NEW OPTICAL EXPLICIT PLETHORA OF THE RESONANT SCHRODINGER'S EQUATION VIA TWO RECENT COMPUTATIONAL SCHEMES

by

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This research paper investigates the computational solutions of the resonant schrödinger's equation. The modified Khater method and Adomian decomposition method are applyied for construct new analytical traveling and semi-analytical wave solutions. This model describes the pulse phenomena and studied in non-linear optics. For further illustration of our obtained solutions, some distinct types of sketches are given.

Key words: *(3+1) resonant Schrodinger's equation, modified Khater method, Adomian decomposition method, exact and approximate solutions*

Introduction

Marvellous applications in the field of science and engineering have been formulating in non-linear PDE with an-integer order or fractional order [1-15]. Thus, many researchers in different fields have been focusing on studying the exact travelling and solitary wave solutions. These solutions are used to discover more physical, dynamical behaviour, and the nature of non-linear problems and explain the different scientific non-linear phenomena. For achieving to this fundamental goal, many computational and numerical schemes have been being formulated such as the improved and novel expansion method [1-3], extended simplest equation method [4-7], first integral method [8, 9], Khater method which is derived by Khater in 2017 [10-13], the exp -expansion method [14, 15], the Kudryashov method [16], the Jacobi elliptic functions [17, 18], the modified Khater method, [19-22], the generalized Kudryashov method [23, 24], the new extended direct algebraic method [25], the functional variable method [26], Adomian decomposition method [27-29], and the sub equation method [30].

In this paper, with the availability of symbolic computation packages, the resonant non-linear Schrodinger's equation (RNLS) is investigated by employing the modified Khater

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method and Adomian decomposition method to construct analytical and semi-analytical solutions [31-41]. The (3+1)-D RNLS model is given:

$$\iota\mathcal{Q}_t + r_1\nabla^2\mathcal{Q} + r_2|\mathcal{Q}|^2\mathcal{S} + r_2\left(\frac{\nabla^2|\mathcal{Q}|}{|\mathcal{Q}|}\right)\mathcal{S} = 0 \tag{1}$$

where

$$\mathcal{Q} = \mathcal{Q}(x,y,z,t),\ \nabla = \frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z},\ x,y,z\in\mathcal{R}$$

Employing the following traveling wave transformation:

$$\mathcal{Q}(x,y,z,t) \quad \mathcal{G}(\ \ )\mathrm{e}^{\iota\varphi}$$

where

$$\hbar = \cos(s_1)x + \cos(s_2)y + \cos(s_3)z + s_4 t, \varphi = \left\{s_5\left[\cos(s_1)x + \cos(s_2)y + \cos(s_3)z\right] + s_6 t\right\}$$

yields

$$\mathcal{L}_1\ \mathcal{G}'' - \mathcal{L}_2\ \mathcal{G} + r_2\mathcal{G}^3 = 0 \tag{2}$$

where

$$\mathcal{L}_1 = \wp(r_3 + r_1),\ \mathcal{L}_2 = (\wp r_1 s_5 + s_6),\ s_4 = -2\wp r_1 s_5,\ \wp = \cos(s_1)^2 + \cos(s_2)^2 + \cos(s_3)^2$$

Balancing the highest order derivative term and non-linear term in eq. (2), leads:

$$\mathcal{G}'', \mathcal{G}^3 \Rightarrow N + 2 = 3N \Rightarrow N = 1$$

Application

In this part, we apply the modified Khater method and Adomian decomposition method [42-44] to the (3+1) RNLS equation.

Modified Khater method

According to the general solutions suggested by the method, we get the general solution of eq. (2) in the form:

$$\mathcal{G}(\hbar) = a_0 + \sum_{i=1}^{N} a_i\mathcal{K}^{i\,f(\hbar)} + \sum_{i=1}^{N} b_i k^{-i\,f(\hbar)} = a_0 + k^{f(\hbar)}a_1 + k^{-f(\hbar)}b_1 \tag{3}$$

where a_0, a_1, b_1, and k are arbitrary constants while $f(\hbar)$ satisfies the auxiliary equation

$$f'(\hbar) = \frac{1}{\mathrm{Ln}(\mathcal{K})}\left(\varrho\mathcal{K}^{-f(\hbar)} + \chi + \delta\mathcal{K}^{f(\hbar)}\right),\ \text{where}\ \varrho,\chi,\delta\ \text{are orbitary constans}$$

