RESEARCHES ON LEAKAGE WITH LUBRICATING OIL USING A COMPOSITE MULTI-PHASE LATTICE BOLTZMANN METHOD

by

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Original scientific paper
https://doi.org/10.2298/TSCI200605135L

In this paper, a novel model to investigate leakage of gaseous working fluid in pressured devices with lubricating oil was created with lattice Boltzmann method and Shan-Chen multi-phase model. A method to adapt actual pressure-density relation into the lattice via a self-adapting timestep and simplify the simulation of compressible fluid was developed. A model to simulate two-phased leakage with lubricating oil was created with a combination of Shan-Chen model and passive scalar model. The model can realize the phase distribution simulation in the leakage field without causing the pressure and the inter-phase interactions to overlap. This model is also able to be combined with other multi-phase models. After a group of preliminary tests of the model, the characteristics of phase distribution and leakage were investigated qualitatively. Five types of phase distribution in the simulation results were classified, which are: uniformed distribution, sphered drips, gas channel, blocked channel, and slug bubbles. The results of simulations show good conformance with actual leakage patterns. Preliminary discussions about the leakage features are made upon the results. However, these simulation results are only qualitative and cannot show the quantitative features in leakages. More experimental investigations should be carried out to realize correlations to the model.

Key words: *lattice Boltzmann method, passive scalar model, Shan-Chen model, leakage, lubricating oil*

Introduction

Leakage of air [1] and other refrigerants occurs in pressured devices through structural gaps could cause solemn mass and energy loss in any refrigeration or heat pump systems, especially in systems utilizing gaseous refrigerant such as CO_2 [2]. Even with high isentropic efficiency, the relative energy loss in expanders from leakage is still more than 30% in a scroll expander [3]. Experiments and static analysis have shown that lubricating oil has a significant effect on reducing leakage by filling the leaking gap [4-7]. However, only a few investigations considered using the microscopic mechanism in the modelling of leakage reduction. Therefore, it is necessary to investigate the mechanism of lubricant in the leakage reduction by numerical methods, such as the lattice Boltzmann method (LBM).

The LBM is an emerged numerical technique and became a powerful competitor to the conventional CFD methods, especially in dealing with multi-phase flows and complex

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boundary conditions. The LBM is based on the kinetic theory of gases and describes the fluid-flow physics at a mesoscopic scale [8, 9]. The method is widely used to solve many problems in fluid, heat and mass transfer and has shown a significant advantage in parallel computing [10]. In general, there are four types of multi-phase LBM models [11]. Among them, the pseudo-potential model developed by Shan and Chen is widely used to solve inter-phase problems due to its simplicity [12].

The focus of this work is to establish a basic simulation model for the working fluid leakage through the gaps of pressured devices. A pressure-based leakage model is developed by adding the Shan-Chen phase term as a passive scalar to the pressure-motivated leakage model.

Basic analyses

The leakages mentioned in this paper are mostly straight structural gaps between the connected parts of the pressured equipment. In various devices, the lubricant is distributed in different ways in the working volume. In this work, the lubricant is assumed to be uniformly distributed in the device, and the leaking gas is assumed to be static in the device, which means there is no lateral velocity at the inlet of the leakage section.

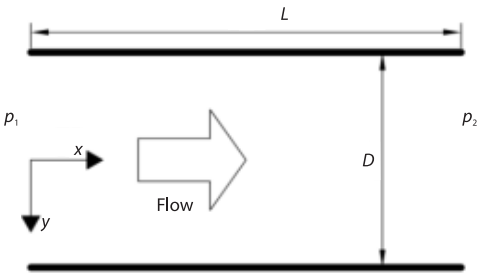

Figure 1. Sketch of the system

Since the system is complex and has many controlling factors, especially in the inter-phase interactions, a few assumptions were made to simplify the problem. We considered the leakage without lubricating oil as an ideal 2-D Poiseuille flow with a compressible gas. The leakage with lubricating oil is considered as a pressure-driven flow, which means that the pressure difference between the inlet and the outlet of the gap is the driving force of the flow through the gaps.

