FRACTAL CALCULUS FOR REFRIGERATED TRANSPORTATION OF PERISHABLE FOODS
Energy Consumption and Energy Saving

by

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This paper studies the frozen pork transportation in short and long distances, the loss of cooling capacity and the energy consumption in the cold chain transportation are analyzed experimentally and numerically. It finds that the material for freezer insulation plays in important role in energy consumption and energy saving during refrigerated transportation, an optimal freezer can reduce temperature rise by 13.88% and the heat flow by 71.05% compared with the traditional ones. A fractal model is established to reveal the thermal property of the freezer.

Key words: *fractal dimension, refrigerated transportation, energy consumption, energy saving, fractal calculus, fractal derivative, biomimetic design, wool, polar bear hair, perishable foods*

Introduction

With the fast development of China's national economy and the quick improvement of people's living standards, a cold chain of foods has been formed in China, and the transportation volume of refrigerated (frozen) foods has been improved greatly. Foods preservation has attracted increasing attention from academia and industry communities. In view of the growing demand for vehicle-mounted refrigerated (frozen) foods, a large amount of perishable foods are deteriorated during transportation and storage each year. Therefore, it has become a core issue to ensure the quality of refrigerated (frozen) foods and transport safety while saving energy and reducing energy loss. It is of great significance to study the energy consumption analysis and energy saving of refrigerated transportation of perishable foods**.** This paper aimed at solving the key technical bottlenecks by mimicking natural fibers like polar bear hairs and wools, an optimal freezer with fractal-like porous structure is discussed.

Energy consumption during refrigerated transportation

This paper used frozen pork transportation as an example to calculate the cooling loss and heat flux change in short and long distance transportations from the aspects of loading, transportation, and unloading.

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The two refrigerated transportation routes from Shanghai to Kunshan (about 1 hour drive by transportation, 65 km) and from Shanghai to Chuzhou (about 5 hours and 30 minutes by transportation, 375 km) in China are taken as the research objects. Two samples are used in temperature control comparison experiment to analyze and study the effect of two refrigerated vehicles on the cooling effect of frozen pork: refrigerated vehicle with better thermal insulation performance and refrigerated vehicle with slightly weaker insulation performance.

The heat flow of refrigerated goods (vehicle-mounted frozen pork) consists of four parts:
– the heat flow of foods,
– the heat flow of packaging materials and transportation means,
– the heat flow of breathing when the goods were cooled, and
– the heat of breathing when the goods were refrigerated.

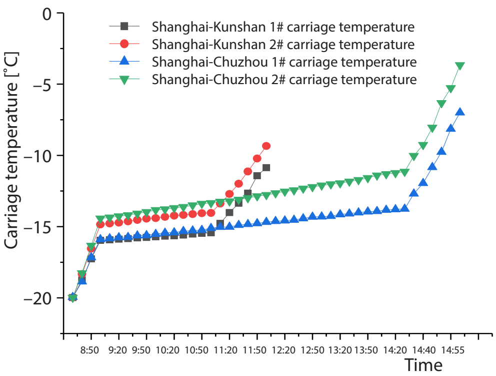

Figure 1. Temperature variation curve of vehicle-mounted frozen pork

The heat flow of *Shanghai to Kunshan* frozen pork is calculated as 59.169 kW and 85.26 kW, respectively. The heat flow of *Shanghai to Chuzhou* frozen pork is 94.62 kW and 151.91 kW, respectively.

In the process of short-distance and long-distance refrigerated transportation, the curve of vehicle-mounted frozen pork temperature change with time is shown in fig. 1. It is obvious that the temperature of second ($2^{\#}$) vehicle raised 10.81% compared with that of first ($1^{\#}$) partner, and the heat flow increased by 71.05%.

