# FIXED POINT THEOREM FOR COMPATIBLE MAPPINGS IN INTUITIONISTIC FUZZY 3-METRIC SPACES

by

***Hassan M. ABU-DONIA*[a], *Hany A. ATIA*[a], *and Omnia M. A. KHATER*[a,b*]**

[a] Department of Mathematics, Faculty of Science, Zagazig University, Zagazig, Egypt
[b] Department of Basic Science, Zagazig Higher Institute of Engineering and Technology, Zagazig, Egypt

Original scientific paper
https://doi.org/10.2298/TSCI20S1371A

*In this paper, we introduced the concept of weak compatible of type (α) and asymptotically regular defined on intuitionistic fuzzy 3-metric space and proved the uniqueness and existence the fixed point theorem for five mappings from a complete intuitionistic fuzzy 3-metric space into itself under weak compatible of type (α) and asymptotically regular. The used definitions and theorem show the practice of our main idea.*

Key words*: fixed point, intuitionistic fuzzy 3-metric space, vompatible mappings*

## Introduction

Functional analysis science which is divided into two main parts (linear and non-linear), is a branch of mathematical analysis where it based on the study of vector spaces endowed with some kind of limit-related structure (*e. g.*, inner product, norm, topology*, etc.*) and the linear functions defined on these spaces and respecting these structures in a suitable sense. Studying the formulation properties of the transformations functions, operators between function spaces and spaces of functions are considered as the historical roots of this branch of science. The importance of this science is clearly showing in studying the integral and differential equations [1-10].

Zadeh [11] introduced the theory of fuzzy sets and showed successful application in many fields. The concept of fuzzy metric spaces in different ways was presented by Kramosil and Michalek [12] and proved fixed point theorems. Park [13] introduced the concept of intuitionistic fuzzy metric space by using continuous $t$-norm and continuous $t$-conorm. Abu-Donia *et al*. [14] studied the common fixed-point theorems in intuitionistic fuzzy metric space and intuitionistic, $(\phi, \varphi)$-contractive mappings. The definition of 2-metric space was described by Gahler [15] and proved some applications. Sharma [16] introduced the concept of fuzzy metric spaces. Mursaleen and Lohani [17] modified the concept of intuitionistic fuzzy metric spaces to intuitionistic fuzzy 2-metric spaces. Saadati and Park [18] defined the concept of weak compatible mapping in intuitionistic fuzzy 2-metric spaces. Abu-Donia *et al*. [19] proved some fixed-point theorems in fuzzy 2-metric space under $\psi$-contractive mappings. Chauhan *et al*. [20] gave the concepts of intuitionistic fuzzy 3-metric spaces. Nigam and Pagey [21] defined the concept of asymptotically regular defined on intuitionistic fuzzy 2-metric spaces. Abu-Donia *et al*. [22] presented fixed-point theorem by using $\psi$-contraction and $(\phi, \varphi)$-contraction mapping in probabilistic 2-metric space.

[*] Corresponding author, e-mail: Omnia.khater1693@yahoo.com

In this paper, we establish the concept of asymptotically regular defined on intuitionistic fuzzy 3-metric spaces and presented the idea of weak compatible of type ($\alpha$) and proved the common fixed-point theorem in five mappings under some contractive mappings.

*Definition 1*. [23] A binary operation $*:[0, 1]^4 \to [0, 1]$ is continuous $t$-norm if $*$ satisfies the following conditions:

i. $*$ is commutative and associative,
ii. $*$ is continuous,
iii. $a_1 * 1 = $ a, for all $a \in [0, 1]$, and
iv. $a_1 * b_1 * c_1 * d_1 \le a_2 * b_2 * c_2 * d_2$ whenever $a_1 \le a_2$, $b_1 \le b_2$, $c_1 \le c_2$, $d_1 \le d_2$, and for all $a_1$, $b_1$, $c_1$, $d_1$, $a_2$, $b_2$, $c_2$, $d_2 \in [0, 1]$.

For example $a*b*c*d = \min\{a, b, c, d\}$.

