LUMP SOLUTIONS FOR THE DIMENSIONALLY REDUCED VARIABLE COEFFICIENT B-TYPE KADOMTSEV-PETVIASHVILI EQUATION

by

Yanni ZHANG[a,b], Xin ZHAO[c], and Jing PANG[a,b*]

[a] College of Science, Inner Mongolia University of Technology, Hohhot, China
[b] Inner Mongolia Key Laboratory of Statistical Analysis Theory for Life Data and Neural Network Modelling, Inner Mongolia University of Technology, Hohhot, China
[c] Department of Basic, Officers College of PAP, Chengdu, China

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Based on Hirota bilinear formulation, the lump solutions to dimensionally reduced generalized variable coefficient B-type Kadomtsev-Petviashvili equation are obtained. The solution process is figured out and the solution properties are illustrated graphically. The present method can be extended to other non-linear equations.

Key words: *generalized variable coefficient B-type Kadomtsev-Petviashvili equation, Hirota bilinear form, lump solution*

Introduction

Non-linear evolution equations can model various non-linear phenomena, and have been extremely studied in plasma physics, non-linear optics, fluid mechanics, and thermodynamics [1-5]. So far many techniques for constructing solutions have been proposed, such as Painleve test [6], Darboux transformation [7], Hirota method [8], Backlund transformation [9], and Bell polynomials [10]. Through these methods, many kinds of solutions are presented. Solitons have been at the forefront of integrable systems for many years, other kinds of solutions also attract much attention. As a kind of rational function solutions, lump solutions, which localize in all directions in the space, have many applications to non-linear PDE. Recently, a hot topic is the Kadomtsev-Petviashvili (KP)-type equations [11-14], such as the (2+1)-D B-type KP equation, the (3+1)-D B-type KP equation, which can be transformed into a generalized bilinear equation. Multi-component and higher-order extensions of lump solutions exhibit diverse soliton phenomena, particularly the (3+1)-D case always leads to multiple wave solutions and lump solutions. The aim of this study is to use the Hirota bilinear forms to generate the generalized (3+1)-D variable coefficient B-type KP equation [15-17]:

$$P_{BKP}(u) = a(t)u_{xxxy} + \rho a(t)(u_x u_y)_y + (u_x + u_y + u_z)_t + b(t)(u_{xx} + u_{zz}) = 0 \tag{1}$$

where u is the wave amplitude function of the scaled space co-ordinates x, y, z, and retarded time co-ordinate t, $a(t)$ and $b(t)$ are the real function of t and ρ is a real non-zero constant.

* Corresponding author, e-mail: pang_j@imut.edu.cn

Hirota bilinear form and lump solutions

It is clear that Hirota bilinear operator D_x is just the special case of the generalized bilinear operator [12-15]. Substituting $u = (6/\rho)(\ln f)_x$ into eq. (1) and simplifying the equation, we obtain the Hirota bilinear form:

$$B_{BKP}(f) = [a(t)D_{x,3}^3 D_{y,3} + D_{x,3}D_{t,3} + D_{y,3}D_{t,3} + D_{z,3}D_{t,3} + b(t)(D_{x,3}^2 + D_{z,3}^2)]f \cdot f =$$
$$= 6a(t)(f_{xx}f_{xt} - f_{xxt}f_x) + 2(f_{xt}f - f_x f_t) + 2(f_{yt}f - f_y f_t) +$$
$$+2(f_{zt}f - f_z f_t) + 2b(t)(f_{xx}f - f_x^2 + f_{zz}f - f_z^2) = 0 \tag{2}$$

where f is a function of x, y, z, and t, the bilinear differential operators $D_{3,x}$, $D_{3,t}$, $D_{3,y}$, $D_{3,z}$, $D_{3,x}^2$, and $D_{3,z}^2$ are the Hirota bilinear operators when $p = 3$.