Freezer temperature simulation

An uniform temperature field of the refrigerated freezer is assumed [1, 2], and the following assumptions are made:
– the frost-prone and icing areas in the freezer are closed areas,
– the influence of frozen products and brackets on the flow field in the freezer is ignored, and
– the vehicle freezer size is 5150 mm × 2215 mm × 2250 mm, the evaporator size is 580 mm × 1000 mm × 150 mm, and the air outlet size 1000 mm × 150 m.

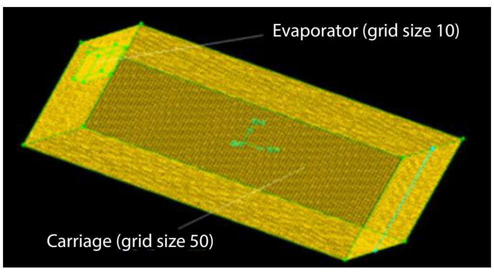

Figure 2. The grid model

The temperature in the freezer to –18 °C and the ambient temperature is 36 °C, the wind speed and temperature of the air outlet are set as and –22 °C, respectively, and the equivalent diameter is 260 mm, the required cooling capacity of the freezer is about 2000 W. The grid model is shown in fig. 2.

Three samples are simulated:
– Vacuum insulation materials with thicknesses of 50 mm and 80 mm, respectively, the numerical results are given in figs. 3-5,
– polyurethane foam with thicknesses of 80 mm and 100 mm, respectively, the numerical results are shown in figs. 6-8, respectively, and
– polystyrene foam with thicknesses of 80 mm and 100 mm, the simulated results are illustrated in figs. 9-11.

