THERMAL RADIATION AND MAGNETOHYDRODYNAMICS FLOW OVER A BLACK ISOTHERMAL PLATE

by

Andreas RAPTIS*

Department of Mathematics, University of Loannina, Loannina, Greece

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We study the effects of the thermal radiation and an induced magnetic field on the flow over a black isothermal plate for an optically thin gray fluid. The flowing medium absorbs and emit radiation, but scattering is not included. Numerical solutions are obtained for different values of radiation parameter, Prandtl number, Grashof number and magnetic Prandtl number.

Key words*: thermal radiation, MHD flow*

Introduction

The radiative flows of an electrically conducting fluid with high temperature in the presence of a magnetic field are encountered in electrical power generation, astrophysical flows, solar power technology, space vehicle re-entry nuclear engineering applications and other industrial areas.

The MHD flow and heat transfer for two types of viscoelastic fluid over a stretching sheet with energy dissipation, internal heat source and thermal radiation analyzed by Chen [1]. Hayat *et al*. [2] carried out the convection flow of a viscoelastic fluid with thermal radiation and convective conditions. The thermal radiative hydromagnetic-flow of Jeffrey nanofluid by an exponentially stretching sheet was studied by Hussain *et al*. [3]. Ramzan and Bilal [4] investigated the time dependent MHD nanosecond grade fluid-flow induced by permeable vertical sheet with mixed convection and thermal radiation. Babu and Narayana [5] investigated Joule heating effects on MHD mixed convection of a Jeffrey fluid over a stretching sheet with power law heat flux in the presence thermal radiation. The thermal radiation effect in MHD flow of Powell-Eyring nanofluid induced by a stretching cylinder was examined by Hayat *et al*. [6]. Saidulu and Lakshman [7] presented the MHD flow of Casson fluid with slip effects over an exponentially porous stretching sheet in the presence of thermal radiation, viscous dissipation and heat source/sink. The effect of the frictional heating on radiative ferrofluid-flow over a slendering stretching sheet with aligned magnetic field was analyzed by Reddy *et al*. [8]. Chaudhary *et al.* [9] presented a computational modelling of partial slip effects on hydromagnetic boundary-layer flow past an exponential stretching surface in presence of thermal radiation. Ibrahim *et al.* [10] investigated the influence of Joule heating and heat source on radiative MHD flow over a stretching porous sheet with power-law heat flux. The effects of thermal radiation and Joule heating on magnetohydrodynamic Marangoni convection over a flat surface analyzed by Khaled [11]. In all these studies the fluid was assumed to be optically thick and using the Rosseland approximation for radiation.

* Author's e-mail: araptis@uoi.gr

In this paper, we have studied the effects of the thermal radiation and an induced magnetic field on the flow over a black isothermal plate for an optically thin gray fluid. The flowing medium absorbs and emit radiation, but scattering is not included.

Mathematical formulations

We consider the 2-D steady flow of a viscous, incompressible and electrically conducting fluid over an infinite black heat vertical and porous plate. All the fluid properties are considered constant except that the influence of the density variation with temperature has been considered only in the body-force term. The x'-axis is along the plate in the upward direction and the y'-axis normal to the plate. The plate is electrically non-conducting and the external applied magnetic field H_0 is perpendicular to the plate. In the region of the plate the magnetic field is of the form $H = (H'_{x'}, H_0, 0)$, the thermal radiation heat flux at the x'-direction is considered negligible in comparison the y'-direction. Under the previous assumptions, the magnetohydromagnetic flow in the presence thermal radiation is governed by [12-14]:

– Continuity equation

$$\frac{\mathrm{d}v'}{\mathrm{d}y'} = 0 \tag{1}$$

– Momentum equation

$$v'\frac{\mathrm{d}u'}{\mathrm{d}y'} = \mathrm{g}\beta\left(T' - T'_\infty\right) + \nu\frac{\mathrm{d}^2u'}{\mathrm{d}y'^2} + \frac{\mu}{\rho}\frac{\mathrm{d}H'_{x'}}{\mathrm{d}y'} \tag{2}$$

– Energy equation

$$v'\frac{\mathrm{d}T'}{\mathrm{d}y'} = \frac{k}{\rho c_p}\frac{\mathrm{d}^2T'}{\mathrm{d}y'^2} - \frac{1}{\rho c_p}\frac{\mathrm{d}q_{r,y'}}{\mathrm{d}y'} \tag{3}$$