Figure 1 shows the sketch of the leakage system with the co-ordinate system.

Numerical method

Lattice Boltzmann method

This research utilizes a typical D2Q9 LBM model [9]:

$$f_i\left(\mathbf{x}+\mathbf{e}_i\Delta t, t+\Delta t\right)-f_i\left(\mathbf{x},t\right)=-\frac{1}{\tau}\left[f_i\left(\mathbf{x},t\right)-f_i^{\mathrm{eq}}\left(\mathbf{x},t\right)\right] \tag{1}$$

where f_i is the particle distribution function, f_i^{eq} – the corresponding equilibrium distribution, $\mathbf{x}$ – the co-ordinate vector of the lattice node, $\mathbf{e}_i$ – the particle velocity vector, Δt – the time step, t – is the time, and τ – the relaxation time of the model.

The particle speed indicated by each velocity vector is summarized in tab. 1.

Table 1. The D2Q9 particle speeds

i	1	2	3	4	5	6	7	8	9
$\mathbf{e}_i$	$(-c.c)$	$(0, c)$	$(c.c)$	$(-c, 0)$	(0.0)	$(c.0)$	$(-c, -c)$	$(0, -c)$	$(c, -c)$

where $c = \Delta x/\Delta t$ and Δx is the lattice spacing.

Pressure-driven LBM model

A single-phase, incompressible fluid-flow in a rectangular gap driven by the difference of pressure is called the Poiseuille flow, where the maximum velocity, u_{max} is:

$$u_{\max} = \frac{\Delta p}{8\mu L} D^2 \tag{2}$$

where Δp is the pressure difference and $\mu = \rho\nu$ is the dynamic viscosity, which varies slightly with changes in the temperature, and is considered constant in this work. For isothermal flow (constant temperature flow), the density is proportional to the pressure, *i.e.*, $\rho \propto p$ (ideal gas $p/\rho = RT$), L and D are the length and the width of the gap, respectively.

In this work, the pressure gradient and other related issues are simplified to ensure a constant relaxation time in the lattice Boltzmann model. The variation of the density with pressure is not that significant. Also, the variation of kinematic viscosity with pressure is ignored:

$$\mathrm{d}p / \mathrm{d}x = \text{constant} \tag{3}$$

which is the same as the situation in an incompressible fluid.

In LBM modeled ideal leakage without oil, after enough number of iterations, the maximum velocity is:

$$u'_{\max} = \frac{\Delta p'}{8\rho'_2 \alpha X} Y^2 = \frac{c_s^2\left(\rho'_1 - \rho'_2\right)}{8c_0^2\rho'_2\alpha X} Y^2 \tag{4}$$

where $\Delta p' = (\rho'_1 - \rho'_2)c_s^2/c_o^2$ is the non-dimensional pressure in the lattice units, $X = L/\Delta x$ and $Y = D/\Delta x$ are the number of lattices in x- and y-directions, respectively. The ρ'_2 is the non-dimensional density at the outlet of the duct, which is set as 1 in this work. The non-dimensional kinematic viscosity is:

$$\alpha = \frac{\nu\Delta t}{\Delta x^2} \tag{5}$$

To properly express the density in the lattice units, we get the non-dimensional density at the inlet:

$$\rho'_1 = \frac{\Delta p \rho'_2}{c_s^2 \rho_2} + \rho'_2 = \frac{\Delta p}{c_s^2 \rho_2} + 1 \tag{6}$$

To ensure the linear relationship between the actual density and the non-dimensional density (pressure as well), the timestep should be properly calculated. The density ratio can be expressed:

$$\frac{\rho'_1}{\rho_1} = \left(\frac{p_1 - p_2}{c_s^2 \rho_2} + 1\right) / \rho_1 = \frac{\rho'_2}{\rho_2} = \frac{1}{\rho_2} \tag{7}$$