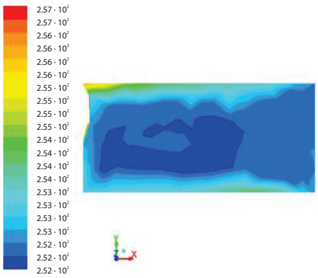

Figure 3. Temperature distribution with thickness of 50 mm

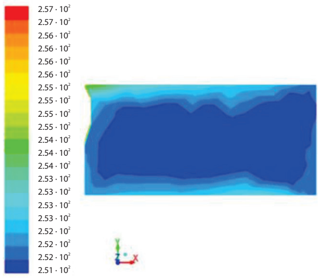

Figure 4. Temperature distribution with thickness of 80 mm

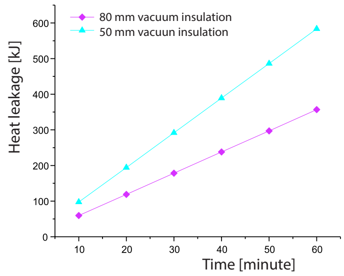

Figure 5. Curve of heat leakage with time

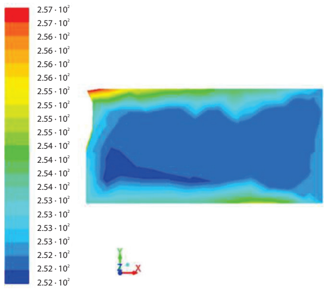

Figure 6. Temperature distribution with thickness of 80 mm

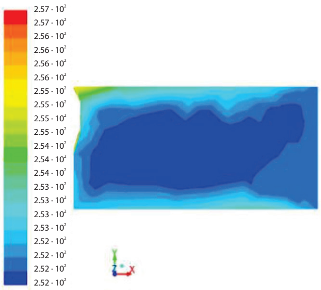

Figure 7. Temperature distribution with thickness of 100 mm

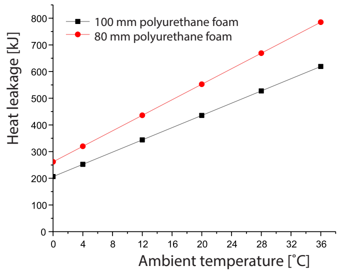

Figure 8. Curve of heat leakage with time

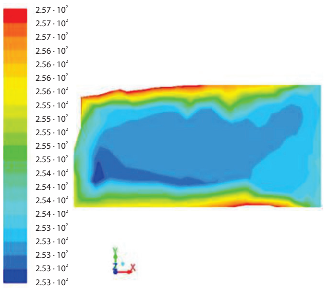

Figure 9. Temperature distribution with thickness of 80 mm

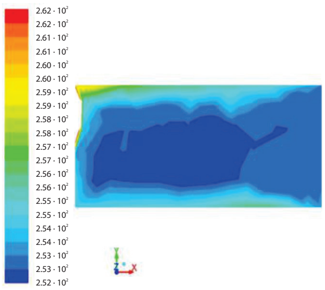

Figure 10. Temperature distribution with thickness of 100 mm

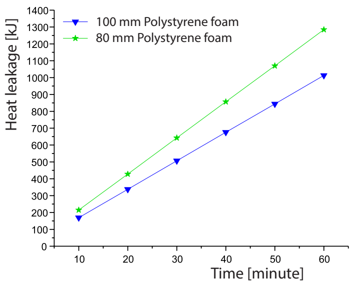

Figure 11. Curve of heat leakage with time

According to fig. 5, the heat leakage of in unit time was 266.65 kJ less than that of 50 mm insulation materials, and the energy saving was about 38.80%. According to fig. 8, the heat leakage of the 100 mm unit of polyurethane foam over 80 mm per unit time was reduced by 166 kJ and energy saving was about 21.15%. According to fig. 11, the heat leakage of the 100 mm compared with 80 mm polystyrene foam in the unit time was reduced by 271.72 kJ and energy saving was about 21.15%.

Theoretical analysis

We use 1-D fractal model to describe the heat conduction of the freezer, which is considered as a fractal-like porous structure. The heat equation for the freezer can be expressed [3-8]:

$$\frac{\partial T}{\partial t^{\alpha}}+\frac{\mathrm{d}}{\mathrm{d}x^{\beta}}\left(k\frac{\mathrm{d}T}{\mathrm{d}x^{\beta}}\right)=Q \tag{1}$$

where k is thermal conduction coefficient, Q – the source term, the fractal derivatives are defined [3-8]:

$$\frac{\partial T}{\partial t^{\alpha}}(t_0,x)=\Gamma(1+\alpha)\lim_{t-t_0\to\Delta t}\frac{T(t,x)-T(t_0,x)}{(t-t_0)^{\alpha}} \tag{2}$$

$$\frac{\partial T}{\partial x^{\beta}}(t,x_0)=\Gamma(1+\beta)\lim_{x-x_0\to\Delta x}\frac{T(t,x)-T(t,x_0)}{(x-x_0)^{\beta}} \tag{3}$$

where α and β are the fractal dimensions in time and space, respectively, Δx – the minimal porous size of the freezer, and Δt – the time needed for the heat to be transferred through Δx:

$$s=t^{\alpha} \tag{4}$$

$$\xi=x^{\beta} \tag{5}$$

Equation (1) becomes:

$$\frac{\partial T}{\partial s}+\frac{\mathrm{d}}{\mathrm{d}\xi}\left(k\frac{\mathrm{d}T}{\mathrm{d}\xi}\right)=Q \tag{6}$$

Introducing a complex variable λ defined:

$$\lambda=\xi-cs \tag{7}$$

Equation (6) becomes:

$$k\frac{\mathrm{d}^2T}{\mathrm{d}\lambda^2}-c\frac{\mathrm{d}T}{\mathrm{d}\lambda}-Q=0 \tag{8}$$

Its solution can be expressed:

$$T=a\exp\left(\frac{c}{k}\lambda\right)-\frac{Q}{c}\lambda \tag{9}$$

In view of eq. (7), we have:

$$T=a\exp\left[\frac{c}{k}(\xi-cs)\right]+\frac{Q}{c}(\xi-cs) \tag{10}$$

Finally we obtain:

$$T=a\exp\left[\frac{c}{k}(x^{\beta}-ct^{\alpha})\right]+\frac{Q}{c}(x^{\beta}-ct^{\alpha}) \tag{11}$$

