WHEN MATHEMATICS MEETS THERMAL SCIENCE
The simpler is the better

by

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Mathematics is an important tool to dealing with a complex problem, but it might be too abstract and elusive to be directly applied by engineers. This paper shows mathematics should be simple but effective for thermodynamic problems, the simpler is the better. The two-scale fractal is used as an example to show the importance of application of mathematics to practical problems, and a fast-slow law is suggested to deal with many discontinuous problems by fractal calculus.

Keywords: *fractal geometry, porous medium, fractal derivative, two-scale fractal dimension, slow-fast law, Brachistochrone curve*

Introduction

Mathematics for medicine is called as iatromathematics, and it is called as biomathematics for biology. The integration of modern mathematics and thermal science always leads to the research frontiers, and we can call it as thermomathematics.

Mathematics was invented in order to make a practical problem much easy to be solved, it is established under a rigorous assumption of continuous space and time, however, the continuum assumption becomes invalid for many phenomena, and some thermodynamic problems cannot be effectively modelled by the traditional differential models, for example, snow's thermal insulation property [1].

Mathematics in thermal science

Just consider a drop of red ink on the surface of a moving water [2]. If water is considered as a continuum, the Navier-Stokes equations can predicted approximately the motion of the red ink along the streamline for a steady flow, but the continuum model cannot describe the penetration process of the red ink into water, there would be no space for the red ink to penetrate into water under the continuum assumption, and the penetration process would become quite randomly. However, if we consider water as a porous medium, and the motion of the red ink on a molecule's scale becomes determinative. This two-scale analysis led to the birth of two-scale mathematics, two-scale thermodynamics and fractal calculus [3-10].

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Mathematics for thermodynamic problems should be simple but effective, the simpler is the better. Now fractional calculus and fractal calculus are widely used to deal with non-continuum problems, but many such models are established a fractal space or a fractal time.

Two-scale fractal

But what is a fractal problem? It is of self-similarity that means on any scales, the problem is self-similar. Can we say a porous medium is a fractal medium? Can we consider an unsmooth boundary problem as a fractal problem? The answer is no. But we can say the non-continuum problem or an un-smooth problem is two adjacent cascades of a fractal pattern, on a large scale we have an approximate continuum, and we have a porous medium or unsmooth boundary on a smaller scale. This leads to a new definition of fractal dimensions [3-5]:

$$\alpha = \frac{\alpha_0 V}{V_0} \tag{1}$$

where α is called as the two-scale fractal dimension on a small scale, α_0 – the integer dimension on a large scale for the continuum assumption, V and V_0 are measured volume/area/length on different scales, respectively.

Consider two adjacent levels of a Koch curve as illustrated in fig. 1(a). A large scale of L leads to a 1-D problem, on the other hand, a smaller scale of $L/3$ reveals the unsmooth property of the curve, so the two-scale fractal dimension is:

$$\alpha = \frac{1\times 4}{3} = 1.3333 \tag{2}$$

while the Hausdorff fractal dimension of the Koch curve is:

$$\alpha = \frac{\ln 4}{\ln 3} = 1.2618 \tag{3}$$

The difference between two values are small, the latter is for an exact fractal pattern, while the former is for a real curve. For a real curve, it might be random as shown in fig. 1(b), the two-scale fractal dimension can be calculated by measuring its real length, which is $3.42L/3$, so the fractal dimension is $3.42/3 = 1.14$.

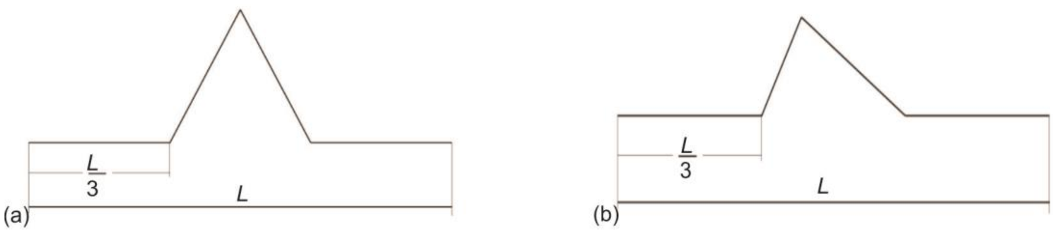

Figure1. Two-scale fractal; (a) an adjacent two levels of a Koch curve and (b) a practical curve

The two-scale fractal is the foundation of the fractal calculus, which can describe many phenomena beyond the traditional differential models, it can describe many slow-fast change phenomena, and we suggest a slow-fast law for simple treatment of a complex problem.

The slow-fast law

The slow-fast law (extremely slow-extremely fast law) implies that a slow change at initial stage must lead to a fast change at terminal stage, and vice versa; an extremely fast change at initial stage must result in an extremely slow change at the terminal stage, examples of the slow-fast law are thermal properties of cocoon [11], polar bear hairs [8, 9], and snow ball [1]. A sudden earthquake, a sudden avalanche and a sudden gas explosion arise in, respectively, an extremely slow change of tectonic plate's motion, an extremely slow motion of the snow drift, and an extremely slow concentration change of a flammable gas. Other examples include the fractal current law [12], the fractal rheological law [13], and fast-slow convection-diffusion process [14]. As an illustrating example, we consider the well-known brachistochrone problem.

Brachistochrone curve

Brachistochrone problem can be solved by the variational theory, however the solution process is not accessible to engineers, its exact solution is a cycloid. The two possible solutions are the straight line AB and the discontinuous line AOB as illustrated in fig. 2. The former predicts a constant accelerated speed, and it does not follow the fast-slow law. The latter sees a high accelerated speed at the initial stage, while it becomes constant (zero accelerated speed) at the terminal stage, which also contradicts the requirement for the low slowly accelerated speed at the terminal point. An approximate solution is illustrated in fig. 2 with a relatively high accelerated speed at the initial stage and a relatively low accelerated speed at the terminal stage.

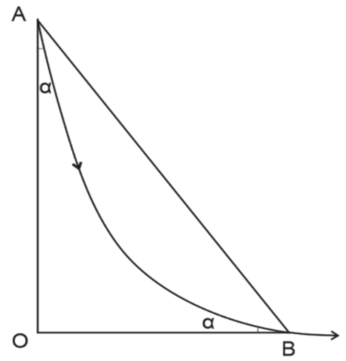

Figure 2. Brachistochrone curve

Conclusion

This paper shows importance of mathematics in thermal science, and this issue collects many mathematical methods for various problems arising thermal science, meeting research frontiers in both mathematics and thermal science.

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