We get:

$$\Delta t = \Delta x \sqrt{\frac{\rho_1 - \rho_2}{3\left(p_1 - p_2\right)}} \tag{8}$$

When assuming the density is in proportion the pressure ($p_1/\rho_1 = p_2/\rho_2$), there is:

$$\Delta t = \Delta x \sqrt{\frac{\rho_2}{3p_2}} \tag{9}$$

The bounce-back boundary is utilized at the solid, non-permeable boundaries, *i.e.*:

$$\mathbf{u} = 0 \tag{10}$$

$$f_i'(\mathbf{x},t) = f_{\text{opp}}(\mathbf{x},t) - \frac{1}{\tau}\left[f_{\text{opp}}(\mathbf{x},t) - f_{\text{opp}}^{\text{eq}}(\mathbf{x},t)\right] \tag{11}$$

The subscript opp means the opposite velocity vector to the direction *i*.

Extrapolations are used in simulating the inlet and outlet boundaries of the domain:

$$\mathbf{u}(\mathbf{x},t) = 2\mathbf{u}\left[\mathbf{x} + (c,0),t\right] - \mathbf{u}\left[\mathbf{x} + (2c,0),t\right] \tag{12}$$

and

$$\mathbf{u}(\boldsymbol{x},t) = 2\mathbf{u}\left[\boldsymbol{x} - (c,0),t\right] - \mathbf{u}\left[\mathbf{x} - (2c,0),t\right] \tag{13}$$

In addition, the non-dimensional densities at the inlet and outlet are set to ρ_1' and ρ_2', respectively.

For the fully developed ideal Poiseuille flow [13] in a duct, the velocity distribution can be expressed:

$$u = u_{\max}\left(1 - \frac{4y^2}{D^2}\right), -\frac{D}{2} < y < \frac{D}{2} \tag{14}$$

For compressible fluid, suppose the outlet density is ρ_2, we have:

$$u = \frac{\rho_2}{\rho}u_{\max}\left(1 - \frac{4y^2}{D^2}\right), -\frac{D}{2} < y < \frac{D}{2} \tag{15}$$

To verify the prediction of the developed model, a single-phase leakage model using CO_2 as the working fluid is created, where the mesh is uniformly discretized in square lattices. The comparison of the simulation results after 5000 time steps when the simulation is converged (residual error less than 10^{-9}) with the theoretical results of ideal Poiseuille flow calculated from eq. (15) is shown in fig. 2. It is shown that the error of the model is acceptable, with a max error of 2.24% at the outlet area, and 1.41% at the center line.

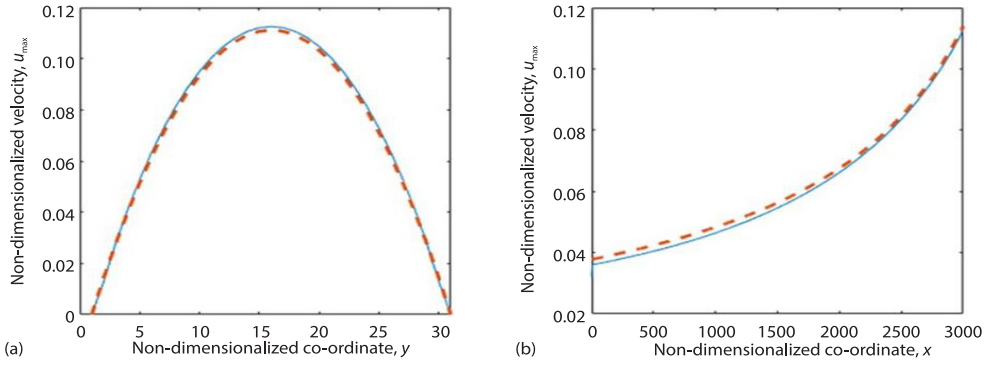

Figure 2. Verification of the single-phase model (dashed: theoretical and thin: simulated); (a) $x = 3000$ (X) and (b) $y = 15$ ($0.5Y$)

Shan-Chen model integration

The leakage is mainly driven by the pressure gradient. The inter-phase interactions are a secondary feature to affect the phase distribution. Simply utilizing Shan-Chen model will cause one of these features becomes dominant, resulting into the effect of either inter-phase interactions or pressure difference, and not both.

