL Multiphysics®. Initially un-optimized numerical and simulated data is presented in comparison with the available relevant evidence. Then experimental observations using lab grade test apparatus for the discussed geometry is made available to access and compare its validity. Finally, optimized geometry with enhanced thermal competences is reported considering the effect of variable flow rate in combination with the length of sink to attain minimization of objective function *i.e.* thermal resistance offered by the geometry. Results are summarized as follows.

- Optimized model for velocity distribution contours confirms lesser by-pass flow adaption leading towards the more heat dissipation.
- About 12% enhancement in pressure drop and up to 51% improvement in thermal resistance is reported for the optimized plate fin heat sink as per manifested data in comparison with the experimental observations.

Nomenclature

A – total area of base plate, [m^2]
A_b – exposed area, [m^2]
A_f – total fin area, [m^2]
D_h – hidraulic diametar [$= 2HW_c(H + W_c) - 1$], [m]
f – friction factor in fully developed flow
h_T – heat transfer co-efficient, [Wm^{-2}K^{-1}]
L^* – heat sink dimensioless length [$= (L/D_h)\text{Re}_{D_h}$]
Nu – Nusselt nuber [$= hd(k - 1)$]
ΔP – pressure drop, [Pa]
Re – Reynolds nuber [$= \rho U_{max} D_h(\mu - 1)$]
R_{th} – thermal resistance, [WK^{-1}]
W_c – fin spacing, [mm]

Greek symbols

η – fin efficiency
μ – dynamic viscosity, [kgm^{-1}s^{-1}]
ρ – density, [kgm^{-3}]

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Paper submitted: December 27, 2020
Paper revised: January 20, 2021
Paper accepted: February 8, 2021