STATISTICAL INFERENCE ON THE ACCELERATED COMPETING FAILURE MODEL FROM THE INVERSE WEIBULL DISTRIBUTION UNDER PROGRESSIVELY TYPE-II CENSORED DATA

by

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In this paper, the parameter estimation is discussed by using the maximum likelihood method when the available data have the form of progressively censored sample from a constant-stress accelerated competing failure model. Normal approximation and bootstrap confidence intervals for the unknown parameters are obtained and compared numerically. The simulation results show that bootstrap confidence intervals perform better than normal approximation. A thermal stress example is discussed.

Key words: *constant-stress accelerated competing failure model, thermal stress, inverse Weibull distribution, Markov chain Monte-Carlo method*

Introduction

As the statistical analysis of engineering, econometric and other fields, the competing failure model plays an important role. In fact, many conditions can lead to the failure of a life test, which may be caused by one of several factors. Usually failure time is defined to be the earliest occurrence among all these risks, so there is much literature for the competing failure model. Several scholars have given statistical inference based on the competing failure model [1-4]. Because many competing failure products with high reliability can work for a long time, so various accelerated life tests (ALT) are widely used to study the product lifetimes with long-life in order to reduce the testing time and cost in the experiment. In ALT, the units are run at higher-than-use stresses, such as thermal, voltage and mechanical stress and so on. In this paper, an accelerated thermal ageing technique is used with the application of the Arrhenius model. Under the accelerated competing failure model, Wu *et al.* [5] made inference in Weibull distribution. Constant-stress testing is a widely used in ALT [6-8]. Moreover, in order to save time and reduce cost in ALT, censoring is considered in reliability experiments. The main advantage of censoring is to speed up the experiment and get the effective number of failures. The Type-I, Type-II, progressively Type-I and progressively Type-II censoring are the most common censoring schemes (CS) which are applied in ALT [9-13]. The two-parameter inverse Weibull (IW) distribution with upside-down bathtub shaped failure rate is popular as the lifetime distribution in a life-testing when the data indicate non-monotone or unimodal hazard functions. Extensive work has been done on the IW distribution [14-16].

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In this paper, we discuss a constant-stress accelerated competing failure model under progressively Type-II censoring when the lifetime distribution of the different risk factors is independently IW.

Model description and basic assumption

Model description

For constant-stress accelerated life test (CS-ALT) with the normal stress level S_0 and k accelerated stress levels $S_1, S_2, \cdots, S_k$ $(S_1 < S_2 < \cdots < S_k)$, n test units are randomly divided into k groups, and the size of the i^{th} group is n_i $(i = 1, 2, \cdots, k)$. Suppose that there are two independent causes of failure, and each unit failure is caused only by one of two failure factors. Under stress, S_i, the progressively Type-II censoring life test can be described. The first failure takes place, R_{i1} units are progressively removed from the remaining survived units, we get the sample $(t_{i1}, \delta_{i1}, R_{i1})$. Similarly, following the second failure takes place, R_{i2} of the remaining units are progressively removed from the remaining survived units and so on. When the m_i^{th} $(m_i < n_i)$ failure t_{im_i} occurs, all the remaining units R_{im} are removed and the test terminates. Here $m_i, R_{i1}, R_{i2}, \cdots, R_{im_i}$ $(m_i + R_{i1} + R_{i2} + \cdots + R_{im_i} = n_i)$ are prefixed constants. The final observation sample is $(t_{i1}, \delta_{i1}, R_{i1}), (t_{i2}, \delta_{i2}, R_{i2}), \ldots, (t_{im_i}, \delta_{im_i}, R_{im_i})$, $i = 1, 2, \cdots, k$, where $t_{i1}, t_{i2}, \ldots, t_{im_i}$ are order statistics, $\delta_{im_i} \in \{1, 2\}$. Let:

$$I_j(\delta_{im_i}) = \begin{cases} 1, & \delta_{im_i} = j \\ 0, & \delta_{im_i} \neq j \end{cases}$$

that is the indicator function.