Substituting eq. (3) and its derivatives into eq. (2). Collecting all terms of the same power of $\mathcal{K}^{f(\hbar)}$. Solving the obtained algebric system by any computer software program, leads:

Family 1.

$$a_1 \to \frac{2\delta a_0}{\chi},\ b_1 \to 0,\ \mathcal{L}_1 \to -\frac{2a_0^2 r_2}{\chi^2},\ \mathcal{L}_2 \to \frac{-4\delta\varrho a_0^2 r_2 + \chi^2 a_0^2 r_2}{\chi^2}$$

Family 2.

$$a_1 \to 0,\ b_1 \to \frac{2\varrho a_0}{\chi},\ \mathcal{L}_1 \to -\frac{2a_0^2 r_2}{\chi^2},\ \mathcal{L}_2 \to \frac{-4\delta\varrho a_0^2 r_2 + \chi^2 a_0^2 r_2}{\chi^2}$$

According to the value of parameters in *Family 1 and 2* the explicit wave solutions of eq. (1) are given:

– for $\beta^2 - 4\alpha\sigma < 0,\ \sigma \neq 0$

$$\mathcal{Q}_{1,1}(x,y,x,t) = \mathrm{e}^{\iota\varphi}\frac{\sqrt{4\delta\varrho - \chi^2}\,a_0\tan\left[\frac{1}{2}\hbar\sqrt{4\delta\varrho - \chi^2}\right]}{\chi} \tag{4}$$

$$\mathcal{Q}_{1,2}(x,y,x,t) = \mathrm{e}^{\iota\varphi}\frac{\sqrt{4\delta\varrho - \chi^2}\cot\left[\frac{1}{2}\hbar\sqrt{4\delta\varrho - \chi^2}\right]a_0}{\chi} \tag{5}$$

– for $\beta^2 - 4\alpha\sigma > 0,\ \sigma \neq 0$

$$\mathcal{Q}_{1,3}(x,y,x,t) = -\mathrm{e}^{\iota\varphi}\frac{\sqrt{-4\delta\varrho + \chi^2}\,a_0\tanh\left[\frac{1}{2}\hbar\sqrt{-4\delta\varrho + \chi^2}\right]}{\chi} \tag{6}$$

$$\mathcal{Q}_{1,4}(x,y,x,t) = -\mathrm{e}^{\iota\varphi}\frac{\sqrt{-4\delta\varrho + \chi^2}\coth\left[\frac{1}{2}\hbar\sqrt{-4\delta\varrho + \chi^2}\right]a_0}{\chi} \tag{7}$$

– for $\chi = \delta = \kappa$ and $\varrho = 0$

$$\mathcal{Q}_{1,5}(x,y,x,t) = -a_0\,\mathrm{e}^{\iota\varphi}\,\coth\left[\frac{\kappa}{2}\hbar\right] \tag{8}$$

– for $\varrho = 0,\ \chi \neq 0$, and $\delta \neq 0$

$$\mathcal{Q}_{1,6}(x,y,x,t) = -a_0\,\mathrm{e}^{\iota\varphi}\left(1 + \frac{4}{-2 + \mathrm{e}^{\hbar\chi}\delta}\right) \tag{9}$$

– for $\chi 2 - 4\varrho\delta = 0$

$$\mathcal{Q}_{1,7}(x,y,x,t) = -a_0\mathrm{e}^{\iota\varphi}\left(-1 + \hbar\frac{4\delta\varrho(2 + \hbar\chi)}{\chi^3}\right) \tag{10}$$