In this study, we construct positive quadraic function solution the dimensionally reduced Hirota bilinear equation for the case $z = x$, and a quadratic function is supposed:

$$f = g^2 + h^2 + a_9$$
$$g = a_1 x + a_2 y + a_3 t + a_4 \tag{3}$$
$$h = a_5 x + a_6 y + a_7 t + a_8$$

where $a_i (1 \le i \le 9)$ are all real parameters which we need to determine later.

The dimensionally reduced Hirota bilinear equation in (2+1)-D with $z = x$ reads:

$$6a(t)(f_{xx}f_{xt} - f_{xxt}f_x) + 4(f_{xt}f - f_x f_t) +$$
$$+2(f_{yt}f - f_y f_t) + 4b(t)(f_{xx}f - f_x^2) = 0 \tag{4}$$

Through the link between f and u, substituting eq. (3) into eq. (4), we obtain:

$$a_1 = a_1,\ a_2 = a_2,\ a_3 = -\frac{2b(t)[2a_1^3 + a_1^2 a_2 - a_2 a_5^2 + 2a_1 a_5 (a_6 + a_5)]}{2(a_1 + a_2)^2 + 2(a_5 + a_6)^2},\ a_4 = a_4$$
$$a_5 = a_5,\ a_6 = a_6,\ a_7 = -\frac{2b(t)[2a_5(a_1^2 + a_1 a_2 + a_5^2) + a_6(a_5^2 - a_1^2)]}{2(a_1 + a_2)^2 + 2(a_5 + a_6)^2},\ a_8 = a_8 \tag{5}$$
$$a_9 = \frac{3b(t)(a_1^2 + a_5^2)^2[2a_1^2 + a_1 a_2 + a_5 a_6 + 2a_5^2)]}{a(t)(a_2 a_5 - a_1 a_6)^2}$$

which needs to satisfy a determinant condition:

$$a_2 a_5 - a_1 a_6 = \begin{vmatrix} a_2 & a_1 \\ a_5 & a_6 \end{vmatrix} \ne 0$$
$$a_1 a_2 + a_5 a_6 = \begin{vmatrix} a_1 & a_5 \\ -a_6 & a_2 \end{vmatrix} \succ 0 \tag{6}$$

A class of positive quadratic function solutions to eq. (4):

$$f = [a_1 x + a_2 y - \frac{2[2a_1^3 + a_1^2 a_2 - a_2 a_5^2 + 2a_1 a_5(a_6 + a_5)]t}{2(a_1 + a_2)^2 + 2(a_5 + a_6)^2} + a_4]^2 +$$
$$+[a_5 x + a_6 y - \frac{2[2a_5(a_1^2 + a_1 a_2 + a_5^2) + a_6(a_5^2 - a_1^2)]t}{2(a_1 + a_2)^2 + 2(a_5 + a_6)^2} + a_8]^2 +$$
$$+\frac{3(a_1^2 + a_5^2)^2[2a_1^2 + a_1 a_2 + a_5 a_6 + 2a_5^2)]}{(a_2 a_5 - a_1 a_6)^2} \tag{7}$$