*Definition 2*. [23] A binary operation $\diamond:[0, 1]^4 \to [0, 1]$ is continuous $t$-norm if $\diamond$ satisfies the following conditions:

i. $\diamond$ is commutative and associative,
ii. $\diamond$ is continuous,
iii. $a \diamond 0 = a$, for all $a \in [0, 1]$,
iv. $a_1 \diamond b_1 \diamond c_1 \diamond d_1 \le a_2 \diamond b_2 \diamond c_2 \diamond d_2$ whenever $a_1 \le a_2$, $b_1 \le b_2$, $c_1 \le c_2$, $d_1 \le d_2$ and for all $a_1$, $b_1$, $c_1$, $d_1$, $a_2$, $b_2$, $c_2$, $d_2 \in [0, 1]$.

For example $a \diamond b \diamond c \diamond d = \max\{a, b, c, d\}$.

*Definition 3.* [23] Let $(X, M, N,*, \diamond)$ is said to be intuitionistic fuzzy 3-metric space if $X$ is an arbitrary set, $*$ is continuous $t$-norm, $\diamond$ is continuous $t$-conorm, and $M$, $N$ are intuitionistic fuzzy sets on $X^4 \times [0, \infty) \to [0, 1]$ satisfying the conditions:

i. $M(x, y, z, w, t) + N(x, y, z, w, t) \le 1$,
ii. $M(x, y, z, w, 0) = 0$,
iii. $M(x, y, z, w, t) = 1$, for all t > 0. Only when at least two of the three simplexes $(x, y, z, w)$ degenerate,
iv. $M(x, y, z, w, t) = M(x, z, y, w, t) = M(w, z, y, x, t) = M(w, z, x, y, t)$,
v. $M(x, y, z, w, t_1 + t_2 + t_3 + t_4) \ge M(x, y, z, w, t_1) * M(x, y, z, w, t_2) * M(x, y, z, w, t_3) * M(x, y, z, w, t_4)$,
vi. $M(x, y, z, w, .):[0, \infty) \to [0, 1]$ is left continuosus,
vii. $\lim_{t\to\infty} M(x, y, z, w, t) = 1$,
viii. $N(x, y, z, w, 0) = 1$,
ix. $N(x, y, z, w, t) = 0$, for all $t > 0$. Only when at least two of the three simplexes $(x, y, z, w)$ degenerate,
x. $N(x, y, z, w, t) = N(x, z, y, w, t) = N(w, z, y, x, t) = N(w, z, x, y, t)$
xi. $N(x, y, z, w, t_1 + t_2 + t_3 + t_4) \le N(x, y, z, w, t_1) \diamond N(x, y, z, w, t_2) \diamond N(x, u, z, w, t_3) \diamond N(u, y, z, w, t_4)$,
xii. $N(x, y, z, w,.):[0, \infty) \to [0, 1]$ is right continuosis, and
xiii. $\lim_{t\to\infty} N(x, y, z, w, t) = 0$.

For all $x$, $y$, $z$, $w$, $u \in$ X, and $t$, $t_1$, $t_2$, $t_3$, $t_4 > 0$. The values $M(x, y, z, w, t)$ and $N(x, y, z, w, t)$ may be interpreted the degree of nearness and non-nearness that the volume of the quadrilateral enlarged $(x, y, z, w)$ concerning $t$, respectively.

*Definition 4*. [23] Let $(X, M, N,*, \diamond)$ be an intuitionistic fuzzy 3-metric space. Then a sequence $\{x_n\}$ in $X$ is said to be convergent to a point $x \in X$ for all $t > 0$, $\lim_{n\to\infty} M(x_n, x, z, w, t) = 1$, and $\lim_{n\to\infty} N(x_n, x, z, w, t) = 0$.

*Definition 5*. [23] Let $(X, M, N,*, \diamond)$ be an intuitionistic fuzzy 3-metric space. Then a sequence $\{x_n\}$ in $X$ is said to be Cauchy sequence if, for all $t > 0$ and and $p > 0$, $\lim_{n\to\infty} M(x_{n+p}, x_n, z, w, t) = 1$, and $\lim_{n\to\infty} N(x_{n+p}, x_n, z, w, t) = 0$.