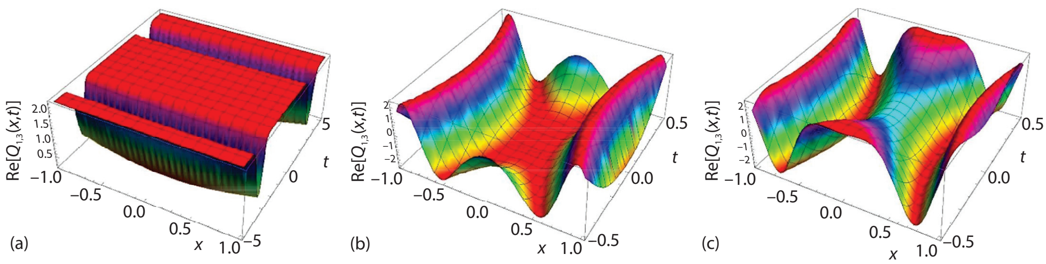

Figure 1. Solitary wave solutions of absolute, real, and imaginary value of eq. (6) in 3-D for $s_1 = 10$, $s_2 = 12$, $s_3 = 13$, $s_4 = 14$, $s_5 = 15$, $z = 9$, $y = 8$, $a_0 = 7$, $\delta = 1$, $\varrho = 2$, $\chi = 3$

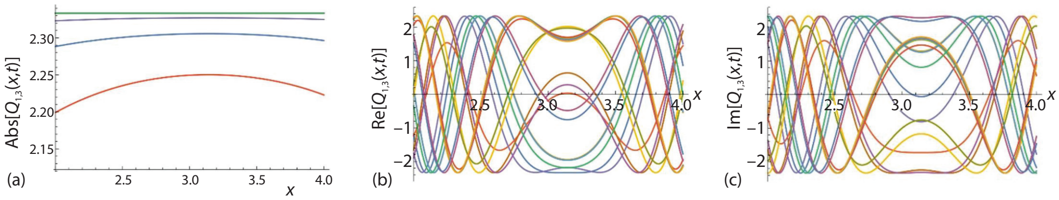

Figure 2. Solitary wave solutions of absolute, real, and imaginary value of eq. (6) in 2-D for $s_1 = 10$, $s_2 = 12$, $s_3 = 13$, $s_4 = 14$, $s_5 = 15$, $z = 9$, $y = 8$, $a_0 = 7$, $\delta = 1$, $\varrho = 2$, $\chi = 3$

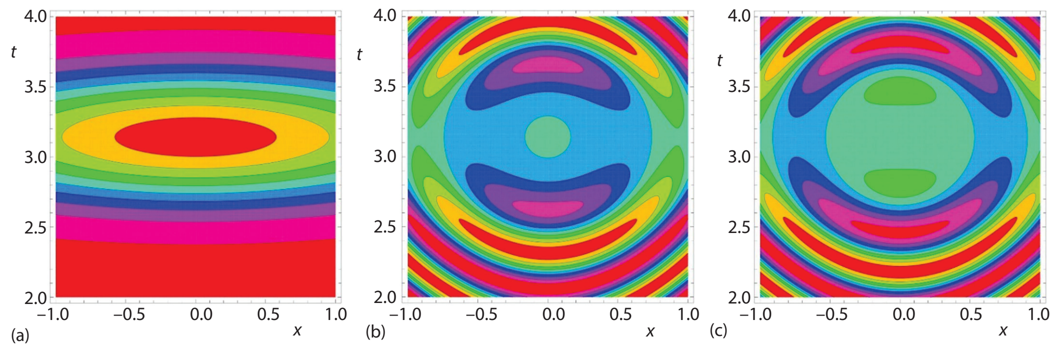

Figure 3. Solitary wave solutions of absolute, real, and imaginary value of eq. (6) in contour plot for $s_1 = 10$, $s_2 = 12$, $s_3 = 13$, $s_4 = 14$, $s_5 = 15$, $z = 9$, $y = 8$, $a_0 = 7$, $\delta = 1$, $\varrho = 2$, $\chi = 3$

According to the value of parameters in *Family 2*, we get the solitary wave solutions of eq. (1):