Therefore, before introducing Shan-Chen model into the leak model, a passive scalar model [14] is utilized to simulate the pressure-driven flow and the oil-CO_2 phase interactions simultaneously.

Another equilibrium distribution is introduced:

$$g_i^{\mathrm{eq}} = \omega_i \rho \rho_{\mathrm{p}} \left[1 + 3\frac{\mathbf{e}_i \mathbf{u}}{c_0^2} + \frac{9}{2}\frac{\left(\mathbf{e}_i \mathbf{u}\right)^2}{c_0^4} - \frac{3}{2}\frac{\mathbf{u}^2}{c_0^2} \right] \tag{16}$$

where ρ_{p} is the passive scalar of phase density that diffuses in the simulation field as well as being transported by the migration of density, ρ. In this research, ρ_{p} is the phase distribution scalar.

The collision step and the streaming step of the function g:

$$g_i'\left(\mathbf{x},t\right) = g_i\left(\mathbf{x},t\right) - \frac{1}{\tau_g}\left[g_i\left(\mathbf{x},t\right) - g_i^{\mathrm{eq}}\left(\mathbf{x},t\right) \right] \tag{17}$$

$$g_i\left(\mathbf{x} + \mathbf{c}_i \Delta t, t + \Delta t\right) = g_i'\left(\mathbf{x},t\right) \tag{18}$$

After every timestep, the varied ρ_p can be calculated:

$$\rho_{\mathrm{p}} = \sum_{i=1}^{9} g_i \,/\, \rho \tag{19}$$

In this research, the density varies between the density of CO_2 and that of the lubricating oil, which means that the variable ρ is not suitable to indicate actual density anymore. The ρ is defined as a non-dimensional pseudo-density to simulate the motivation effect of pressure p in this work.

In double-phased fluid with constant pressure and unit volume, V, assuming there is a liquid volume, V_{l}, the average density of the fluid is calculated:

$$\rho_{\mathrm{mix}} = \frac{\rho_{\mathrm{l}} V_{\mathrm{l}} + \rho_{\mathrm{g}}\left(V - V_{\mathrm{l}}\right)}{V} = \rho_{\mathrm{g}} + \frac{\rho_{\mathrm{l}} - \rho_{\mathrm{g}}}{V} V_{\mathrm{l}} \tag{20}$$

where ρ_{l} and ρ_{g} is the liquid density and the gas density for a constant pressure. This equation indicates the first-order relation between the averaged mixture density and the liquid volume in a constant pressure.

The product $\rho\rho_{\mathrm{p}}$ is defined as the non-dimensional mixture density, taking the place of the density, ρ, in the previous chapter (without phase distribution features), and the ρ_{p} is a variable which is only determined by the volume ratio of the liquid or gas phase:

$$\frac{V_{\mathrm{l}}}{V} = \frac{1}{\rho_{\mathrm{p,l}} - \rho_{\mathrm{p,g}}}\left(\rho_{\mathrm{p}} - \rho_{\mathrm{p,g}}\right) \tag{21}$$

$$\frac{V_{\mathrm{g}}}{V} = 1 - \frac{V_{\mathrm{l}}}{V} = \frac{1}{\rho_{\mathrm{p,l}} - \rho_{\mathrm{p,g}}}\left(\rho_{\mathrm{p,l}} - \rho_{\mathrm{p}} - \rho_{\mathrm{p,g}}\right) \tag{22}$$

where $\rho_{\mathrm{p,l}}$ and $\rho_{\mathrm{p,g}}$ are the ρ_{p} values for pure gas and pure liquid densities.