It is obvious that:

$$\frac{\partial T}{\partial t}=-ct^{\alpha-1}\left\{a\exp\left[\frac{c}{k}(x^{\beta}-ct^{\alpha})\right]+\frac{Q}{c}(x^{\beta}-ct^{\alpha})\right\} \tag{12}$$

$$\frac{\partial T}{\partial t}(0,0)=\begin{cases}-c, & \alpha=1\\ 0, & \alpha>1\\ \infty, & \alpha<1\end{cases} \tag{13}$$

For a continuum freezer ($\alpha=1$), the temperature change with time is a constant. However, when $\alpha>1$, the initial temperature change is extremely small, on the other hand if $\alpha<1$

the temperature will be suddenly change at the initial stage. For our study, it requires $\alpha > 1$ for energy consumption and energy salving for a long-distance transportation.

For vacuum insulation, α is larger for the freezer with 80 mm thickness than that with 50 mm, so the former sees a better energy saving than the latter as shown in fig. 5. Similarly for the polyurethane foam and the polystyrene foam, the higher thickness implies higher value of α, as a result, more energy saving is predicted than those with lower thickness, see figs. 8 and 11.

Now we consider the initial stage when temperature changes little, under this condition, eq. (3) becomes:

$$\frac{\mathrm{d}}{\mathrm{d}x^{\beta}}\left(k\frac{\mathrm{d}T}{\mathrm{d}x^{\beta}}\right)=Q \tag{14}$$

Its solution reads:

$$T=\frac{Q}{k}x^{2\beta}+T_0 \tag{15}$$

It is obvious that:

$$\frac{\partial T}{\partial x}=\frac{\beta}{k}Qx^{2\beta-1} \tag{16}$$

$$\frac{\partial^2 T}{\partial x^2}=\frac{\beta(2\beta-1)}{k}Qx^{2\beta-2} \tag{17}$$

$$\frac{\partial^3 T}{\partial x^3}=\frac{\beta(2\beta-1)(2\beta-2)}{k}Qx^{2\beta-3} \tag{18}$$

In case $\beta > 1.5$, we have:

$$\frac{\mathrm{d}T}{\mathrm{d}x}(t,0)=\frac{\mathrm{d}^2T}{\mathrm{d}x^2}(t,0)=\frac{\mathrm{d}^3T}{\mathrm{d}x^3}(t,0)=0 \tag{19}$$

Equation (19) implies that the temperature change on the freezer boundary is extremely small when $\beta > 1.5$. It was reported that $\beta = 1.618$ for wools [9] and polar bear hairs [10-15], enabling the animals extremely good thermal property.

Discussion and conclusions

Snow and cocoon have good thermal insulation property [7, 8] due to their hierarchical structure, wools and pole bear hairs also have good thermal property due to their special geometric structure with fractal dimension of 1.618 [9-16], all those natural phenomena can be used for biomimetic design of freezers in future, and the fractal theory and fractal calculus [5, 6] provide us with good mathematical tools to theoretical analysis of the thermal properties of the freezers.

The cold storage plate refrigerated vehicle is suitable for the transportation of frozen pork over short distances (driving time 3 hours), and the solar-semiconductor refrigerated vehicle is suitable for long distances (driving time was between 3~12 hours) transport of frozen pork. An economical operation optimization mode suitable for keeping frozen pork and reducing energy consumption is proposed: 100 mm thick polystyrene foam is selected as the thermal insulation material for frozen pork short-distance transportation. Long-distance transportation could choose:

- The freezer with good thermal insulation performance and 50 mm thick vacuum insulation material.

- The 100 mm thick polyurethane foam freezer to ensure the frozen pork storage quality during transportation.

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