– for $\beta^2 - 4\alpha\sigma < 0$, $\sigma \neq 0$

$$\mathcal{Q}_{2,1}(x,y,x,t) = a_0 e^{\imath\varphi}\left(1 - \frac{4\delta\varrho}{\chi^2 - \chi\sqrt{4\delta\varrho - \chi^2}\tan\left[\frac{1}{2}\hbar\sqrt{4\delta\varrho - \chi^2}\right]}\right) \tag{11}$$

$$\mathcal{Q}_{2,2}(x,y,x,t) = a_0 e^{\imath\varphi}\left(1 - \frac{4\delta\varrho}{\chi^2 - \chi\sqrt{4\delta\varrho - \chi^2}\cot\left[\frac{1}{2}\hbar\sqrt{4\delta\varrho - \chi^2}\right]}\right) \tag{12}$$

– for $\beta^2 - 4\alpha\sigma > 0$, $\sigma \neq 0$

$$\mathcal{Q}_{2,3}(x,y,x,t) = a_0 e^{\imath\varphi}\left(1 - \frac{4\delta\varrho}{\chi^2 + \chi\sqrt{-4\delta\varrho + \chi^2}\tanh\left[\frac{1}{2}\hbar\sqrt{-4\delta\varrho + \chi^2}\right]}\right) \tag{13}$$

$$\mathcal{Q}_{2,4}(x,y,x,t) = a_0 e^{\imath\varphi}\left(1 - \frac{4\delta\varrho}{\chi^2 + \chi\sqrt{-4\delta\varrho + \chi^2}\coth\left[\frac{1}{2}\hbar\sqrt{-4\delta\varrho + \chi^2}\right]}\right) \tag{14}$$

– for $\chi = \varrho/2 = \kappa$ and $\delta = 0$

$$\mathcal{Q}_{2,5}(x,y,x,t) = a_0 \mathrm{e}^{\iota\varphi}\left(1+\frac{4}{-2+\mathrm{e}^{\kappa\hbar}}\right) \tag{15}$$

– for $\delta = 0$, $\chi \neq 0$, and $\varrho \neq 0$

$$\mathcal{Q}_{2,6}(x,y,x,t) = a_0 \mathrm{e}^{\iota\varphi}\left(1-\frac{2\varrho}{\varrho-\mathrm{e}^{\hbar\chi}\chi}\right) \tag{16}$$

– for $\chi^2 - 4\varrho\delta = 0$

$$\mathcal{Q}_{2,7}(x,y,x,t) = \frac{2a_0\mathrm{e}^{\iota\varphi}}{2+\hbar\chi} \tag{17}$$

where

$$\hbar = \cos(s_1)x+\cos(s_2)y+\cos(s_3)z+s_4 t,\ \varphi = \left\{s_5\left[\cos(s_1)x+\cos(s_2)y+\cos(s_3)z\right]+s_6 t\right\}$$

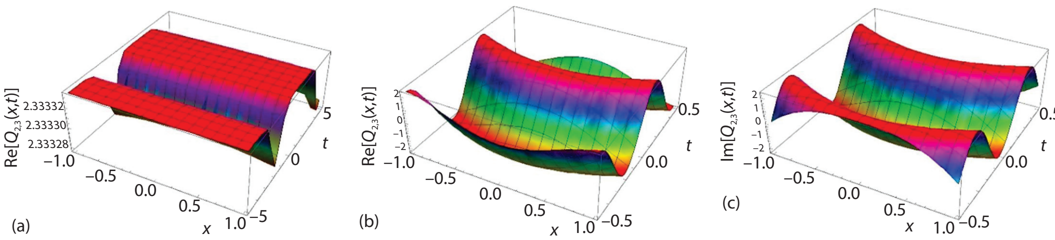

Figure 4. Solitary wave solutions of absolute, real, and imaginary value of eq. (13) in 3-D for $s_1 = 5$, $s_2 = 6$, $s_3 = 7$, $s_4 = 4$, $s_5 = 10$, $z = 9$, $y = 8$, $a_0 = 7$, $\delta = 1$, $\varrho = 2$, $\chi = 3$