The core of Shan-Chen model is the forcing term determined by the phase distribution:

$$\mathbf{F}(\mathbf{x},t) = -G\psi(\mathbf{x},t)\left(\sum_{i=1}^{4}\omega_i\psi(\mathbf{x}+\mathbf{e}_i\Delta t,t)\mathbf{e}_i + \sum_{i=6}^{9}\omega_i\psi(\mathbf{x}+\mathbf{e}_i\Delta t,t)\mathbf{e}_i\right) \tag{23}$$

where G is the interaction strength parameter that determines the values of the two non-dimensional densities $\rho_{p,l}$ and $\rho_{p,g}$.

The ψ is the pseudo-potential of the interaction, which is taken as [15]:

$$\psi = 1 - \exp\left(-\rho_p\right) \tag{24}$$

The force factor works in two ways. After the calculation of macroscopic velocity **u**, the factor **F** directly act on the calculated **u** as an acceleration:

$$\mathbf{u}' = \mathbf{u} + \beta\mathbf{F} \tag{25}$$

where β is the pressure impact coefficient that indicates the impact of inter-phase interaction on the pressure distribution. The value of β could be gained from test simulations combined with experimental results.

After the collision step, the impact of the force factor, **F**, on the phase distribution function, g, is calculated:

$$g_i^F = \left(1 - \frac{1}{0.5\tau_g}\right)\left[\frac{3(\mathbf{e}_i - \mathbf{u})}{c_0} + 9\frac{\mathbf{e}_i}{c_0}\frac{(\mathbf{e}_i\mathbf{u})}{c_0^2}\right]\mathbf{F} \tag{26}$$

Then eq. (17) is modified:

$$g_i'(\mathbf{x},t) = g_i(\mathbf{x},t) - \frac{1}{\tau_g}\left[g_i(\mathbf{x},t) - g_i^{eq}(\mathbf{x},t)\right] + g_i^F \tag{27}$$

The side boundary conditions applied on the walls of the leakage used in function, g, is identical to those used in function, f. In order to simulate the uncertain inlet phase distribution, the phase density in the lattice where $x = 1$ and $x = 2$ set to a value of $\rho_{p,in}$, to meet theassumption that lubricant is uniformly distributed in the device. The $\rho_{p,in}$ is obtained by the inflow lubricant volume ratio:

$$\rho_{p,in} = \rho_g + \frac{\rho_l - \rho_g}{V}V_l = \rho_g + (\rho_l - \rho_g)v \tag{28}$$

where $v = V_l/V$.

At the outlet, a fully developed flow is assumed to ensure the stability of outlet phase density:

$$\rho_p(\mathbf{x},t) = \rho_p\left[\mathbf{x} - (c,0),t\right] \tag{29}$$

Test model and simulation results

A test model is built with the constant parameters summarized in tab. 2. The inflow phase distribution scalar $\rho_{p,in}$ is the independent variables in the tests. Note that this model shows only the basic qualitative features of this model, not referring to realistic leakage situations.

The simulation went for 10^6 iteration steps. The resulted types of phase distributions under various inlet ratio of phase content is discussed in detail in the following sections. In the following figures in this section, the horizontal co-ordinates and the vertical co-ordinates indi-

cates the non-dimensional co-ordinates in the simulated leakage field. Blue color indicates low phase density, corresponding to the gas phase, while yellow color indicates high phase density, corresponding to the liquid phase (lubricating oil). The color bar on the right shows the relationship between the colors and the values of phase density, which are non-dimensional.

Five types of phase distribution are presented.