– Induced magnetic field equation

$$v'\frac{\mathrm{d}H'_{x'}}{\mathrm{d}y'} = H_0\frac{\mathrm{d}^2u'}{\mathrm{d}y'^2} + \frac{1}{\sigma\mu}\frac{\mathrm{d}^2H'_{x'}}{\mathrm{d}y'^2} \tag{4}$$

where u', v' are the components of the velocity parallel and perpendicular to the plate, T' – the fluid temperature, T'_∞ – the fluid temperature at infinity, g – the acceleration due to gravity, β – the coefficient of volume expansion, ν – the kinematic viscosity, μ – the magnetic permeability, ρ – the fluid density, k – the thermal conductivity, c_p – the specific heat at constant pressure, σ – the electrical conductivity, and $q_{r,y'}$ – the thermal radiative heat flux at y'-direction.

The boundary conditions:

$$\begin{gathered} u' = 0, \quad v' = -v_0, \quad T' = T'_w, \quad H'_{x'} = 0, \quad \text{at } \ y' = 0 \\ u' \to U_0, \quad T' \to T'_\infty, \quad H_{x'} \to 0, \quad \text{as } \ y' \to \infty \end{gathered} \tag{5}$$

where v_0 is the constant suction velocity, U_0 – the constant free stream velocity, and T'_w – the temperature of the plate, where $T'_\infty < T'_w$.

From eq.(1) we take:

$$v' = -v_0 \tag{6}$$

For the case of an optically thin gray fluid the local radiation over a black isothermal plate is expressed [13, 14]:

$$-\frac{\mathrm{d}q_{r,y'}}{\mathrm{d}y'} = 2a\sigma^*\left(T'^4_w + T'^4_\infty - 2T'^4\right) \tag{7}$$

where a is the absorption coefficient and σ^* – the Stefan-Boltzman constant.

We are putting:

$$2T_r'^4 = T_w'^4 + T_\infty'^4 \tag{8}$$

where $T'_\infty < T'_r < T'_w$.

By using eqs. (7) and (8), eq. (3) gives:

$$v'\frac{\mathrm{d}T'}{\mathrm{d}y'} = \frac{k}{\rho c_p}\frac{\mathrm{d}^2T'}{\mathrm{d}y'^2} + \frac{4a\sigma^*}{\rho c_p}\left(T_r'^4 - T'^4\right) \tag{9}$$

We assume that the temperature differences within the flow are sufficiently small such that T'^4 may be expressed as a linear function of the temperature T'. This is accomplished by expanding in a Taylor series about T'_r and neglecting higher-order terms, thus:

$$T'^4 \cong 4T_r'^3T' - 3T_r'^4 \tag{10}$$

By using eqs. (10) and (9) gives:

$$v'\frac{\mathrm{d}T'}{\mathrm{d}y'} = \frac{k}{\rho c_p}\frac{\mathrm{d}^2T'}{\mathrm{d}y'^2} + \frac{16a\sigma^* T_r'^3}{\rho c_p}\left(T'_r - T'\right) \tag{11}$$

We introduce the dimensionless quantities:

$$\begin{gathered}
y = \frac{y'v_0}{v},\ u = \frac{u'}{U_0},\ \theta = \frac{T' - T'_r}{T'_w - T'_r},\ H = \left(\frac{\mu}{\rho}\right)^{1/2}\frac{H'_{x'}}{U_0} \\
M = \left(\frac{\mu}{\rho}\right)^{1/2}\frac{H_0}{v_0}\ (\text{magnetic parameter}),\ \mathrm{Gr} = \frac{v g\beta\left(T'_w - T'\right)}{U_0 v_0^2}\ (\text{Grashof number}) \\
\mathrm{Pr}_m = v\sigma\mu\ (\text{magnetic Prandtl number}),\ m = \frac{T'_\infty - T'_r}{T'_w - T'_r},\ (\text{temperature parameter}) \\
S = \frac{16a\sigma^* T_r'^3 v^2}{k v_0^2},\ (\text{radiation parameter}),\ \mathrm{Pr} = \frac{\rho v c_p}{k}\ (\text{Prandtl number})
\end{gathered} \tag{12}$$

Using eq. (6) and the dimensionless quantities eq. (12), eqs. (2), (4), and (9) become:

$$\frac{\mathrm{d}^2u}{\mathrm{d}y^2} + \frac{\mathrm{d}u}{\mathrm{d}y} + M\frac{\mathrm{d}H}{\mathrm{d}y} + \mathrm{Gr}\left(\theta - m\right) = 0 \tag{13}$$

$$\frac{\mathrm{d}^2H}{\mathrm{d}y^2} + \mathrm{Pr}_m\frac{\mathrm{d}H}{\mathrm{d}y} + M\,\mathrm{Pr}_m\frac{\mathrm{d}u}{\mathrm{d}y} = 0 \tag{14}$$

$$\frac{\mathrm{d}^2\theta}{\mathrm{d}y^2} + \mathrm{Pr}\frac{\mathrm{d}\theta}{\mathrm{d}y} - S\theta = 0 \tag{15}$$