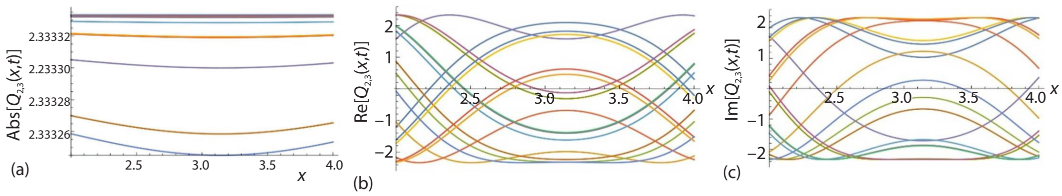

Figure 5. Solitary wave solutions of absolute, real, and imaginary value of eq. (13) in 2-D for $s_1 = 5$, $s_2 = 6$, $s_3 = 7$, $s_4 = 4$, $s_5 = 10$, $z = 9$, $y = 8$, $a_0 = 7$, $\delta = 1$, $\varrho = 2$, $\chi = 3$

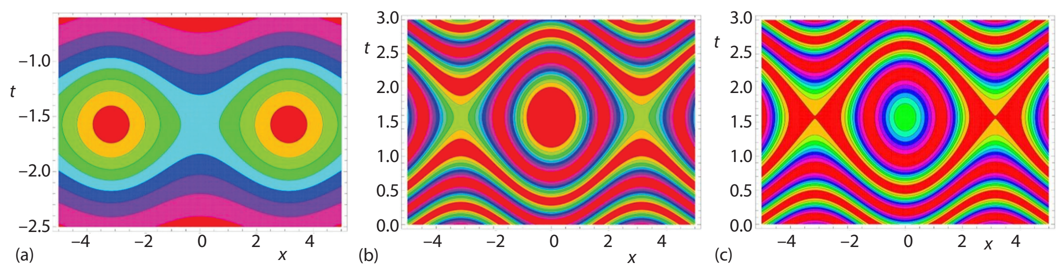

Figure 6. Solitary wave solutions of absolute, real, and imaginary value of eq. (13) in contour plot for $s_1 = 5$, $s_2 = 6$, $s_3 = 7$, $s_4 = 4$, $s_5 = 10$, $z = 9$, $y = 8$, $a_0 = 7$, $\delta = 1$, $\varrho = 2$, $\chi = 3$

Adomian decomposition method

Applying the Adomian decomposition method on eq. (2) enables rewriting it to be in the form:

$$L\mathcal{G}(h)+R\mathcal{G}(h)+N\mathcal{G}(h)=0 \tag{18}$$

where (L, R, and N) represent a differential operator, a linear operator and non-linear term, respectively.

Using the inverse operator (L^{-1}) on (18), we get:

$$\sum_{i=0}^{\infty}\mathcal{G}_i(h)=\mathcal{G}(0)+\mathcal{G}'(0)h+\frac{\mathcal{L}_2}{\mathcal{L}_1}L^{-1}\left[\sum_{i=0}^{\infty}\mathcal{G}_i(h)\right]-\frac{r_1}{\mathcal{L}_1}L^{-1}\left(\sum_{i=0}^{\infty}A_i\right) \tag{19}$$

Under the following condition [$\delta = 1$, $\varrho = 2$, $\chi = 3$, $a_0 = 5$, $r_2 = 4$] on eq. (6), we get:

$$\mathcal{G}_0=-\frac{5h}{6} \tag{20}$$

$$\mathcal{G}_1=\frac{5h^3}{72}-\frac{h^5}{192} \tag{21}$$

$$\mathcal{G}_2=-\frac{h^5}{576}+\frac{h^7}{16128}-\frac{25h^8}{64512}+\frac{h^{10}}{55296} \tag{22}$$

$$\mathcal{G}_3=\frac{31h^7}{48384}-\frac{67h^9}{1161216}+\frac{5h^{10}}{2322432}+\frac{h^{11}}{337920}-\frac{h^{12}}{14598144}-\frac{h^{13}}{12779520} \tag{23}$$

According to eqs. (20)-(23), we get an approximate solution of eq. (2):

$$\begin{aligned}\mathcal{G}(h)=&-\frac{5h}{6}+\frac{5h^3}{72}-\frac{h^5}{144}+\frac{17h^7}{24192}-\frac{25h^8}{64512}-\frac{67h^9}{1161216}+\\&+\frac{47h^{10}}{2322432}+\frac{h^{11}}{337920}-\frac{h^{12}}{14598144}-\frac{h^{13}}{12779520}+\ldots\end{aligned} \tag{24}$$

In tab. 1, we discuss the exact and approximate solutions of the (3+1) RNLS equation show the value of the absolute error between them.