Table 2. Parameters of the double-phase test model

Parameters	Actual value	Non-dimensional value	Parameters	Non-dimensional value
Δx	0.12 μm	–	χ	0.9264
L	0.3 mm	2500	τ	1.0698
D	3.0 μm	25	G	–5.5
p_1	0.3 MPa	–	β	0.1
p_2	0.1 MPa	–	γ	48.67
ρ_1	5.022 kg/m^3	3.0064	$\rho_{p,l}$	2.2437
ρ_2	1.661 kg/m^3	1	$\rho_{p,g}$	0.0461
μ	$1.599 \cdot 10^{-5}$ Pa·s	–	$(V_l/V)_{in}$	0.3431
u_{max}	46.902 m/s	0.1110	$\rho_{p,in}$	Variable
Δt	0.264 ns	–		

Uniformed distribution

Under $\rho_{p,in}$ is too low or too high, the inter-phase interactions are merely active due to the low or high liquid existence. Oil content of the whole leakage field hardly varies from the inlet liquid volume ratio.

Sphered drips

Under low value of $\rho_{p,in}$, the liquid phase starts to gather, but not enough to stay stable on the borders. The high velocity in the middle of the path drags the liquid into the centerline, making them gathered into sphered drips, as shown in fig. 3. The drips took the area with higher velocity and leaves the gas to areas with lower velocity, which reduces the mean velocity of the leaking gas and reduces gas leakage.

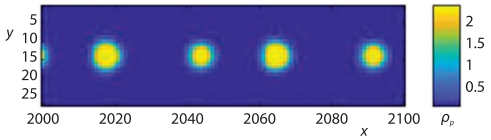

Figure 3. Partial figure of spherical droplets

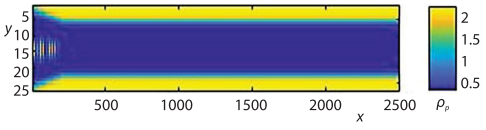

Figure 4. Compressed figure of gas channel (length compressed to 1/20)

Gas channel

Under moderate $\rho_{p,in}$, the liquid phase is enough to gather on the side borders and stay stable as tight liquid films with low velocity. Much of the lubricant flows from the inlet will be gathered to reinforce the liquid films, leaves a narrow gas channel in the middle of the path with a relatively high velocity, as shown in fig. 4. The oil films could reduce gas leakage by occupying much of the leakage field. On some occasions, the feature of gas channel will appear along with sphered drips forming in the centerline of the channel.

Blocked channel

Under high $\rho_{p,in}$, the oil film will not be able to accommodate some of the redundant oil drips, leaving them to be dragged into the centerline again. The process has caused some blockades of oil in the gas channel, as shown in fig. 5. The leakage reduction is realized mostly by the oil films, which is similar with what happens in the form of gas channel, while the blockades also provided some assistance with the leakage reduction by occupying some areas with higher velocity.

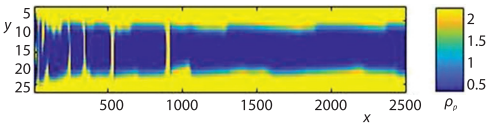

Figure 5. Compressed figure of blocked channel (length compressed to 1/20)

Slug bubbles

Under even higher $\rho_{p,in}$, much more of the oil will be dragged into the centerline, and cut the gas channel into several slug bubbles with approximate sizes, as shown previously in fig. 6. This kind of phase distribution form could be seen in most conducted observations of multiphase leakage or transport in tubes or gaps, which is known as slug flow [16]. The principles of leakage reduction are similar with the that in the form of blocked channel, while much more of the areas with higher velocity is occupied by the lubricant.

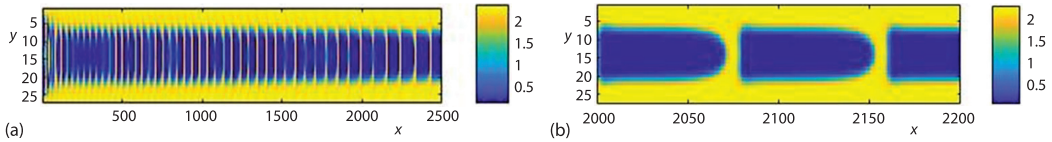

Figure 6. Compressed figures of slug bubbles; (a) length compressed to 1/20 and (b) length compressed to 1/2 - partial

Characteristics of different types of leakage

A group of simulations under wider leakage gap were conducted to further investigate the viability of the model. The parameters in these simulations are suitable for investigating the outlet part of leakage (as investigating full part of leakage requires much larger mesh and computing power) through thin walls on gas pipes or gas tanks with lubricant. The selected parameters in the model are listed in tab. 3.