Basic assumption

- There is just a cause leading to the failure in the life testing. The failure time of the unit is $T = \min\{T_1, T_2\}$.
- The lifetime follows the IW distribution $\text{IW}(\alpha_j, \lambda_{ij})$ with shape parameter, α_i, and scale parameter, λ_{ij}:

$$f_{ij}(t; \alpha_j, \lambda_{ij}) = \alpha_j \lambda_{ij} t^{-(\alpha_j + 1)} \mathrm{e}^{-\lambda_{ij} t^{-\alpha_j}} \tag{1}$$

$$F_{ij}(t; \alpha_j, \lambda_{ij}) = \mathrm{e}^{-\lambda_{ij} t^{-\alpha_j}} \tag{2}$$

with $t > 0, \alpha_j > 0, \lambda_{ij} > 0$.

- The failure causes are the same under different stress levels. So they have the same shape parameters $\alpha_{1j} = \alpha_{2j} = \ldots = \alpha_{kj} = \alpha_j \, (j = 1, 2)$.
- The scale parameter, λ_{ij}, agrees with a log-linear function of stress:

$$\ln \lambda_{ij} = a_j + b_j \varphi(s_i) \; (i = 1, 2, \ldots, k; j = 1, 2) \tag{3}$$

where a_j and b_j are unknown coefficient parameters, $\varphi(s_i)$ is a given decreasing function of stress level s_i. In particular, when the stress is temperature, it offers the Arrhenius model that is $\varphi(s_i) = 1/s_i$, and when voltage is the stress, it turn to be the inverse power law model with $\varphi(s_i) = \ln s_i$.

Maximum likelihood estimates

Likelihood function

Based on progressively Type-II censored competing failure data $(t_{i1},\delta_{i1},R_{i1})$, $(t_{i2},\delta_{i2},R_{i2}),\dots,(t_{im_i},\delta_{im_i},R_{im_i})$, $i=1,2,\dots,k$, the likelihood function can be obtained.

$$L_{ij}=C\prod_{l=1}^{m_i}[f_{ij}(t_{il})]^{I_j(\delta_{il})}[1-F_{ij}(t_{il})]^{1-I_j(\delta_{il})}[1-F_{ij}(t_{il})]^{R_{il}} \tag{4a}$$

By substituting eqs. (1) and (2) into eq. (4a):

$$L_{ij}=C\alpha_j^{n_{ij}}\lambda_{ij}^{n_{ij}}\left(\prod_{l=1}^{m_i}t_{il}^{I_j(\delta_{il})}\right)^{-(\alpha_j+1)}\exp\left[-\sum_{l=1}^{m_i}\lambda_{ij}t_{il}^{-\alpha_j}I_j(\delta_{il})\right]\prod_{l=1}^{m_i}[1-\exp(-\lambda_{ij}t_{il}^{-\alpha_j})]^{[1-I_j(\delta_{il})+R_{il}]} \tag{4b}$$

where C is constant, $n_{ij}=\Sigma_{l=1}^{m_i}I_j(\delta_{il})$ denotes the failure numbers of the j^{th} cause under stress level S_i:

$$L_j=\prod_{i=1}^{k}L_{ij}\propto\alpha_j^{\sum_{i=1}^{k}n_{ij}}\left(\prod_{i=1}^{k}\lambda_{ij}^{n_{ij}}\right)\left[\prod_{i=1}^{k}\prod_{l=1}^{m_i}t_{il}^{I_j(\delta_{il})}\right]^{-(\alpha_j+1)}\cdot$$

$$\cdot\exp\left[-\sum_{i=1}^{k}\sum_{l=1}^{m_i}\lambda_{ij}t_{il}^{-\alpha_j}I_j(\delta_{il})\right]\left\{\prod_{i=1}^{k}\prod_{l=1}^{m_i}[1-\exp(-\lambda_{ij}t_{il}^{-\alpha_j})]^{1-I_j(\delta_{il})+R_{il}}\right\} \tag{5}$$