*Definition 6*. [23] An intuitionistic fuzzy 3-metric space $(X, M, N,*, \diamond)$ is said to be complete if and only if every Cauchy sequence is convergent.

*Lemma 1*. Let $(X, M, N,*, \diamond)$ be an intuitionistic fuzzy 3-metric space. Then $M$ and $N$ are continuous functions on $X^4 \times [0, \infty)$.

*Lemma 2*. Let $(X, M, N,*, \diamond)$ be an intuitionistic fuzzy 3-metric space. If for all $x, y, z, w \in X$, $t > 0$, and a number $k \in (0.1)$ $M(x, y, z, w, kt) \geq M(x, y, z, w, t)$, and $N(x, y, z, w, kt) \leq N(x, y, z, w, t)$, then $x = y$, $y = z$, $z = w$, $w = x$, $w = y$, $z = x$, and $z = y$.

*Definition 7*. Two self-mapping $A$ and $B$ of an intuitionistic fuzzy 3-metric space $(X, M, N,*, \diamond)$ are said to be weakly compatible if $ABx = BAx$ when $Ax = Bx$ for some $x \in X$.

*Definition 8*. Two self-mapping $A$ and $B$ of an intuitionistic fuzzy 3-metric space $(X, M, N,*, \diamond)$ are said to be compatible if $\lim_{n\to\infty} M(ABx_n, BAx_n, z, w, t) = 1$, and $\lim_{n\to\infty} N(ABx_n, BAx_n, z, w, t) = 0$ for all $z, w \in X$ and $t > 0$ whenever $\{x_n\}$ is a sequence in $X$ such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Bx_n = x$ for some $x \in X$.

*Definition 9*. A mapping $A$ from an intuitionistic fuzzy 3-metric space $(X, M, N,*, \diamond)$ is said to be sequentially continuous at $x$ if for every sequence $\{x_n\}$ in $X$, $\lim_{n\to\infty} M(x_n, x, z, w, t) = 1$ and $\lim_{n\to\infty} N(x_n, x, z, w, t) = 0$ implies $\lim_{n\to\infty} M(Ax_n, Ax, z, w, t) = 1$, and $\lim_{n\to\infty} N(Ax_n, Ax, z, w, t) = 0$ for all $z, w \in X$, $t > 0$.

*Definition 10*. Let $A$ and $B$ be two mappings from an intuitionistic fuzzy 3-metric space $(X, M, N,*, \diamond)$ into itself. A sequence $\{x_n\}$ in $X$ is said to be asymptotically $(A\sim B)$ regular if $\lim_{n\to\infty} M(Ax_n, Bx_n, z, w, t) = 1$, $\lim_{n\to\infty} N(Ax_n, Bx_n, z, w, t) = 0$ for all $z, w \in X$, $t > 0$.

*Definition 11*. Let $A$ and $B$ be two mappings from an intuitionistic fuzzy 3-metric space $(X, M, N,*, \diamond)$ into itself. The mappings $A$ and $B$ are said to be weak compatible with type $(\alpha)$ if $\lim_{n\to\infty} M(ABx_n, BBx_n, z, w, t) \geq \lim_{n\to\infty} M(BAx_n, BBx_n, z, w, t)$, and $\lim_{n\to\infty} N(ABx_n, BBx_n, z, w, t) \leq \lim_{n\to\infty} N(BAx_n, BBx_n, z, w, t)$ for all $z, w \in X$, $t > 0$. Whenever $\{x_n\}$ is a sequence in $X$ such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Bx_n = x$ for some $x \in X$.

*Lemma 3*. Let $A$ and $B$ be weak compatible mappings of type $(\alpha)$ from an intuitionistic fuzzy 3-metric space $(X, M, N,*, \diamond)$ into itself. If $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Bx_n = x$ for some$x$ $\in X$. Consequently, we have the following:

- $\lim_{n\to\infty} BAx_n = Ax$ if $A$ is sequentially continuous at $x$,
- $\lim_{n\to\infty} BAx_n = Bx$ if $B$ is sequentially continuous at $x$, and
- $ABx = BAx$, and $Ax = Bx$ if $A$ and $B$ are sequentially continuous at $x$.