Table 3. Parameters of the double-phase model

Parameters	Actual value	Non-dimensional value	Parameters	Non-dimensional value
Δx	0.12 μm	–	χ	1.0000
L	0.6 mm	5000	τ	1.0698
D	6.0 μm	50	G	–6.0
p_1	0.3 MPa	–	β	0.1
p_2	0.1 MPa	–	γ	389.79
ρ_1	5.022 kg/m^3	3.0064	$\rho_{p,l}$	2.5726
ρ_2	1.661 kg/m^3	1	$\rho_{p,g}$	0.0066
μ	1.599×10-5 Pa·s	–	$(V_l/V)_{in}$	Variable
u_{max}	93.803 m/s	0.2220		
Δt	0.284 ns	–		

The simulation was conducted under various inflow lubricant volume ratio, that is, under various phase density ρ_p. The obtained relationship between the inflow lubricant volume ratio $v = (V_l/V)_{in}$ and the leakage ratio (LR) is shown in fig. 7, where the LR is calculated:

As shown in fig. 7, it is visible that the LR with lubricant is slightly lower than the LR under the assumption that the leakage would be reduced by a ratio that is identical with the inflow lubricant volume ratio. is likely that the formation of oil drips or oil films helped with the gathering of the oil in the leakage field and reduced the gas leakage with higher efficiency. But on the other hand, it also caused more lubricant loss when the formed drips or oil blockades in the gas channels flows out fast and cannot stay in the leakage field for long.

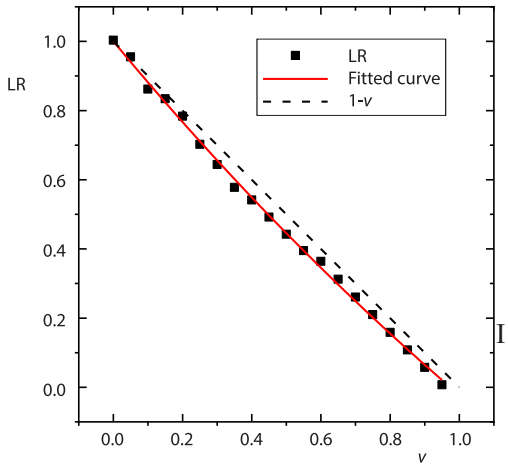

Figure 7. The LR variation with *v*

I t

Limitations of the model

These simulation results are only qualitative and cannot show the quantitative features in leakages with lubricating oil. But these qualitative results can still show some of the properties in the leakages, for example, phase distribution formations.

Shan-Chen model is often used in macroscopic situations. To increase the consistence of the model with actual leakage situations, more investigations should be carried out and could show better results via this composite model combining with multi-phase models more suitable for microscopic situations.

Due to the difficulties in the measurement of the gas leakage and the observation of the phase distribution in these microscopic dimensions, the related experiments are hard to conduct, and the verification of the model is still unavailable. These experiments should be conducted under better conditions later to verify the model and help with the model parameters.

Conclusions

In this paper, a novel model to investigate leakage of gaseous working fluid in pressured devices with lubricating oil was created with LBM and Shan-Chen multi-phase model as follows.

- A method to adapt actual pressure-density relation into the lattice via a self-adapting time-step and simplify the simulation of compressible fluid was developed.
- A model to simulate two-phased leakage with lubricating oil was created with a combination of Shan-Chen model and passive scalar model.
- A simple method to calculate leakage in lattices to indicate the effect of lubricating oil in reducing leakage is created.
- Five types of phase distribution in the simulation results were classified.
- The lubricating oil reduces leakage with higher efficiency due to its gathering features. In most cases, the quantity of oil flowing out of the leakage field is more than the oil flowing in, causing lubricant waste.