The corresponding boundary conditions:

$$\begin{gathered}
u = 0,\quad H = 0,\quad \theta = 1,\quad \text{at}\ \ y = 0 \\
u \to 1,\quad H \to 0,\quad \theta \to m,\quad \text{as}\ \ y \to \infty
\end{gathered} \tag{16}$$

Equations (13)-(15), subject to the boundary conditions (16), constitute a system of differential equations is solved numerically by using two boundary value problems.

Discussion

In order to understand the physical situation of the problem we have computed the numerical values for non-dimensional temperature and non-dimensional velocity for different values of the physical parameters.

Figure 1 shows the effect of the radiation parameter, S, on the non-dimensional temperature, θ, when $m = -0.1$ and $\mathrm{Pr} = 3$. It is observed that the non-dimensional temperature decreases with the increase of the S.

Figure 2 shows the effect of the Prandtl number, on the θ, when $m = -0.1$ and $S = 2$. It is observed that the non-dimensional temperature decreases with the increase of the Prandtl number.

Figure 3 demonstrates the effect of the Prandtl number on the non-dimensional velocity, u, when $M = 0.8$, $\mathrm{Gr} = 5$, $\mathrm{Pr}_m = 2$, $m = -0.1$, and $S = 2$. It is observed that the non-dimensional velocity decreases with the increase of the Prandtl number.

The effect of the S on the u is shown in fig. 4, when $M = 0.8$, $\mathrm{Gr} = 5$, $\mathrm{Pr}_m = 2$, $m = -0.1$, and $\mathrm{Pr} = 1.5$. It is noticed that when the S increases the non-dimensional velocity decreases.

Figure 5 demonstrates the effect of Grashof number, on the u when $M = 0.8$, $\mathrm{Pr}_m = 2$, $m = -0.1$, $S = 2$, and $\mathrm{Pr} = 1.5$. It is observed that the non-dimensional velocity increases with the increase of Grashof number.

Figure 6 demonstrates the effect of magnetic Prandtl number on the u when $M = 0.8$, $\mathrm{Gr} = 2$, $m = -0.1$, $S = 2$, and $\mathrm{Pr} = 1.5$ It is observed that the non-dimensional velocity decreases with the increase of magnetic Prandtl number.

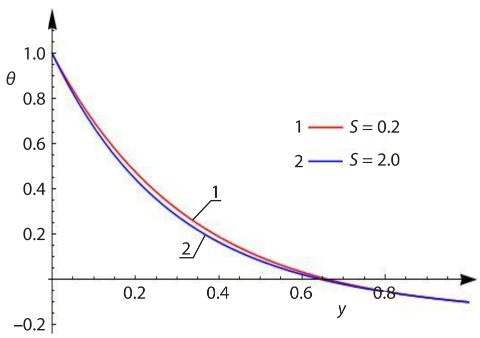

Figure 1. Dimensionless temperature profiles for different values of *S*

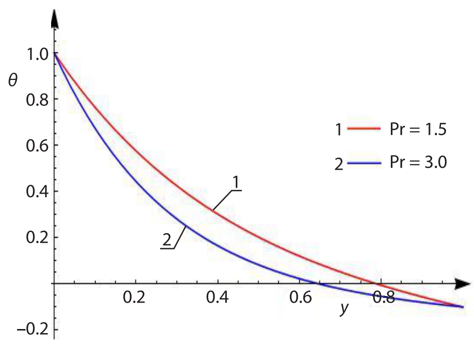

Figure 2. Dimensionless temperature profiles for different values of the Prandtl number

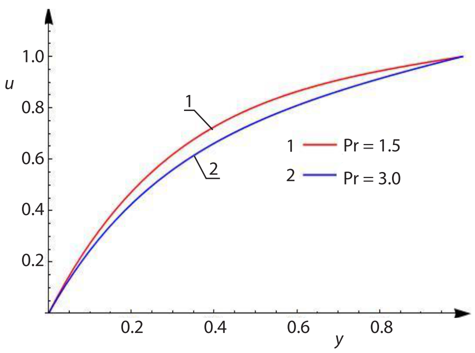

Figure 3. Dimensionless velocity profiles for different values of the Prandtl number