Table 1. Shows for increasing the value h, the absolute error increases gradually; that means the Adomian decomposition method gives more accurate solutions for the values near to zero

Value of h	Exact solution	Approximate solution	Absolute error
0.01	0.00833326	0.00833326	$1.73472 \cdot 10^{-18}$
0.02	0.0166661	0.0166661	$6.93889 \cdot 10^{-18}$
0.03	0.0249981	0.0249981	$2.56739 \cdot 10^{-16}$
0.04	0.0333289	0.0333289	$2.5327 \cdot 10^{-15}$
0.05	0.041658	0.041658	$1.51129 \cdot 10^{-14}$
0.06	0.049985	0.049985	$6.4948 \cdot 10^{-14}$
0.07	0.0583095	0.0583095	$2.22801 \cdot 10^{-13}$
0.08	0.0666311	0.0666311	$6.4812 \cdot 10^{-13}$
0.09	0.0749494	0.0749494	$1.66225 \cdot 10^{-12}$
0.1	0.083264	0.083264	$3.85979 \cdot 10^{-12}$

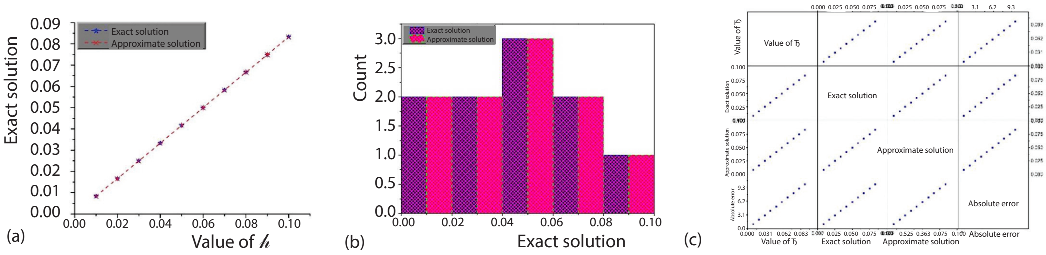

Figure 7. Exact and approximate wave solutions according to the shown values in tab. 1, show the accuracy of the our obtained solutions.

Conclusion

In this paper, the (3+1) RNLS equation have been investigated via the modified Khater method. Some new distinct types of computational solutions have been obtained. These solutions have been used to evaluate the initial and boundary conditions of the model. Furthermore, the Adomian decomposition method have been applied for construct the semi-analytical wave solutions based on these conditions, Some solitary and approximate solutions are sketched to investigate more of the physical properties of this model figs. 1-7. The performance of both methods shows useful and powerful in studying many of non-linear partial differential equations.

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Reference

[1] Attia, R. A. M., *et al*., Optical Wave Solutions of the Higher-Order Non-Linear Schrodinger Equation with the Non-Kerr Non-Linear Term Via Modified Khater Method, *Modern Physics Letters B*, *34* (2020), 2050044
[2] Ali, A. T., *et al.*, Abundant Numerical and Analytical Solutions of the Generalized Formula of Hirota-Satsuma Coupled KdV System, *Chaos, Solitons and Fractals*, *131* (2020), 109473
[3] Houwe, A., *et al*., Chirped Solitons in Discrete Electrical Transmission-Line, *Results in Physics*, *18* (2020), 103188
[4] Khater, M. M. A., *et al*., Analytical and Semi-Analytical Ample Solutions of the Higher-Order Non-Linear Schrodinger Equation with the Non-Kerr Non-Linear Term, *Results in Physics*, *16* (2020), 103000
[5] Khater, M. M. A., *et al*., On New Computational and Numerical Solutions of the Modified Zakharov–Kuznetsov Equation Arising in Electrical Engineering, *Alexandria Engineering Journal*, *59* (2020), 3, pp. 1099-1105
[6] Houwe, A., *et al.*, Chirped Solitons in Negative Index Materials Generated in Kerr Non-Linearity, Results in Physics, *17* (2020), 103097
[7] Khater, M. M. A., *et al*., Analytical and Numerical Solutions for the Current and Voltage Model on an Electrical Transmission-Line with Time and Distance, *Physica Scripta*, *95* (2020), 055206
[8] Yue, C., *et al*., Computational Simulations of the Couple Boiti-Leon-Pempinelli (BLP) System and the (3+1)-Dimensional Kadomtsev-Petviashvili (KP) Equation, *AIP Advances*, *10* (2020), 045216
[9] Houwe, A., *et al*., Complex Traveling Wave and Soliton Solutions to the Klein-Gordon-Zakharov Equations, *Results in Physics*, *17* (2020), 103127
[10] Khater, M. M. A., *et al*., Abundant New Solutions of the Transmission of Nerve Impulses of an Excitable System, *The European Physical Journal Plus*, *135* (2020), Feb., 251
[11] Abdel-Aty, A.-H., *et al.*, Effect of the Spin-Orbit Interaction on Partial Entangled Quantum Network, *Lecture Notes in Electrical Engineering*, *285* (2014), 529