$$L=\prod_{j=1}^{2}L_j \tag{6}$$

Point estimation

Let $\Theta_j=(\alpha_j,\lambda_{1j},\lambda_{2j},\dots,\lambda_{kj})\ (j=1,2)$, the log-likelihood function can be written based on eq. (5):

$$l_j=\ln L_j\propto\sum_{i=1}^{k}n_{ij}\ln\alpha_j+\sum_{i=1}^{k}n_{ij}\ln\lambda_{ij}-(\alpha_j+1)\left\{\sum_{i=1}^{k}\sum_{l=1}^{m_i}[I_j(\delta_{il})\ln t_{il}]\right\}-$$

$$-\sum_{i=1}^{k}\sum_{l=1}^{m_i}\lambda_{ij}t_{il}^{-\alpha_j}I_j(\delta_{il})+\sum_{i=1}^{k}\sum_{l=1}^{m_i}[1-I_j(\delta_{il})+R_{il}]\ln[1-\exp(-\lambda_{ij}t_{il}^{-\alpha_j})] \tag{7}$$

Taking the first partial derivative of eq. (7) with respect to α_j, λ_{ij} and equating them to zeros:

$$\frac{\partial l_j}{\partial\alpha_j}=\frac{\sum_{i=1}^{k}n_{ij}}{\alpha_j}-\sum_{i=1}^{k}\sum_{l=1}^{m_i}I_j(\delta_{il})\ln t_{il}+\sum_{i=1}^{k}\sum_{l=1}^{m_i}\lambda_{ij}t_{il}^{-\alpha_j}I_j(\delta_{il})\ln t_{il}-$$

$$-\sum_{i=1}^{k}\sum_{l=1}^{m_i}\frac{[1-I_j(\delta_{il})+R_{il}]\exp(-\lambda_{ij}t_{il}^{-\alpha_j})\lambda_{ij}t_{il}^{-\alpha_j}\ln t_{il}}{1-\exp(-\lambda_{ij}t_{il}^{-\alpha_j})}=0 \tag{8}$$

$$\frac{\partial l_j}{\partial \lambda_{ij}} = \frac{n_{ij}}{\lambda_{ij}} - \sum_{i=1}^{m_i} t_{il}^{-\alpha_j} I_j(\delta_{il}) + \sum_{i=1}^{m_i} \frac{[1 - I_j(\delta_{il}) + R_{il}]\exp(-\lambda_{ij} t_{il}^{-\alpha_j}) t_{il}^{-\alpha_j}}{1 - \exp(-\lambda_{ij} t_{il}^{-\alpha_j})} = 0 \tag{9}$$

with $i = 1, 2, \ldots, k,\ j = 1, 2.$

It is obviously seen that it is hard to get the closed from solutions, we can obtain the maximum likelihood estimates of the parameters α_j and λ_{ij} $(i = 1, 2, \ldots, k,\ j = 1, 2)$ form eqs. (8) and (9) by the Newton-Raphson iteration.

Asymptotic confidence intervals

According to the asymptotic likelihood theory, we get the information matrix of Θ_j, the elements of which are negative second derivatives of l_j:

$$I_{ii} = -\frac{\partial^2 l_j}{\partial \lambda_{ij}^2} = \frac{n_{ij}}{\lambda_{ij}^2} + \sum_{i=1}^{m_i} \frac{[1 - I_j(\delta_{il}) + R_{il}]\exp(-\lambda_{ij} t_{il}^{-\alpha_j}) t_{il}^{-2\alpha_j}}{[1 - \exp(-\lambda_{ij} t_{il}^{-\alpha_j})]^2} \tag{10}$$