After a group of preliminary tests of the model, the characteristics of phase distribution and leakage were investigated qualitatively. Five types of phase distribution in the simulation results were classified.

There are still limitations in this study and requires further work as follows.

- These simulation results are only qualitative and cannot show the quantitative features in leakages with lubricating oil.
- More experimental investigations should be carried out to realize correlations to the model.

Acknowledgment

This research is supported by Science and Technology Program of Tianjin, China [grant number 16YFZCGX00080] and State Key Laboratory of Air-conditioning Equipment and System Energy Conservation, Gree Electric Appliances, Inc. of Zhuhai, China [Grant No. ACSKL2018KT02].

Nomenclature

c – lattice speed, [ms^{-1}]
c_s – lattice sound speed, [ms^{-1}]
D – gap width, [m]
$\mathbf{e}_i$ – particle velocity vector towards direction i, [ms^{-1}]
$\mathbf{F}$ – inter-phase force vector, [–]
f_i – particle distribution function towards direction i, [–]
f_i' – temporary post-collision particle distribution function towards direction i, [–]
f_i^{eq} – equilibrium particle distribution function towards direction i, [–]
f_{opp} – particle distribution function towards the opposite direction of i, [–]
f_{opp}^{eq} – equilibrium particle distribution function towards the opposite direction of i, [–]
G – phase interaction strength parameter, [–]
g_i – phase distribution function towards direction i, [–]
g_i' – temporary post-collision phase distribution function towards direction i, [–]
g_i^{eq} – equilibrium phase distribution function towards direction i, [–]
g_i^F – term of the inter-phase interactions in the phase distribution towards direction i, [–]
L – gap length, [m]
p – pressure, [Pa]
Δp – actual pressure gradient, [Pam^{-1}]
$\Delta p'$ – non-dimensional pressure gradient, [–]
p_1 – actual inlet pressure, [Pa]
p_2 – actual outlet pressure, [Pa]
t – time, [s]
Δt – timestep length, [s]
$\mathbf{u}$ – velocity vector, [–]
$\boldsymbol{u}'$ – velocity vector modified with inter-phase interactions, [–]
u_x – axial velocity (through the gap), [ms^{-1}]
u_y – lateral velocity (orthogonal with the gap), [ms^{-1}]
u_{max} – actual maximum velocity in a Poiseuille flow, [ms^{-1}]
u'_{max} – maximum non-dimensional velocity in a Poiseuille flow, [–]
V – volume, [m^3]
V_l – liquid volume, [m^3]
V_g – gas volume, [m^3]
v – inflow lubricant volume ratio, [–]
$\mathbf{x}$ – co-ordinate vector, [–]
Δx – lattice spacing, [m]
x – axial co-ordinate, [m]
X – number of lattices in x direction, [–]
y – lateral co-ordinate, [m]
Y – number of lattices in y direction, [–]

Greek symbols

α – non-dimensional kinematic density, [–]
β – pressure impact coefficient, [–]
γ – density ratio between liquid and gas, [–]
μ – dynamic viscosity, [Pas]
ν – kinematic viscosity, [m^{2}s^{-1}]
ρ – density, [kgm^{-3}]
ρ_1 – actual inlet density, [kgm^{-3}]
ρ_2 – actual outlet density, [kgm^{-3}]
ρ'_1 – non-dimensional inlet density, [–]
ρ'_2 – non-dimensional outlet density, [–]
ρ_p – phase density, [–]
$\rho_{p,l}$ – liquid phase density, [–]
$\rho_{p,g}$ – gas phase density, [–]
$\rho_{p,in}$ – inlet phase density, [–]
τ – relaxation time, [–]
χ – velocity correction factor, [–]
ψ – pseudo-potential of phase interaction, [–]
ω_i – weighted coefficient for velocity vector towards direction i, [–]

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Paper submitted: June 5, 2020
Paper revised: November 13, 2020
Paper accepted: February 4, 2021