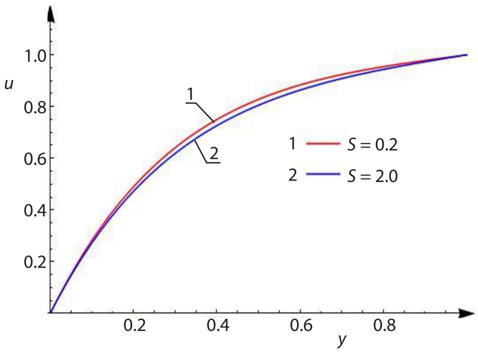

Figure 4. Dimensionless velocity profiles for different values of *S*

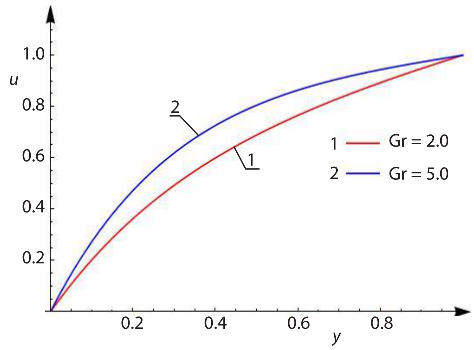

Figure 5. Dimensionless velocity profiles for different values of Grashof number

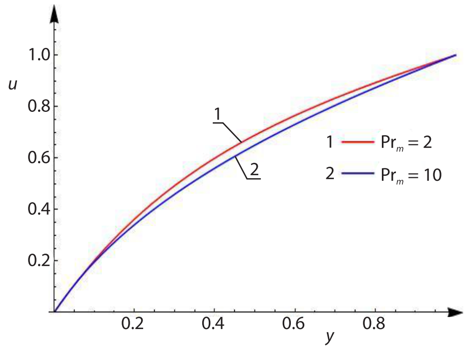

Figure 6. Dimensionless velocity profiles for different values of magnetic Prandtl number

Conclusions

An analysis is performed to study the effects of the thermal radiation and an induced magnetic field on the flow over a black isothermal plate for an optically thin gray fluid. The flowing medium absorbs and emit radiation, but scattering is not included. Numerical solutions are obtained for different values of radiation parameter, Prandtl number, Grashof number and of the of magnetic Prandtl number. The conclusions of the study areas follows.

- The dimensionless temperature decreases with the increase of the radiation parameter.
- The dimensionless temperature decreases with the increase of the Prandtl number.
- The dimensionless velocity decreases with the increase of the Prandtl number.
- The dimensionless velocity decreases with the increase of the radiation parameter.
- The dimensionless velocity increases with the increase of the Grashof number.
- The dimensionless velocity decreases with the increase of magnetic Prandtl number.

Nomenclature

a – absorption coefficient, [m^{-1}]
c_p – specific heat at constant pressure, [Jkg^{-1}K^{-1}]
g – acceleration due to gravity, [ms^{-2}]
H – dimensionless magnetic field $H'_{x'}$, [–]
H_0 – magnetic field at the y'-direction, [Am^{-1}]
$H'_{x'}$ – magnetic field at the x'-direction, [Am^{-1}]
k – thermal conductivity, [Wm^{-1}K^{-1}]
m – temperature parameter, [–]
M – magnetic parameter, [–]
Pr – Prandtl number, [–]
Pr$_m$ – magnetic Prandtl number, [–]
$q_{r,y'}$ – thermal radiative heat flux at y'-direction, [Wm^{-2}]
S – radiation parameter, [–]
T' – fluid temperature, [K]
T'_w – fluid temperature on the plate, [K]
T'_∞ – fluid temperature at infinity, [K]
u' – velocity parallel to the plate, [ms^{-1}]
u – dimensionless velocity parallel to the plate, [–]
v' – velocity perpendicular to the plate, [ms^{-1}]
v_0 – suction velocity, [msec^{-1}]
U_0 – free stream velocity, [msec^{-1}]
x' – axis along of the plate in the upward direction, [m]
y' –axis normal to the plate, [m]
y – dimensionless y', [–]

Greek symbols

β – coefficient of volume expansion, [K^{-1}]
θ – dimensionless temperature, [–]
μ – magnetic permeability, [Hm^{-1}]
ν – kinematic viscosity, [m^{2}s^{-1}]
ρ – fluid density, [kgm^{-3}]
σ – electrical conductivity, [Sm^{-1}]
σ^* – Stefan-Boltzmann constant, [Wm^{-2}K^{-4}]

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