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COMMENTS ON "SIGNIFICANCE OF IMPROVED FOURIER-FICK LAWS IN NON-LINEAR CONVECTIVE MICROPOLAR MATERIAL STRATIFIED FLOW WITH VARIABLE PROPERTIES"

by

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The $\theta(\eta)$ and $\phi(\eta)$ equations are invalid.

Waqas *et al*. [1] studied the effectiveness of temperature-dependent thermal conductivity and improved Fourier-Fick fluxes on the 2-D, steady incompressible micropolar material flow past a stretchable surface. The researchers considered non-linear mixed convection, heat generation and double stratification aspects. Waqas *et al*. [1] presented the energy and concentration equations (eqs. (4) and (5) in ref. [1]) as:

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}+\lambda_1\left[\begin{array}{c} u\frac{\partial u}{\partial x}\frac{\partial T}{\partial x}+v\frac{\partial v}{\partial y}\frac{\partial T}{\partial y}+v\frac{\partial u}{\partial y}\frac{\partial T}{\partial x}+2uv\frac{\partial^2 T}{\partial x\partial y}+u^2\frac{\partial^2 T}{\partial x^2}+ \\ +v^2\frac{\partial^2 T}{\partial y^2}-\frac{Q}{\rho c_p}\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)\end{array}\right]= \tag{1}$$
$$=\frac{1}{\rho c_p}\frac{\partial}{\partial y}\left[K(T)\frac{\partial T}{\partial y}\right]+\frac{Q}{\rho c_p}(T-T_\infty)$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}+\lambda_2\left(\begin{array}{c} u\frac{\partial u}{\partial x}\frac{\partial C}{\partial x}+v\frac{\partial v}{\partial y}\frac{\partial C}{\partial y}+v\frac{\partial u}{\partial y}\frac{\partial C}{\partial x}+ \\ 2uv\frac{\partial^2 C}{\partial x\partial y}+u^2\frac{\partial^2 C}{\partial x^2}+v^2\frac{\partial^2 C}{\partial y^2}\end{array}\right)=D\frac{\partial^2 C}{\partial y^2} \tag{2}$$

From the previous eqs., concentration, C, and temperature, T, depend on x, y.

Waqas *et al*. [1] introduced these variables to convert PDE into ODE (eq. (8) in ref. [1]):

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$$\eta = y\sqrt{\frac{c}{\nu}} \tag{3}$$

$$\theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty} \tag{4}$$

$$\phi(\eta) = \frac{C - C_\infty}{C_w - C_\infty} \tag{5}$$

From eq. (3), the similarity variable, η, depends on y only. From eq. (4), the temperature $\theta(\eta)$ depends on y only while RHS $[T(x, y) - T_\infty]/(T_w - T_\infty)$ depends on x, y. Hence, there is a disagreement between LHS and RHS so that eq. (4) is invalid. From eq. (5), the concentration $\phi(\eta)$ depends on y only while RHS $[C(x, y) - C_\infty]/(C_w - C_\infty)$ depends on x, y. Hence, there is a disagreemen between LHS and RHS so that eq. (5) is invalid.

The same errors were revealed by Pantokratoras [2-5]. As indicated by Pantokratoras [5], the similarity variable, η, was defined by Minkowycz and Sparrow [6] as:

$$\eta = \left[\frac{g\beta(T_w - T_\infty)}{4\nu^2}\right]^{1/4} \frac{y}{x^{1/4}} \tag{6}$$

to agree with their energy equation:

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} \tag{7}$$

Awad [7] revealed recently the same errors.

Reference

[1] Waqas, M., *et. al*., Significance of Improved Fourier-Fick Laws in Non-Linear Convective Micropolar Material Stratified Flow with Variable Properties, *Thermal Science, 23* (2019), 6B, pp. 3809-3815

[2] Pantokratoras, A., Discussion: ‘Homogeneous–Heterogeneous Reactions in Boundary-Layer Flow of a Nanofluid Near the Forward Stagnation Point of a Cylinder,’ (Zhao, Q., Xu, H., and Tao, L., 2017, ASME J. Heat Transfer, 139(3), p. 034502),” *ASME Journal of Heat Transfer, 140* (2018), 10, 105501

[3] Pantokratoras, A., Discussion: ‘Three-Dimensional Stagnation Flow and Heat Transfer of a Viscous, Compressible Fluid on a Flat Plate,’ (Mozayyeni, H. R., and Rahimi, A. B., 2013, ASME J. Heat Transfer, 135(10), p. 101702), *ASME Journal of Heat Transfer, 140* (2018), 11, 115501

[4] Pantokratoras, A., Discussion: “Heat and Mass Transfer Analysis in the Stagnation Region of Maxwell Fluid With Chemical Reaction Over a Stretched Surface,” [T. Hayat, M. I. Khan, M. Imtiaz, and A. Alsaedi, 2018, ASME J. Thermal Sci. Eng. Appl., 10(1), p. 011002], *ASME Journal of Thermal Science and Engineering Applications, 11* (2019), 1, 015502

[5] Pantokratoras, A., Comment on the Paper “Joule Heating and Viscous Dissipation in Flow of Nanomaterial by a Rotating Disk, Tasawar Hayat, Muhammad Ijaz Khan, Ahmed Alsaedi, Muhammad Imran Khan, International Communications in Heat and Mass Transfer, 89 (2017), pp. 190-197”, *International Communications in Heat and Mass Transfer, 103* (2019), Apr., pp. 62-63

[6] Minkowycz, W. J., Sparrow, E. M., Numerical Solution Scheme for Local Nonsimilarity Boundary-Layer Analysis, *Numerical Heat Transfer, Part B: Fundamentals, 1* (1978), 1, pp. 69-85

[7] Awad, M. M., Comments on “Optimization of Entropy Generation and Dissipative Nonlinear Radiative Von Karman's Swirling Flow with Soret and Dufour Effects”, *Journal of Molecular Liquids, 321* (2021), 114749

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