[12] Abdel-Aty, A.-H., *et al*., Quantum Network Via Partial Entangled State, *Journal of Communications*, *9* (2014), 379
[13] Abdel-Aty, A., *et al*., *Characteristics and Distinctive Features of Entanglement in Superconducting Charge Gubits*, Book Title: Quantum Entanglement, Nova Science Publishers Inc., N. Y., USA, 2012, pp. 199-243
[14] Kumar, D., *et al*., Numerical Simulation for System of Time-Fractional Linear and Non-Linear Differential Equations, *Progr. Fract. Differ. Appl*., *5* (2019), 1, pp. 65-77
[15] Abdelkawy, M. A., *et al.*, A Spectral Collocation Method for Coupled System of 2-D Abel Integral Equations of the Second Kind, *Inf. Sci. Lett.*, *8* (2019), 3, pp. 89-93
[16] Akram, G., *et al*., Laguerre Approximations for System of Linear Pantograph Differential Equations, *Mathematical Sciences Letters*, *7* (2018), Jan., pp. 125-131
[17] Qian, L., *et al*., On Breather and Cuspon Waves Solutions for the Generalized Higher-Order Non-Linear Schrodinger Equation with Light-Wave Promulgation in an Optical Fiber, *Num. Comp. Meth. Sci. Eng*, *1* (2019), 2, pp. 101-110
[18] Cherniha, R., *et al*., Exact and Numerical Solutions of a Spatially-Distributed Mathematical Model for Fluid and Solute Transport in Peritoneal Dialysis, *Symmetry*, *8* (2016), 50
[19] Khater, M. M. A., *et al*., Dispersive Long Wave of Non-Linear Fractional Wu-Zhang System Via a Modified Auxiliary Equation Method, *AIP Advances*, *9* (2019), 025003
[20] Osman, M. S., *et al*., A Study of Optical Wave Propagation in the Non-Autonomous Schrodinger-Hirota Equation with Power-Law Non-Linearity, *Results in Physics*, *13* (2019), 102157
[21] Attia, R. A. M., *et al*., Chaos and Relativistic Energy-Momentum of the Non-Linear Time Fractional Duffing Equation, *Mathematical and Computational Applications*, *24* (2019), 10
[22] Khater, M. M. A., *et al*., Lump Soliton Wave Solutions for the (2+1)-Dimensional Konopelchenko-Dubrovsky Equation and KdV Equation, *Modern Physics Letters B*, *33* (2019), 1950199
[23] Benslimane, A., *et al.*, Displacements and Stresses in Pressurized Thick-Walled FGM Cylinders: Exact and Numerical Solutions, *International Journal of Pressure Vessels and Piping*, *168* (2018), Dec., pp. 219-224
[24] Guo, P., Chong-Jun, L., Almost Sure Stability with General Decay Rate of Exact and Numerical Solutions for Stochastic Pantograph Differential Equations, *Numerical Algorithms*, *80* (2019), 2, pp. 1391-1411
[25] Euler, M., Euler, N., On Mobius-Invariant and Symmetry-Integrable Evolution Equations and the Schwarzian Derivative, *Studies in Applied Mathematics*, *143* (2019), 2, pp. 139-156
[26] Attia, R. A., *et al*., Structure of New Solitary Solutions for the Schwarzian Korteweg De Vries Equation and (2+1)-Ablowitz-Kaup-Newell-Segur Equation, *Phys. J*, *1* (2018), 3, pp. 234-254
[27] Inc, M., On Numerical Doubly Periodic Wave Solutions of the Coupled Drienfel'd-Sokolov-Wilson Equation by the Decomposition Method, *Appl. Math. Comput*., *172* (2006), 1, pp. 421-430