$$I_{i(k+1)} = -\frac{\partial^2 l_j}{\partial \lambda_{ij} \partial \alpha_j} = -\sum_{l=1}^{m_i} t_{il}^{-\alpha_j} I_j(\delta_{il}) \ln t_{il} -$$

$$-\sum_{l=1}^{m_i} \frac{[1 - I_j(\delta_{il}) + R_{il}] t_{il}^{-\alpha_j} \ln t_{il} \exp(-\lambda_{ij} t_{il}^{-\alpha_j})[\exp(-\lambda_{ij} t_{il}^{-\alpha_j}) - 1 + \lambda_{ij} t_{il}^{-\alpha_j}]}{[1 - \exp(-\lambda_{ij} t_{il}^{-\alpha_j})]^2} \tag{11}$$

$$I_{(k+1)(k+1)} = -\frac{\partial^2 l_j}{\partial \alpha_j^2} = \frac{1}{\alpha_j^2} \sum_{i=1}^{k} n_{ij} + \sum_{i=1}^{k} \sum_{l=1}^{m_i} \lambda_{ij} t_{il}^{-\alpha_j} I_j(\delta_{il})(\ln t_{il})^2 +$$

$$+\sum_{i=1}^{k} \sum_{l=1}^{m_i} \frac{[1 - I_j(\delta_{il}) + R_{il}]\lambda_{ij}(\ln t_{il})^2 t_{il}^{-\alpha_j} \exp(-\lambda_{ij} t_{il}^{-\alpha_j})[\exp(-\lambda_{ij} t_{il}^{-\alpha_j}) - 1 + \lambda_{ij} t_{il}^{-\alpha_j}]}{[1 - \exp(\lambda_{ij} t_{il}^{-\alpha_j})]^2} \tag{12}$$

$$I_{ij} = I_{ji} = 0 \ \left(i = 1, 2, \ldots, k;\ j = i+1, \quad i+2, \ldots, k\right) \tag{13}$$

$$I_{i(k+1)} = I_{(k+1)i} \ \left(i = 1, 2, \ldots, k\right) \tag{14}$$

In fact, the explicit expressions for the information matrix cannot be given in the closed form. Then, the observed Fisher information matrices:

$$\hat{I}(\Theta_j) = \begin{pmatrix} I_{11} & \cdots & I_{1(k+1)} \\ \vdots & \ddots & \vdots \\ I_{(k+1)1} & \cdots & I_{(k+1)(k+1)} \end{pmatrix}$$

and the approximate asymptotic variance-covariance matrix can be given by $\hat{I}(\Theta_j)^{-1}$.

Therefore, the approximate $100(1-r)\%$ confidence intervals for the parameters α_j, $\lambda_{ij}(i = 1, 2, \ldots, k,\ j = 1, 2)$ are, respectively, expressed:

$$\left[\hat{\lambda}_{ij} - Z_{\gamma/2}\sqrt{\hat{V}_{ii}}, \quad \hat{\lambda}_{ij} + Z_{\gamma/2}\sqrt{\hat{V}_{ii}}\right] \tag{15}$$

$$\left[\hat{\alpha}_j - Z_{\gamma/2}\sqrt{\hat{V}_{(k+1)(k+1)}}, \hat{\alpha}_j + Z_{\gamma/2}\sqrt{\hat{V}_{(k+1)(k+1)}}\right] \tag{16}$$

where $Z_{\gamma/2}$ is the percentile of the standard normal distribution with right probability.

Bootstrap confidence intervals

Compared to the traditional method, the bootstrap method is known to reduce computation. Relying on the observation information other than any subjective assumption, the bootstrap method can expand sample size by simulation so as to compensate the shortage of data. Bootstrap samples are generated.