## Main results

*Theorem 1*. Let $(X, M, N,*, \diamond)$ be a complete intuitionistic fuzzy 3-metric space such that $a * b* c* d= \min\{a, b, c, d\}$ and $a \diamond b \diamond c \diamond d = \max\{a, b, c, d\}$ for all $a, b, c, d \in X$. Let $A$, $B$, $S$, $T$, and $F$ be mapping into itself satisfying the following conditions:

- $AB$ is sequentially continuous,
- the pair $\{F, AB\}$ is weak compatible with type $(\alpha)$,
- there exists a number
  $k \in (0, 1)$ such that $M(Fx, Fy, z, w, kt) \geq M(ABx, Fx, z, w, t) * M(STy, Fy, z, w, t) *1$, $N(Fx, Fy, z, w, kt) \leq N(ABx, Fx, z, w, t) \diamond N(STy, Fy, z, w, t) \diamond 0$,
- There exists an asymptotically $(F\sim AB)$ regular sequence and asymptotically $(F\sim ST)$ proper sequence,
- $M(x, STx, z, w, t) \geq M(x, ABx, z, w, t)$, $N(x, STx, z, w, t) \leq N(x, ABx, z, w, t)$, for all $x, z, w \in X$, $t > 0$, and
- $FB = BF$, $FT = TF$, $AB = BA$, and $ST = TS$.

Then $A$, $B$, $S$, $T$, and $F$ have a unique common fixed point in $X$.

*Proof.* Let $\{x_n\}$ be a sequence in $X$ such that it is an asymptotically $(F{\sim}AB)$ regular and asymptotically $(F{\sim}ST)$ regular. By using (iii) for $m, n \in \mathbb{N}$:

$$M(Fx_n, Fx_m, z, w, kt) \geq M(ABx, Fx_n, z, w, t) * M(STx_m, Fx_m, z, w, t)*1$$
$$N(Fx_n, Fx_m, z, w, kt) \leq N(ABx_n, Fx_n, z, w, t) \diamond N(STx_m, Fx_m, z, w, t) \diamond 0$$

By taking limit $m, n \to \infty$ since asymptotically $(F{\sim}AB)$ regular and asymptotically $(F{\sim}ST)$ regular, we have:

$$\lim_{m,n\to\infty} M(ABx_n, Fx_n, z, w, t) = \lim_{m,n\to\infty} M(STx_m, Fx_m, z, w, t) = 1$$
$$\lim_{m,n\to\infty} N(ABx_n, Fx_n, z, w, t) = \lim_{m,n\to\infty} N(STx_m, Fx_m, z, w, t) = 0.$$

Then, we obtain:

$$\lim_{m,n\to\infty} M(Fx_n, Fx_m, z, w, kt) = 1 \text{ and } \lim_{m,n\to\infty} N(Fx_n, Fx_m, z, w, kt) = 0,$$

for all $z, w \in X$ and $t > 0$ . Then $\{Fx_n\}$ is a Cauchy sequence in $X$. Since $(X, M, N, *, \diamond)$ is complete intuitionistic fuzzy 3-metric space, $\{Fx_n\}$ convergent to a point $u \in X$.

$$M(ABx_n, u, z, w, t) \geq M(ABx_n, u, z, Fx_n, t/4) * M(ABx_n, u, Fx_n, w, t/4) * \\ * M(ABx_n, Fx_n, z, w, t/4) * M(Fx_n, u, z, w, t/4)$$

$$N(ABx_n, u, z, w, t) \leq N(ABx_n, u, z, Fx_n, t/4) \diamond N(ABx_n, u, Fx_n, w, t/4) \diamond \\ \diamond N(ABx_n, Fx_n, z, w, t/4) \diamond N(Fx_n, u, z, w, t/4),$$

since $\{x_n\}$ is asymptotically $(F{\sim}AB)$ regular, we have:

$\lim_{n\to\infty} M(ABx_n, u, z, Fx_n, t/4) = \lim_{n\to\infty} M(ABx_n, u, Fx_n, w, t/4) = \lim_{n\to\infty} M(ABx_n, Fx_n, z, w, t/4) = 1,$
and
$\lim_{n\to\infty} N(ABx_n, u, z, Fx_n, t/4) = \lim_{n\to\infty} N(ABx_n, u, Fx_n, w, t/4) = \lim_{n\to\infty} N(ABx_n, Fx_n, z, w, t/4) = 0,$
also, $\{x_n\}$ convergent to a point $u \in X$. The $\lim_{n\to\infty} M(Fx_n, u, z, w, t/4) = 1$ and $\lim_{n\to\infty} N(Fx_n, u, z, w, t/4) = 0$.

Consequently: $\lim_{n\to\infty} M(ABx_n, u, z, w, t) = 1$, $\lim_{n\to\infty} N(ABx_n, Fx_n, u, z, w, t) = 0$.

Similarity, we have: $\lim_{n\to\infty} M(ABx_n, u, z, w, t) = 1$, $\lim_{n\to\infty} N(ABx_n, Fx_n, u, z, w, t) = 0$.
Since $F$ and $AB$ are a weak compatible mapping of type $(\alpha)$, $AB$ is sequentially continuous by using *Lemma 3*, then we have $\lim_{n\to\infty} F(AB)x_n = ABu$ and $\lim_{n\to\infty} (AB)^2 x_n = ABu$ , by using (iii) there exist a number $k \in (0, 1)$:

$$M[F(AB)x_n, Fx_n, z, w, kt] \geq M[(AB)^2 x_n, F(AB)x_n, z, w, t] * M(STx_n, Fx_n, z, w, t) * 1$$
$$N[F(AB)x_n, Fx_n, z, w, kt] \leq N[(AB)^2 x_n, F(AB)x_n, z, w, t] \diamond N(STx_n, Fx_n, z, w, t) \diamond 0$$

By taking limit $n \to \infty$:

$$M(ABu, u, z, w, kt) \geq M(ABu, ABu, z, w, t) * M(u, u, z, w, t) * 1$$
$$N(ABu, u, z, w, kt) \leq N(ABu, ABu, z, w, t) \diamond N(u, u, z, w, t) * 0$$

Consequentially, $M(ABu, u, z, w, kt) = 1$, $N(ABu, u, z, w, kt) = 0$. By using *Lemma 2*, we have $ABu = u$. Using (v) we have:

$$M(u, STu, z, w, kt) \geq M(u, ABu, z, w, t), \text{ and } N(u, STu, z, w, kt) \leq N(u, ABu, z, w, t).$$

By non-decreasing of $M$ and by non-increasing of $N$, we have:

$$M(u, ABu, z, w, t) \geq M(u, ABu, z, w, kt), \text{ and } N(u, ABu, z, w, t) \leq N(u, ABu, z, w, kt).$$

We obtain:

$$M(u, STu, z, w, kt) \geq M(u, ABu, z, w, kt), N(u, STu, z, w, kt) \leq N(u, ABu, z, w, kt),$$

we have $M(u, ABu, z, w, kt) = 1$, $N(u, ABu, z, w, kt) = 0$.

Consequently, $M(u, STu, z, w, kt) = 1$, $N(u, STu, z, w, kt) = 0$. Using *Lemma 2*, we get $STu = u$. Using (iii) previously, there exist a number $k \in (0, 1)$ such that:

$$M[F(AB)x_n, Fu, z, w, kt] \geq M[(AB)^2 x_n, F(AB)x_n, z, w, t] * M(STu, Fu, z, w, t) * 1$$
$$N[F(AB)x_n, Fu, z, w, kt] \leq N[(AB)^2 x_n, F(AB)x_n, z, w, t] \diamond N(STu, Fu, z, w, t) \diamond 0$$