[28] Inc, M., On Numerical Jacobi Elliptic Function Solutions of the (1+1)-Dimensional Dispersive Long Wave Equation by the Decomposition Method, *Appl. Math. Comput*., *173* (2006), 1, pp. 372-382
[29] Inc, M., Exact Solutions with Solitary Patterns for the Zakharov-Kuznetsov Equations with Fully Non-Linear Dispersion, *Chaos, Solitons and Fractals*, *33* (2007), 5, pp. 1783-1790
[30] Xue, G., *et al*., Darboux Transformation for a Generalized Ablowitz-Kaup-Newell-Segur Hierarchy Equation, *Physics Letters A*, *384* (2020), 126394
[31] Hosseini, K., *et al*., A (3+1)-Dimensional Resonant Non-Linear Schrodinger Equation and Its Jacobi Elliptic and Exponential Function Solutions, *Optik*, *207* (2020), 164458
[32] Korpinar, Z., *et al*., Newoptical Solitons for Biswas-Arshed Equation with Higher Order Dispersions and Fully Non-Linearity, *Optik, 206* (2020), 163332
[33] Inc, M., *et al*., Optical Solitons to the Resonance Non-Linear Schrodinger Equation by Sine-Gordon Equation Method, *Superlattices and Microstructures*, *113* (2018), Jan., pp. 541-549
[34] Aslan, E. C., Inc, M., Optical Soliton Soluions of the NLSE Quadratic-Cubic-Hamiltonian Perturbations and Modulation Instability Analysis, *Optik*, *196* (2019), 162661
[35] Inc, M., *et al*., Optical Solitons and Modulation Instability Analysis of (3+1)-Dimensional Non-Linear Schrodinger Equation, *Superlattices and Microstructures*, *112* (2017), Sept., pp. 296-302
[36] Golmankhaneh, A. K., *et al*., A Review on Local and Non-Local Fractal Calculus, *Num. Com. Meth. Sci. Eng*., *1* (2019), Jan., pp. 19-31
[37] Taiwo, T. J., *et al*., Four-Parameter Potential Function with Negative Energy Bound States, *Inf. Sci. Lett*., *8* (2019), 1, pp. 25-31
[38] Tsega, E. G., A Finite Volume Solution of Unsteady Incompressible Navier-Stokes Equations Using MATLAB, *Num. Com. Meth. Sci. Eng*., *1* (2019), 117

[39] Al-Saif, A. S. J., Abdul-Wahab M. S., Application of New Simulation Scheme for the Non-Linear Biological Population Model, *Num. Com. Meth. Sci. Eng*., *1* (2019), 89
[40] Tasbozan, O., *et al*., New Analytical Solutions and Approximate Solution of the Space-Time Conformable Sharma-Tasso-Olver Equation, *Progr. Fract. Differ. Appl*., *4* (2018), Jan., pp. 519-531
[41] Khater, M. M. A., *et al*., Computational Analysis of a Non-Linear Fractional Emerging Telecommunication Model with Higher-order Dispersive Cubic-Quintic, *Inf. Sci. Lett*., *9* (2020), 2, pp. 83-93
[42] Wazwaz, A., A Reliable Modification of Adomian Decomposition Method, *Applied Mathematics and Computation, 102* (1999), 1, pp. 77-86
[43] Li, W., Pang, Y. Application of Adomian Decomposition Method to Non-Linear Systems, *Adv. Differ Equ*., *2020* (2020), 67
[44] Kulkarni, S., *et.al.*, Application of Adomian Decomposition Method to Solve the Fractional Mathematical Model of Corona Virus, *Journal Math. Comput. Sci*., *10* (2020), 5, pp. 1327-1339

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