- Based on the original data obtain the maximum likelihood estimates of α_j, $\lambda_{ij}(i=1,2,\ldots,k)$, denoted by $\hat{\Theta}_j = (\hat{\alpha}_j, \hat{\lambda}_{1j}, \hat{\lambda}_{2j}, \ldots, \hat{\lambda}_{kj})$ $(j=1,2)$.
- Based on $\hat{\Theta}_j$ $(j=1,2)$ generate a new bootstrap sample, and calculate the new maximum likelihood estimates for parameters, denoted by:

$$\hat{\Theta}_j^{(1)} = \left[\hat{\alpha}_j^{(1)}, \hat{\lambda}_{1j}^{(1)}, \hat{\lambda}_{2j}^{(1)}, \ldots, \hat{\lambda}_{kj}^{(1)}\right] \;\; (j=1,2)$$

- Repeat step $2(N-1)$ times, we can generate N different bootstrap samples for $\hat{\Theta}_j$.
- With the previously acquired bootstrap samples, we now obtain the two sided 100 $(1-r)$% Bootstrap confidence intervals for parameters as follows:

$$\left[\hat{\lambda}_{ij} - Z_{\gamma/2}\sqrt{\hat{V}ar(\lambda_{ij})}, \hat{\lambda}_{ij} + Z_{\gamma/2}\sqrt{\hat{V}ar(\lambda_{ij})}\right] \tag{17}$$

$$\left[\hat{\alpha}_j - Z_{\gamma/2}\sqrt{\hat{V}ar(\hat{\alpha}_j)}, \hat{\alpha}_j + Z_{\gamma/2}\sqrt{\hat{V}ar(\hat{\alpha}_j)}\right] \tag{18}$$

where

$$Var(\hat{\lambda}_{ij}) \approx \frac{1}{N-1}\sum_{l=1}^{N}(\hat{\lambda}_{ij}^{\;l} - \hat{\lambda}_{ij})^2 \text{ and } Var(\hat{\alpha}_j) \approx \frac{1}{N-1}\sum_{l=1}^{N}(\hat{\alpha}_j^{\;l} - \alpha_j)^2$$

Simulation and data analysis

In this subsection, a Monte-Carlo are provide to investigate the proposed methods. Using the algorithm of Balakrishnan and Sandhu [17] and life-stress model [18], the progressively Type-II censored samples are generated under different choice of parameters $\alpha_j, \lambda_{ij}\left(i=1,2,\, j=1,2\right)$. Consider a two-level constant-stress ALT with two competing failure cause under progressively Type-II censoring. By eq. (2), we give $N=1000$, $S_1=200K$, $S_2=250K$, $a_1=1.6$, $b_1=600$, $a_2=-1.5$, $b_2=1000$, so $\ln\lambda_{i1}=1.6+600/S_i$ for failure Cause 1 and $\ln\lambda_{i2}=-1.5+1000/S_i$ for failure Cause 2. We also obtain the initial value for the scale parameters $\lambda_{11}=99.5$, $\lambda_{12}=33.1$, $\lambda_{21}=54.6$, and $\lambda_{22}=12.2$. The initial value for the shape parameters are $\alpha_1=1.2$ and $\alpha_2=0.8$. Moreover the initial sample size n was chosen to be 40, 60, 80. We considered units randomly removed from the experiment with probabilities p = 0.4, 0.5, and 0.6, at each failure in tab. 1.

Furthermore, based on tab. 1, a parametric percentile asymptotic confidence intervals and bootstrap confidence intervals are proposed for parameters $\alpha_j, \lambda_{ij}(i=1,2,\, j=1,2)$. We considered units randomly removed from the experiment with probability $p=0.5$ in tab. 2.