Consequently $M(u, Fu, z, w, kt) \geq M(u, Fu, z, w, t)$, $N(u, Fu, z, w, kt) \leq N(u, Fu, z, w, t)$. By *Lemma 3*, we obtain $u = Fu$. Now, we show that $Bu = u$. Using (iii, vi), we have:

$$M(Bu, u, z, w, kt) = M(FBu, Fu, z, w, kt) \geq M[AB(AB), FAB, z, w, t] * M(STu, Fu, z, w, t) * 1,$$

$$N(Bu, u, z, w, kt) = N(FBu, Fu, z, w, kt) \leq N[AB(AB), FAB, z, w, t] \diamond N(STu, Fu, z, w, t) \diamond 0.$$

Consequently $M(Bu, Fu, z, w, kt) = 1$, $N(Bu, Fu, z, w, kt) = 0$, we obtain $Bu = u$. Consequently, $Au = u$. Now we show that $Tu = u$. Using (iii, vi) we get:

$$M(Tu, u, z, w, kt) = M(FTu, Fu, z, w, kt) = M(Fu, FTu, z, w, kt) \geq$$
$$\geq M[ABu, Fu, z, w, t] * M[ST(Tu), FTu, z, w, t] * 1,$$
$$N(Tu, u, z, w, kt) = N(FTu, Fu, z, w, kt) = N(Fu, FTu, z, w, kt) \leq$$
$$\leq N[ABu, Fu, z, w, t] \diamond N[ST(Tu), FTu, z, w, t] \diamond 0$$

Consequently $M(Tu, u, z, w, kt) = 1$, $N(Tu, u, z, w, kt) = 0$, then we obtain $Tu = u$. Consequently, $Su = u$.

We have $Au = Bu = Tu = Su = Fu = u$.

Then is a common fixed point of $A$, $B$, $S$, $T$, and $F$. Now we show the uniqueness of common fixed points. We suppose and are two common fixed points of the mappings $A$, $B$, $S$, $T$, and $F$. Using (iii) we have:

$$M(Fu, Fv, z, w, kt) \geq M(ABu, Fu, z, w, t) * M(STv, Fv, z, w, t) * 1 \geq$$
$$\geq M(u, u, z, w, t) * M[v, v, z, w, t] * 1 = 1 * 1 * 1 \geq 1,$$
$$N(Fu, Fv, z, w, kt) \leq N(ABu, Fu, z, w, t) \diamond N(STv, Fv, z, w, t) \diamond 0 \leq$$
$$\leq N(u, u, z, w, t) \diamond N[v, v, z, w, t] \diamond 0 = 0 \diamond 0 \diamond 0 \leq 0.$$

Consequently $M(Fu, Fv, z, w, kt) = 1$, $N(Fu, Fv, z, w, kt) = 0$ for all $z, v \in X$, $t > 0$, therefore, $Fu = Fv$, we have $u = v$.

This complete the proof.

*Corollary 1*. Let $(X, M, N, *, \diamond)$ be an entire intuitionistic fuzzy 3-metric space such that $a * b * c * d = \min\{a, b, c, d\}$ and $a \diamond b \diamond c \diamond d = \max\{a, b, c, d\}$ for all $a, b, c, d \in X$. Let $A$, $B$, and $T$ be mappings into itself satisfying the following conditions:

– $A$ is sequentially continuous,
– the pair $\{T, A\}$ is weak compatible with type ($\alpha$),
– there exists a number $k \in (0, 1)$ such that

$$M(Tx, Ty, z, w, kt) \geq M(Ax, Tx, z, w, t) * M(By, Ty, z, w, t) * 1$$
$$N(Tx, Ty, z, w, kt) \leq N(Ax, Tx, z, w, t) \diamond N(By, Ty, z, w, t) \diamond 0,$$

– there exists an asymptotically ($T$~$A$) regular sequence and asymptotically ($T$~$B$) proper sequence.

Then $A$, $B$, and $T$ have a unique common fixed point in $X$.

If we put $S = F = I_x$ in *Theorem 1*, we get the proof.

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Paper submitted: June 10, 2020
Paper revised: July 15, 2020
Paper accepted: July 18, 2020