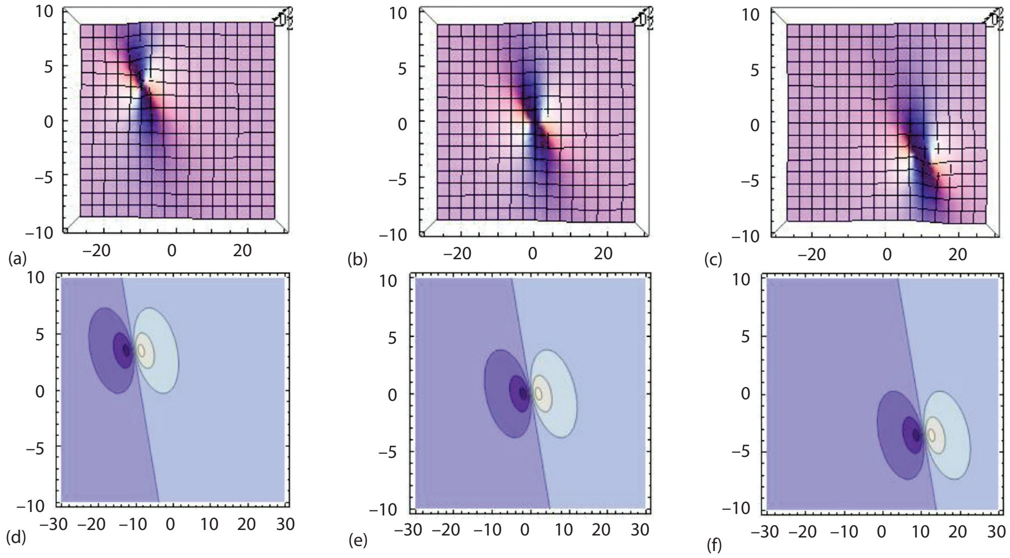

Figure 1. Profiles with $a_1 = 1$, $a_2 = 2$, $a_4 = 0$, $a_5 = 1$, $a_6 = -1$, $a_8 = 0$, when $t = -15, 0, 15$; (a)-(c) 3-D plots and (d)-(f) contour plots

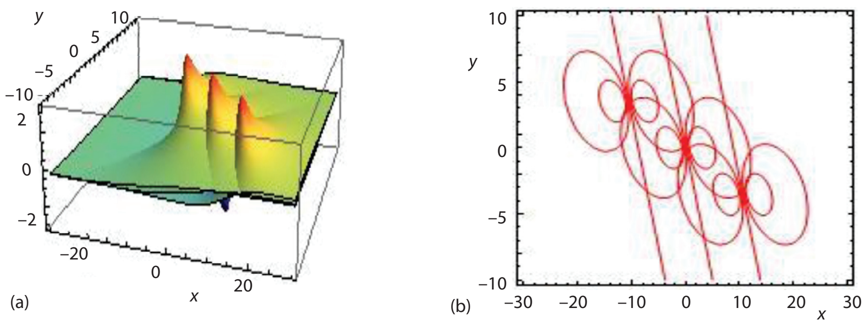

Figure 2. Profile of the solution 3-D plot together; (a) with the time $t = -15, 0, 15$ (a) and the contour plot about the moving path described by the straight line (b)

therefore, we obtain:

$$u = \frac{12(a_1 g + a_5 h)}{\rho f} \tag{8}$$

where the functions g, h are given:

$$g = a_1 x + a_2 y - \frac{2b(t)[2a_1^3 + a_1^2 a_2 - a_2 a_5^2 + 2a_1 a_5 (a_6 + a_5)]t}{2(a_1 + a_2)^2 + 2(a_5 + a_6)^2} + a_4$$
$$h = a_5 x + a_6 y - \frac{2b(t)[2a_5(a_1^2 + a_1 a_2 + a_5^2) + a_6(a_5^2 - a_1^2)]t}{2(a_1 + a_2)^2 + 2(a_5 + a_6)^2} + a_8 \tag{9}$$

The solution involves six parameters a_1, a_2, a_4, a_5, a_6, and a_8. All six involved parameters are arbitrary and the rest are demanded to satisfy the conditions eq. (6). The solutions defined by eq. (6) are analytic and only if the parameter $a_9 \succ 0$. It is easy to observe that at any given time, t, there are various possibilities to take appropriate parameters to obtain lump solutions. We choose a set of parameters to obtain a lump solution and use the 3-D plots and contour plots to show the properties as shown in figs. 1 and 2.

Conclusion

In this paper, via the Bell polynomials and Hirota method, we have derived the Hirota bilinear formulation and then by searching for positive quadratic function solutions, lump solutions to the dimensionally reduced equations are presented with $p = 3$.We obtain different lump solutions, and the result provides some important information on the relevant fields in non-linear science. Furthermore, we have performed the procedures on a generalized form of eq. (1). Many kinds of interaction solutions can be constructed.

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