Table 1. The estimates of the parameters in different schemes

n	p_1	n_1	n_2	α_1	α_2	λ_{11}	λ_{12}	λ_{21}	λ_{22}
40	0.4	24	16	0.691	0.350	39.582	11.816	100.947	4.835
60	0.4	36	24	0.759	0.464	56.656	14.473	95.832	6.154
80	0.4	48	32	0.890	0.487	68.124	17.881	74.457	6.814
40	0.5	24	16	0.813	0.551	45.116	18.112	77.062	6.297
60	0.5	36	24	0.925	0.682	63.170	18.990	68.430	7.302
80	0.5	48	32	1.094	0.711	72.010	21.846	60.074	8.905
40	0.6	24	16	0.796	0.459	45.074	15.753	82.756	5.309
60	0.6	36	24	0.844	0.535	57.216	17.601	79.022	6.712
80	0.6	48	32	0.912	0.634	67.517	19.057	69.467	7.801

In tab. 1, we can find that the maximum likelihood estimation of parameters are closer to the truth values with the increase of sample size. But when the p = 0.5 increases to the p = 0.6, the estimation accuracy decreases. From tab. 2 shows, we can clearly see that the confidence lengths of bootstrap confidence intervals are smaller than that of the asymptotic confidence intervals for all cases. The Bootstrap method is superior to the maximum likelihood estimation in the average confidence lengths of the 95% interval for parameters.

Table 2. The estimates of the parameters and 95% intervals of the parameters

Parameter	Estimates	n = 40	n = 60	n = 80
α_1	ACI	[0.499, 1.236]	[0.579, 1.428]	[0.641, 1.336]
	BCI	[0.574, 1.174]	[0.633, 1.130]	[0.866, 1.369]
α_2	ACI	[0.235, 1.958]	[0.553, 1.724]	[0.512, 1.612]
	BCI	[0.338, 1.219]	[0.626, 1.701]	[0.645, 1.359]
λ_{11}	ACI	[0,93.219]	[14.703, 128.562]	[35.882, 106.376]
	BCI	[35.286, 85.932]	[27.962, 118.025]	[43.205, 90.358]
λ_{12}	ACI	[5.355, 58.928]	[9.134, 43.014]	[10.412, 40.627]
	BCI	[7.690, 43.615]	[9.720, 37.592]	[13.088, 34.988]
λ_{21}	ACI	[22.345, 156.365]	[30.200, 112.289]	[39.056, 107.025]
	BCI	[53.707, 123.970]	[43.834, 107.718]	[40.036, 92.532]
λ_{22}	ACI	[0.800, 17.087]	[2.064, 15.540]	[3.329, 13.544]
	BCI	[1.780, 14.311]	[2.105, 13.492]	[4.036, 11.695]

Illustrative example

In this section, areal data is discussed. The data set about insulated system of electromotor form Nelson [19]. The original data consists of three failure modes: the Turu, Phase, and Ground. For the proposed method, the paper use the data from 220 °C and 240 °C with the Turu and Ground failure causes are presented in tab. 3. Using the aforementioned approach, we get the parameters as $a_1 = 11.6$, $b_1 = 9.8$, $a_2 = -1.5$, $b_2 = -2.4$.

Table 3. Insulated system failure time data with its cause of failure

Temperature	Failure times and failure causes (1 = Turu; 2 = Ground)
220 °C	1764(1), 2436(1), 2436(2), 2436(1), 2436(2), 2436(1), 3180(1), 3180(1), 3180(1), 3180(1)
240 °C	1175(2), 1175(2), 1521(1), 1569(1), 1617(1), 1665(1), 1665(1), 1713(1), 1761(1), 1953(1)

Conclusion

In this paper, we discuss a constant-stress accelerated competing failure model. The maximum likelihood estimates of unknown parameters are given by establishing the likelihood equations. The confidence intervals of the parameters are established by the approximate method and the Bootstrap method. The simulation results show that the maximum likelihood estimation of parameters are closer to the truth values with the increase of sample size, and $p = 0.5$ is the best randomly removed probability. The Bootstrap method is superior to the maximum likelihood estimation in the average confidence lengths of the 95% interval for parameters, mainly because of the approximate interval of the maximum likelihood estimation is based on the approximate variance covariance matrix, and the approximation is good when the sample size is large enough. A real thermal stress data set was presented to illustrate the application proposed method in practice.

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